

# Blind Separation of Fetal ECG from Single Mixture using SVD and ICA

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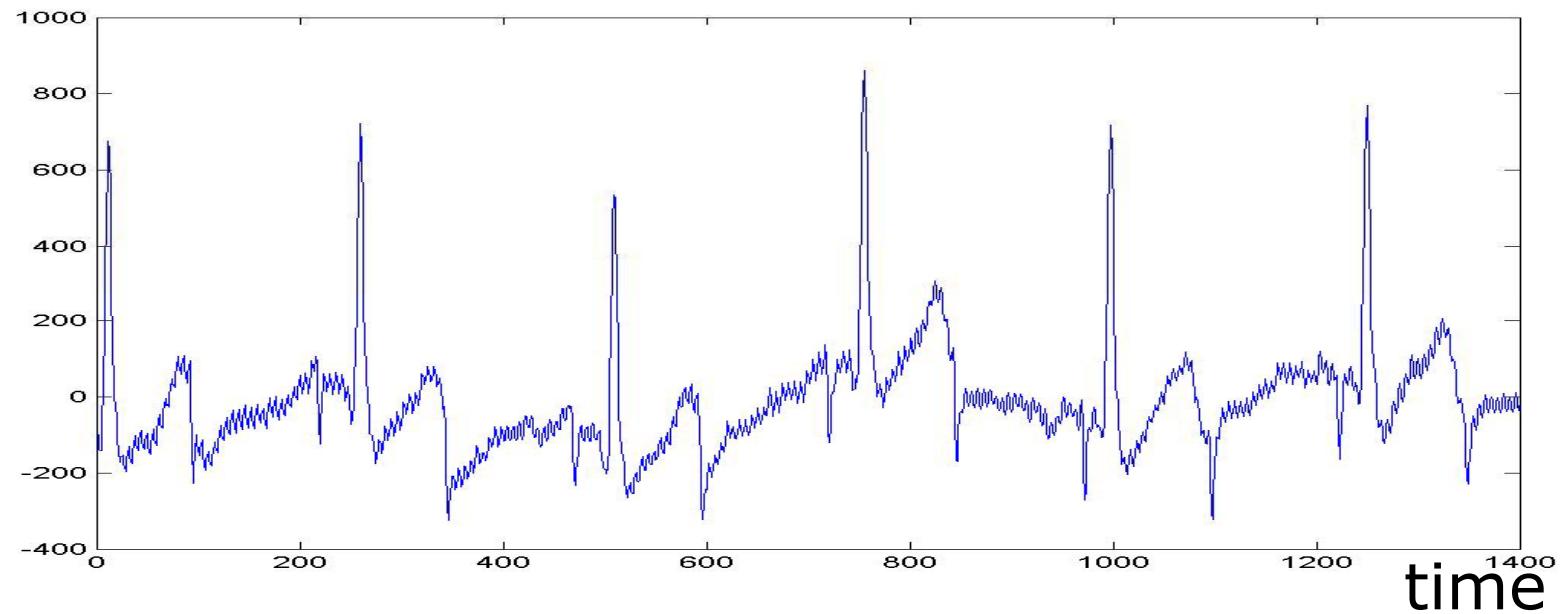
Ee-Chien Chang

School of Computing

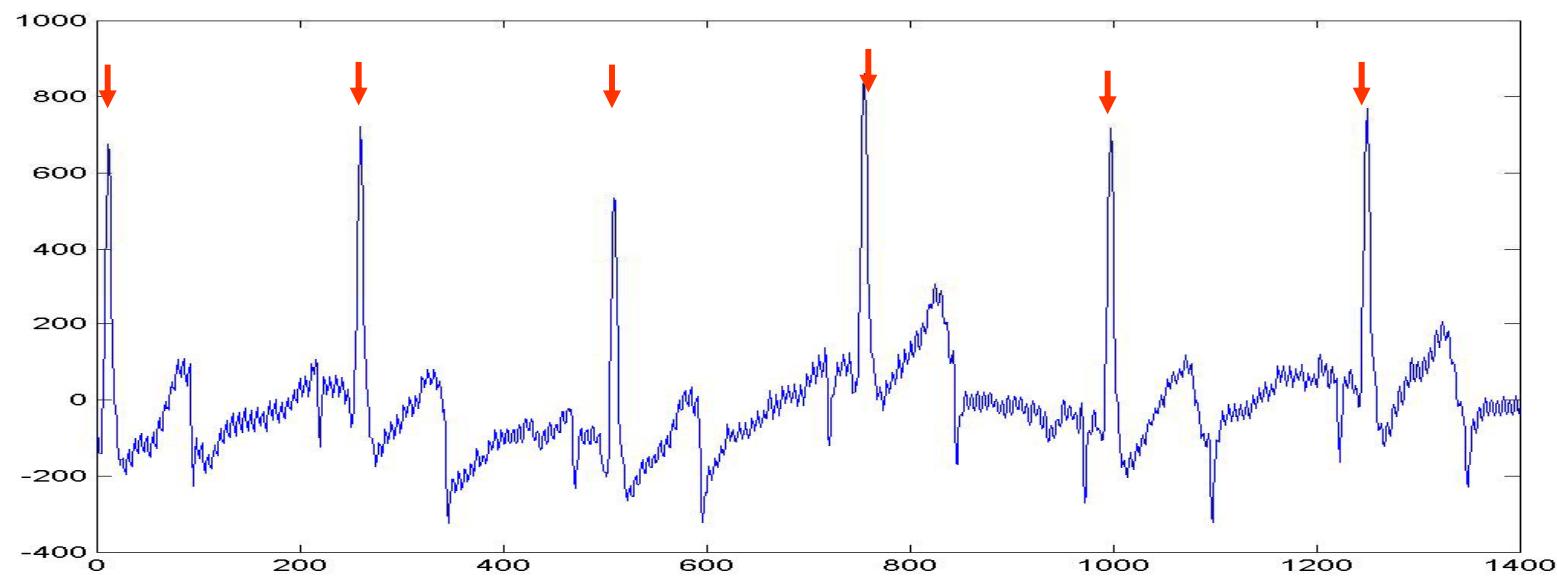
National University of Singapore

Lonce Wyse

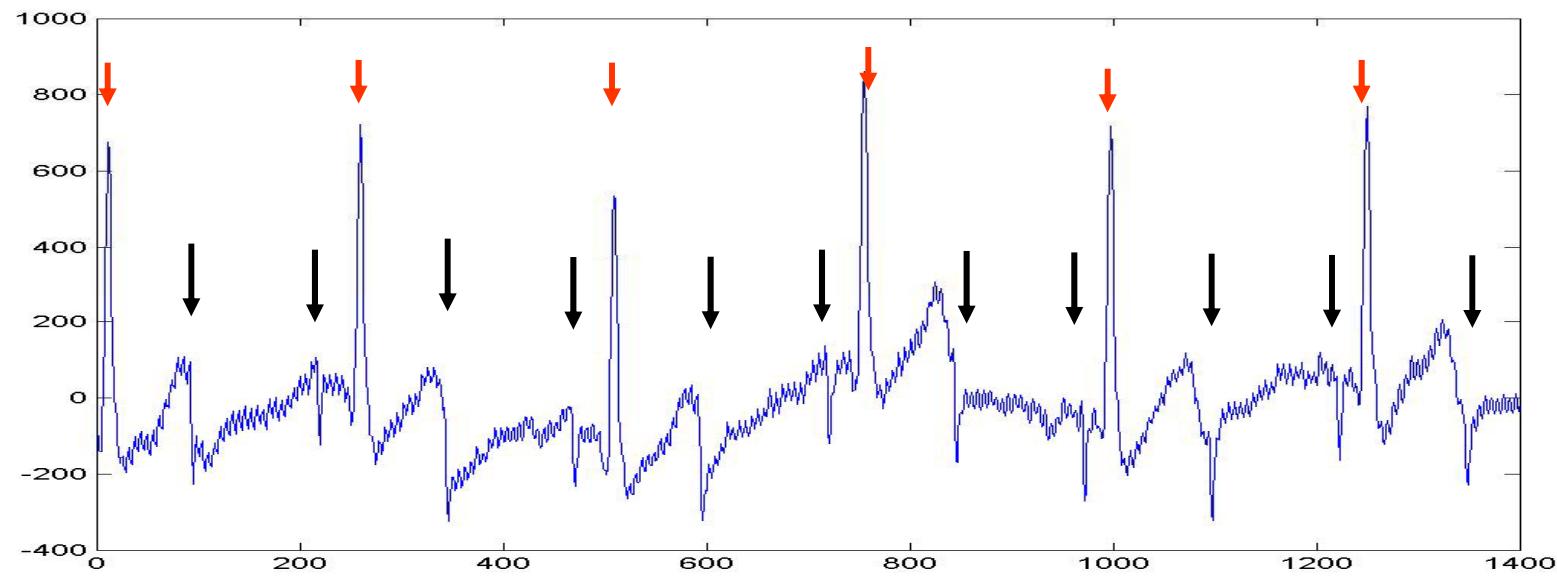
Institute for Infocomm Research



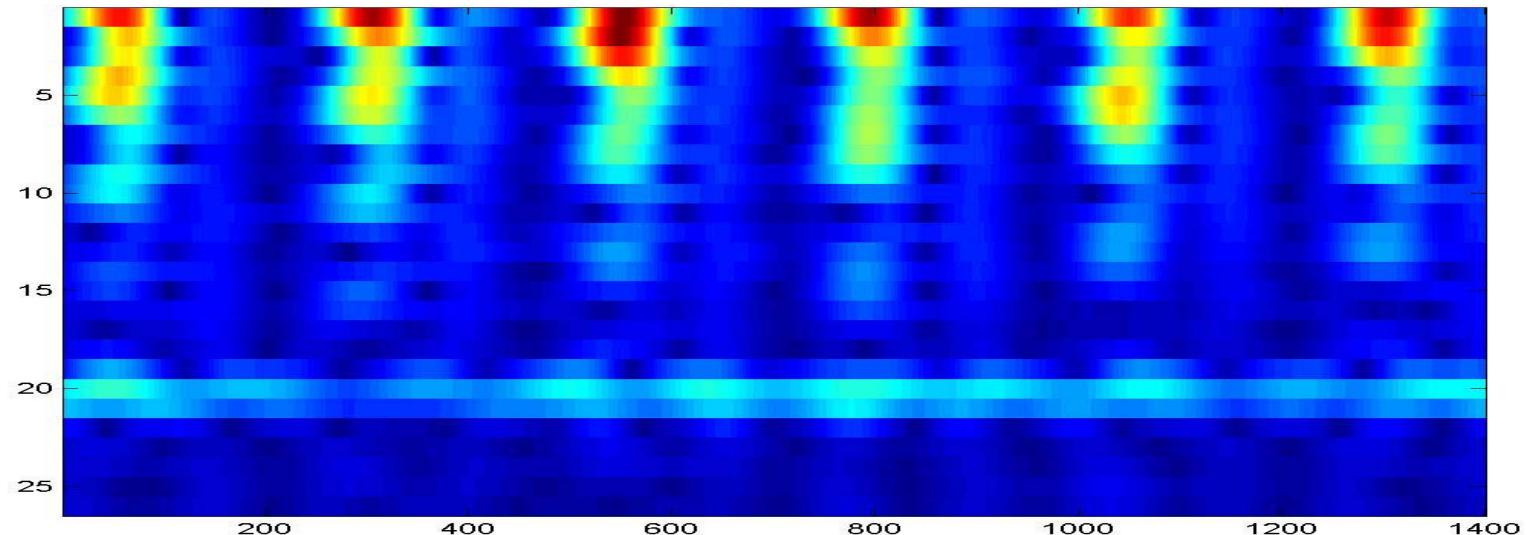
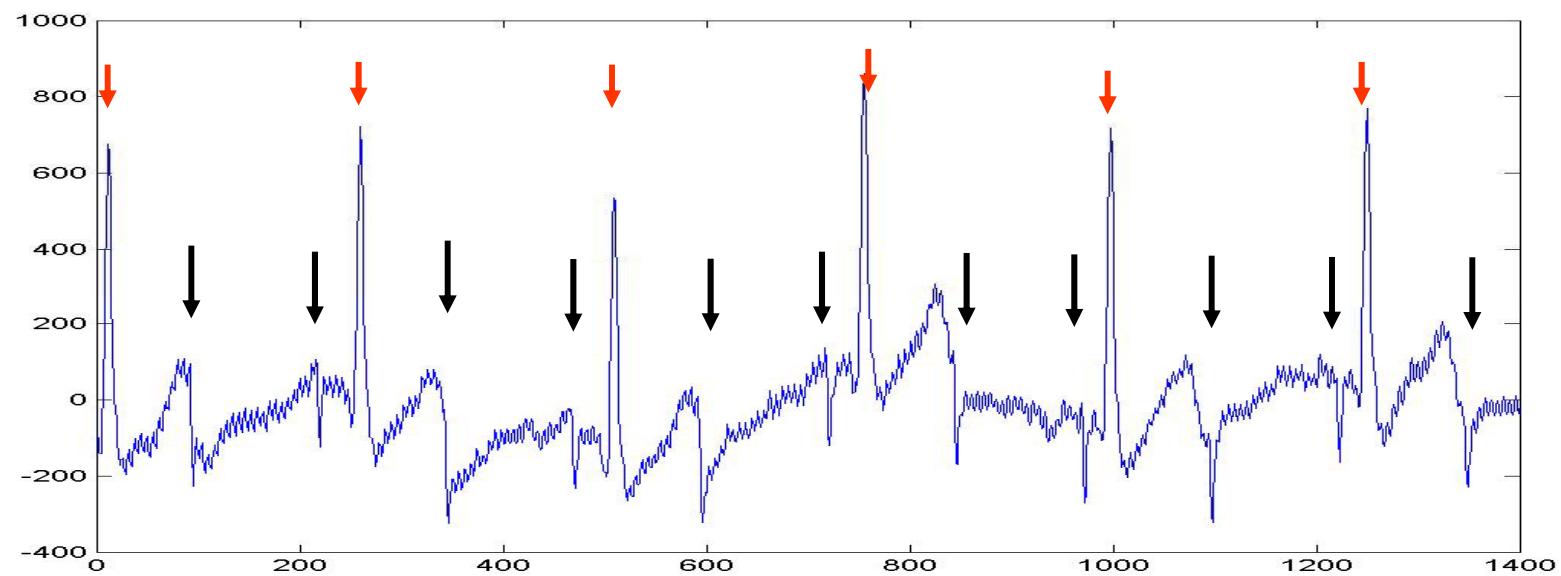
Mixed ECG's for a period of roughly 4.6 seconds

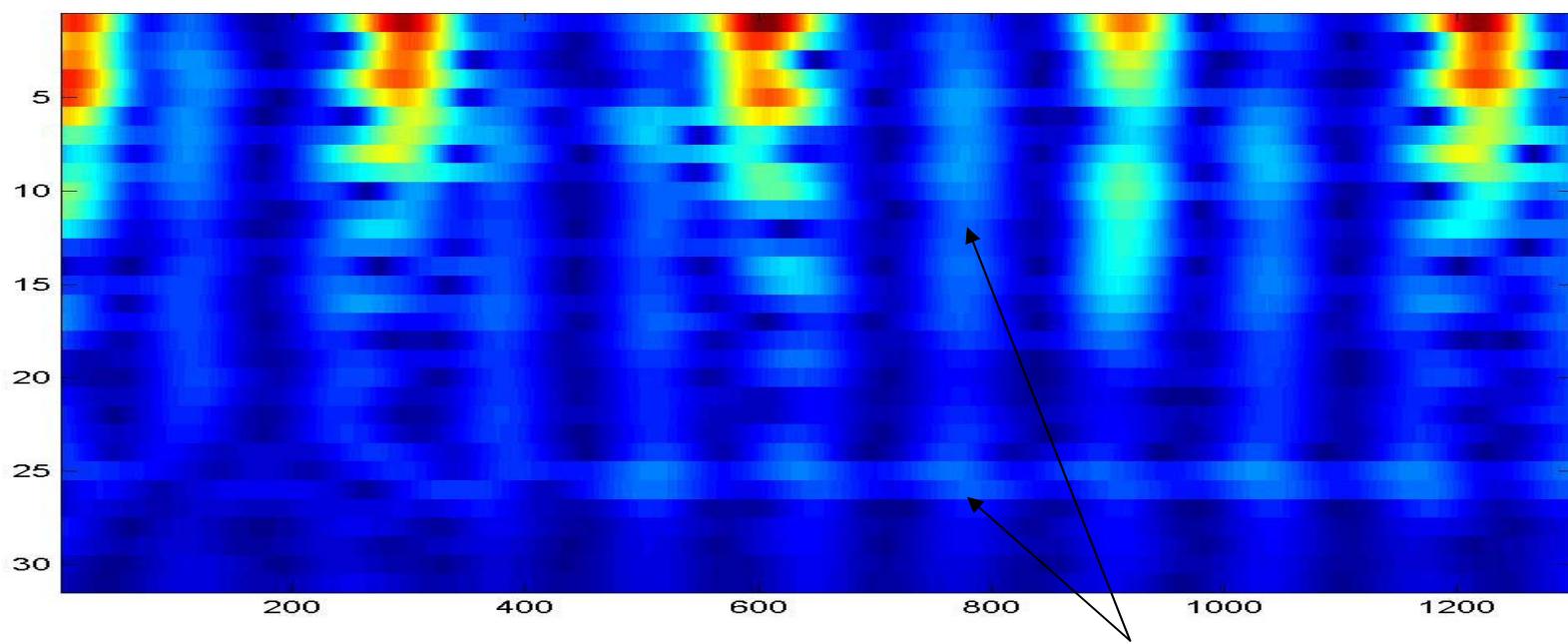
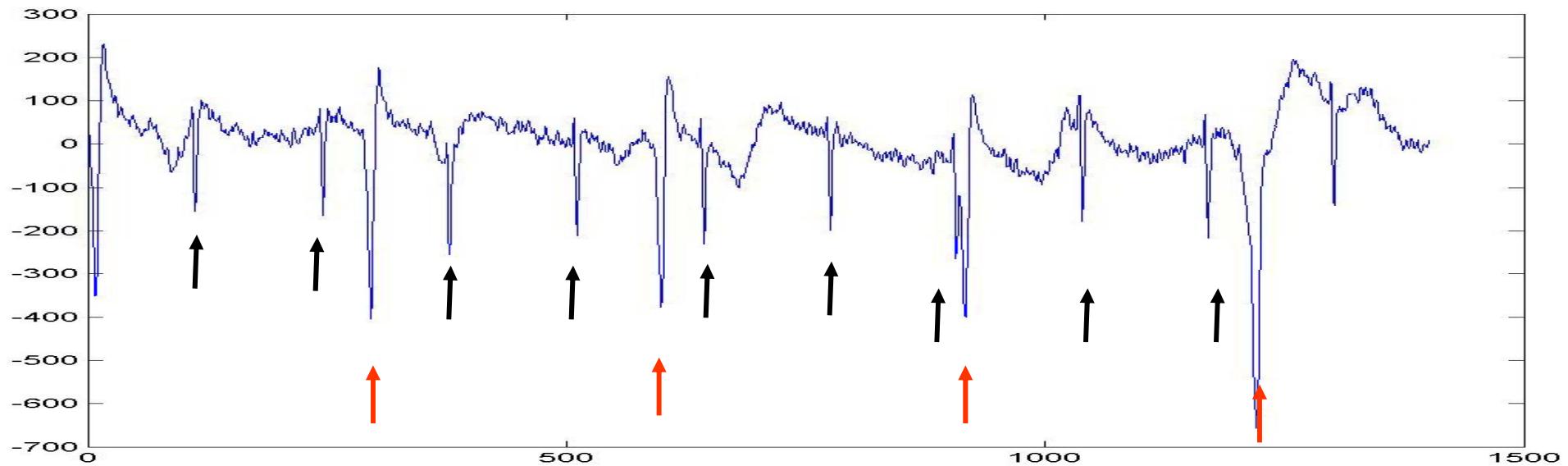


Maternal heartbeat (R)

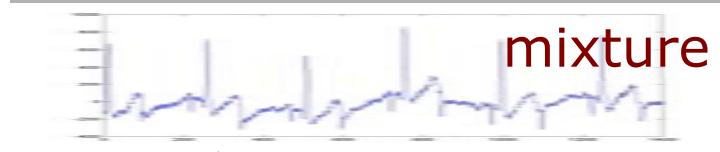


Fetal heartbeat

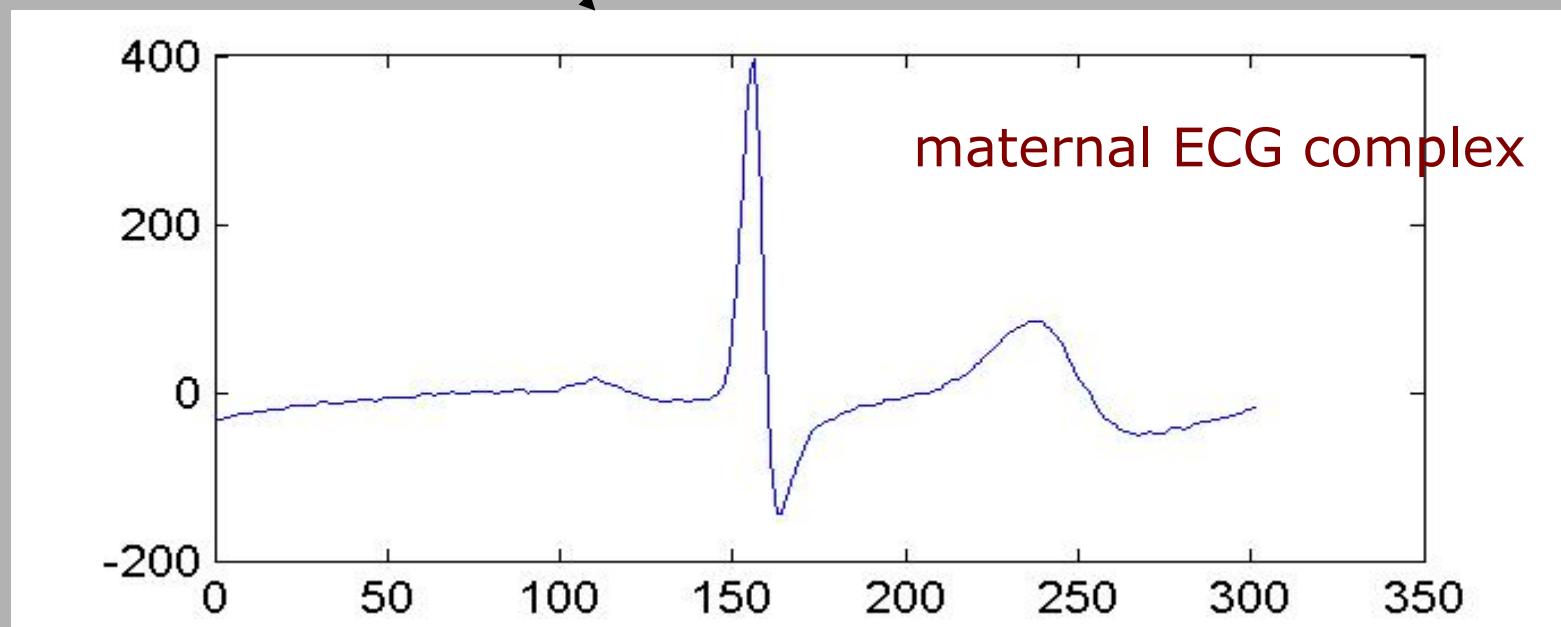
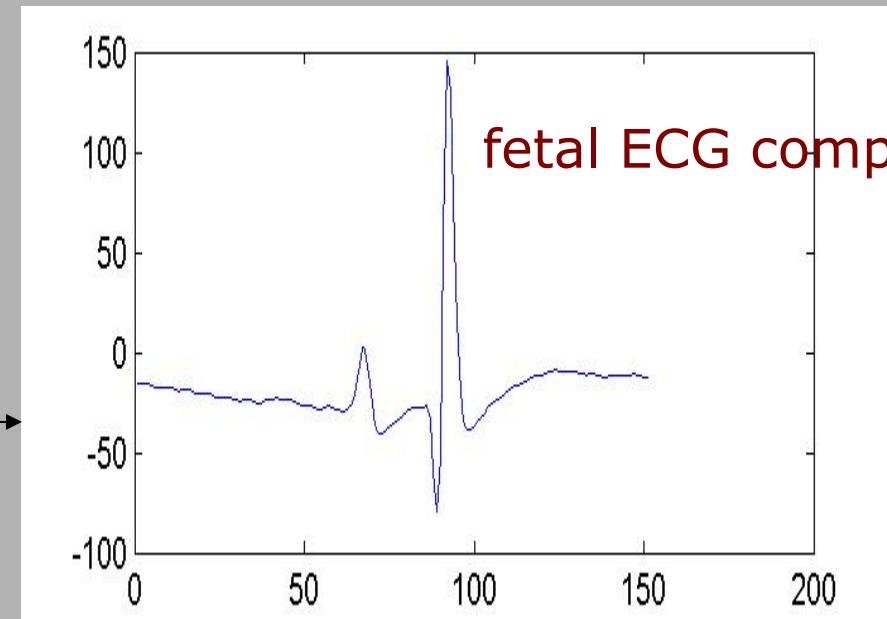




# Goal: separation from a single mixture

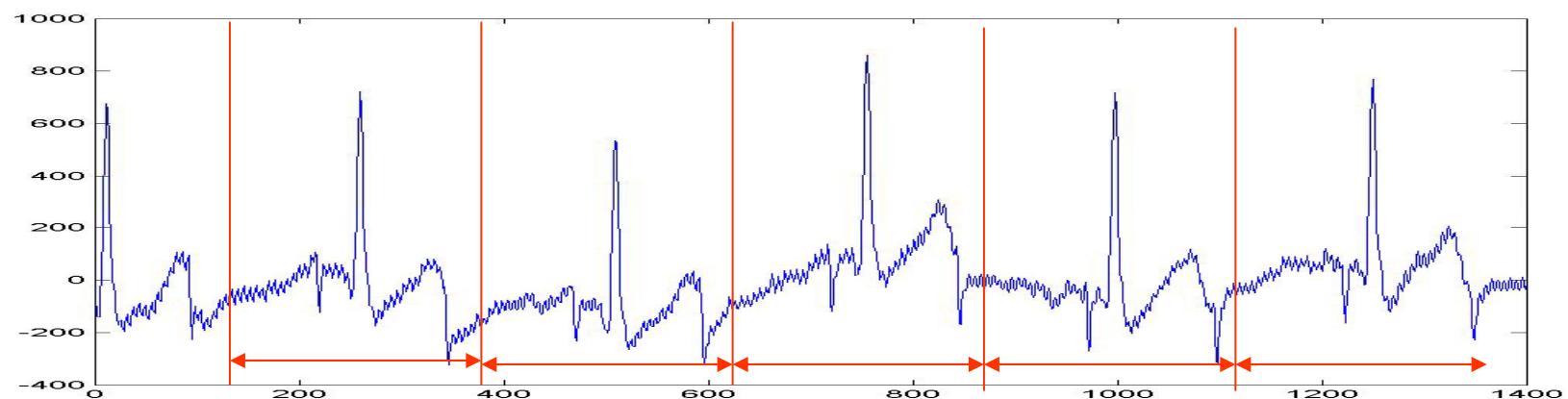


Separation



# One Approach

1. Find the occurrences of maternal heartbeat by identifying the peak
2. Find the maternal ECG complex by “averaging”
3. Subtract the maternal ECG complex from the mixture.
4. Repeat the above for fetal ECG.

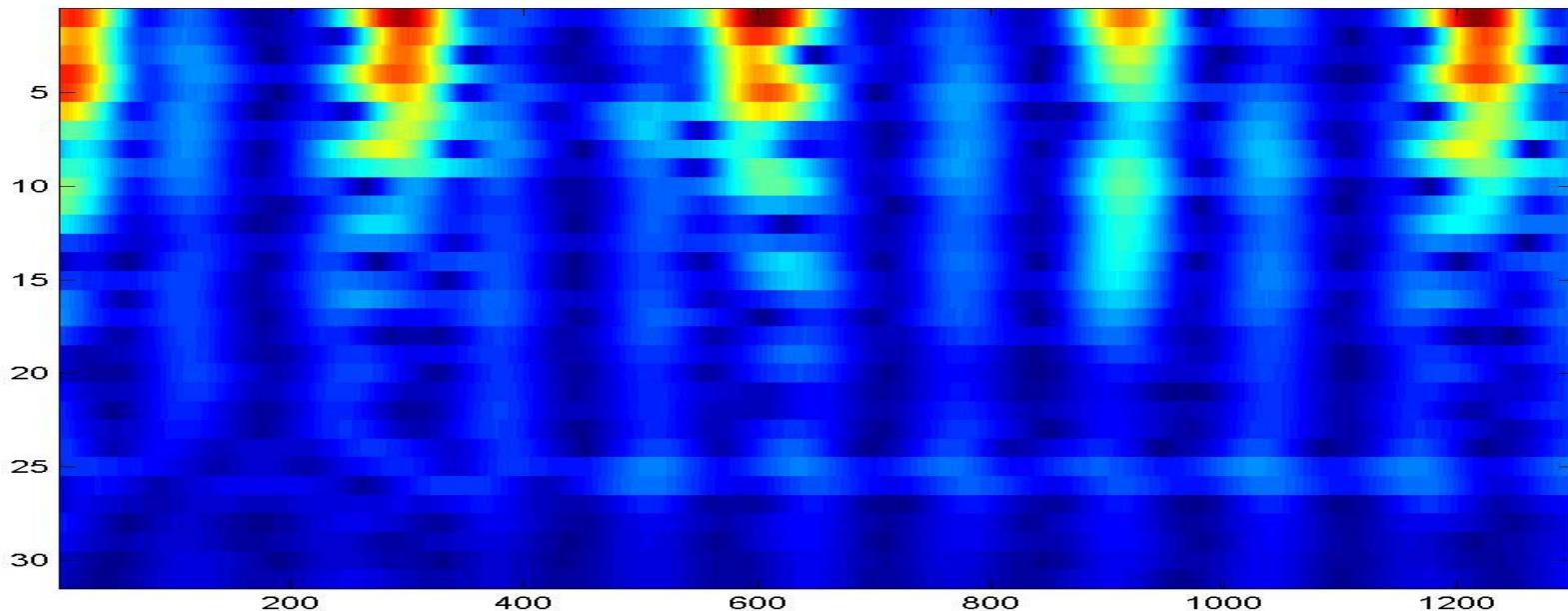


Disadvantage: require significant fine-tuning in step 1 and 3.

# Main Idea

- Identify the heart beat in the spectrogram.
- Observation: with the right window size used in the spectrogram, a ECG complex in the spectrogram can be viewed as a separable function.

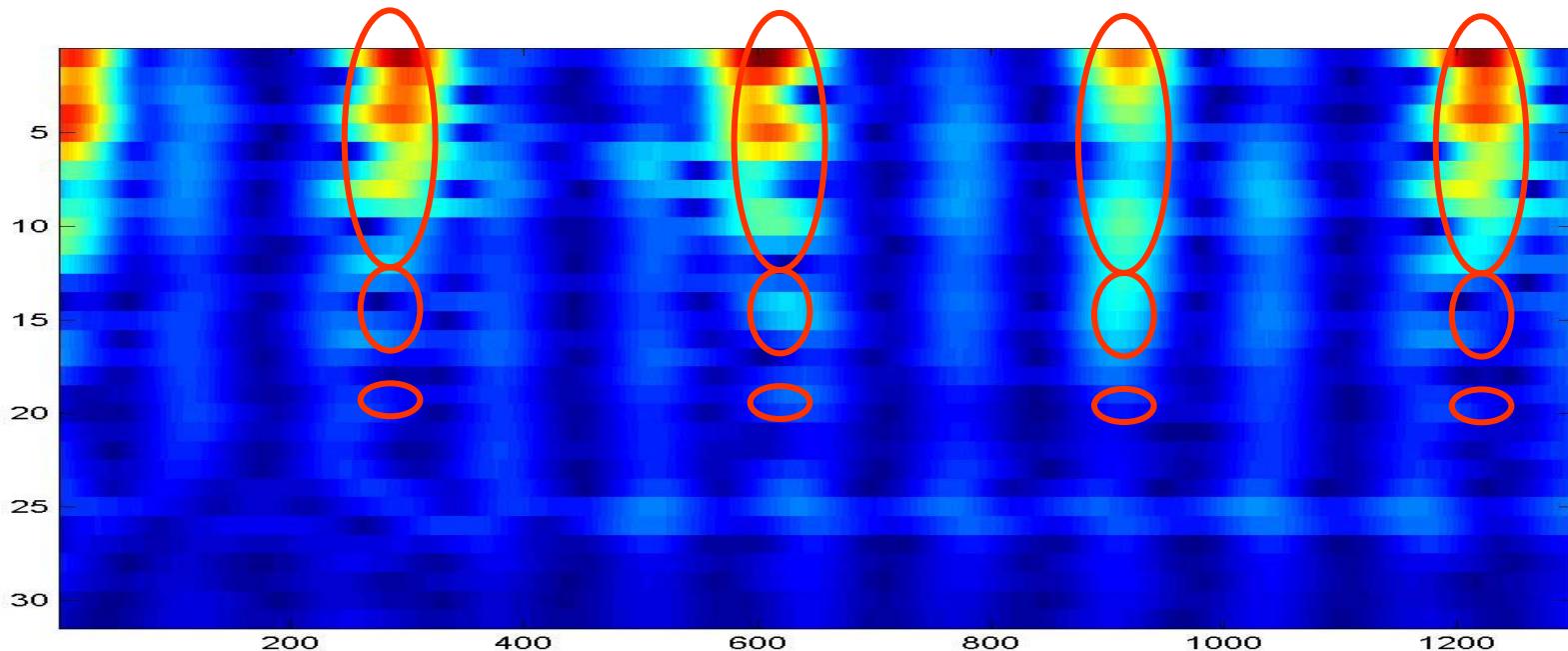
$$f(s,t) = f(s) f(t)$$



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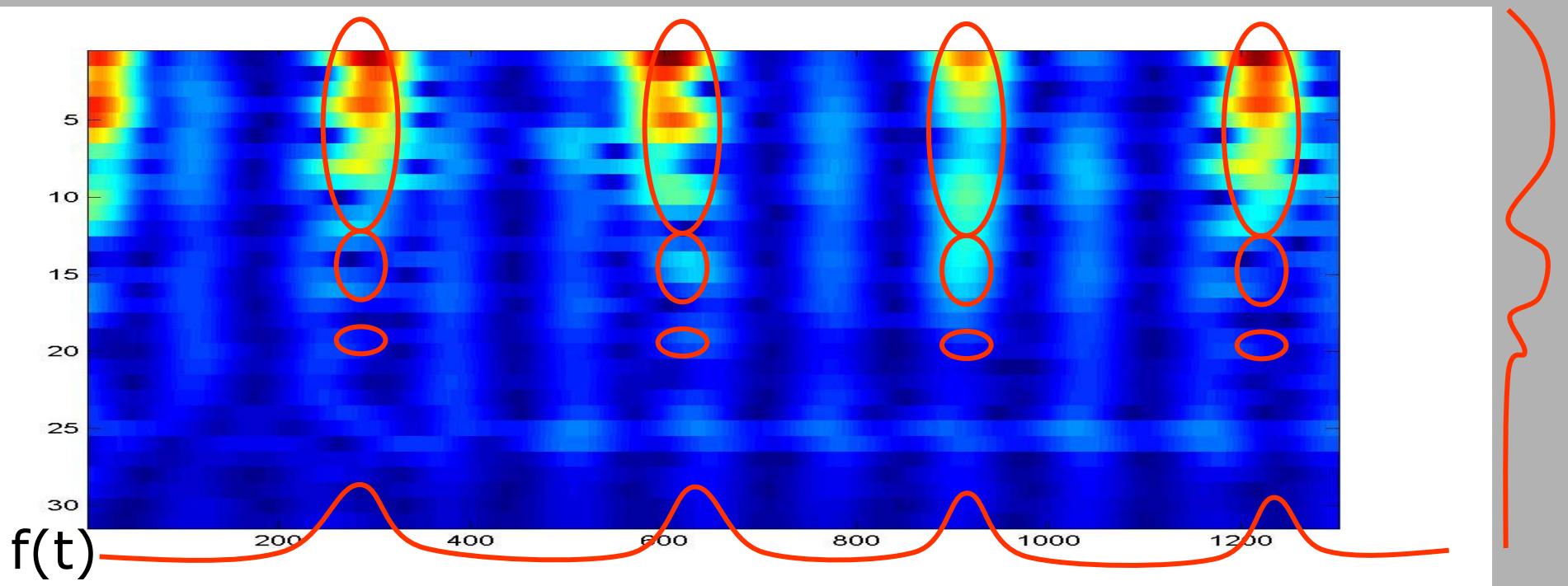


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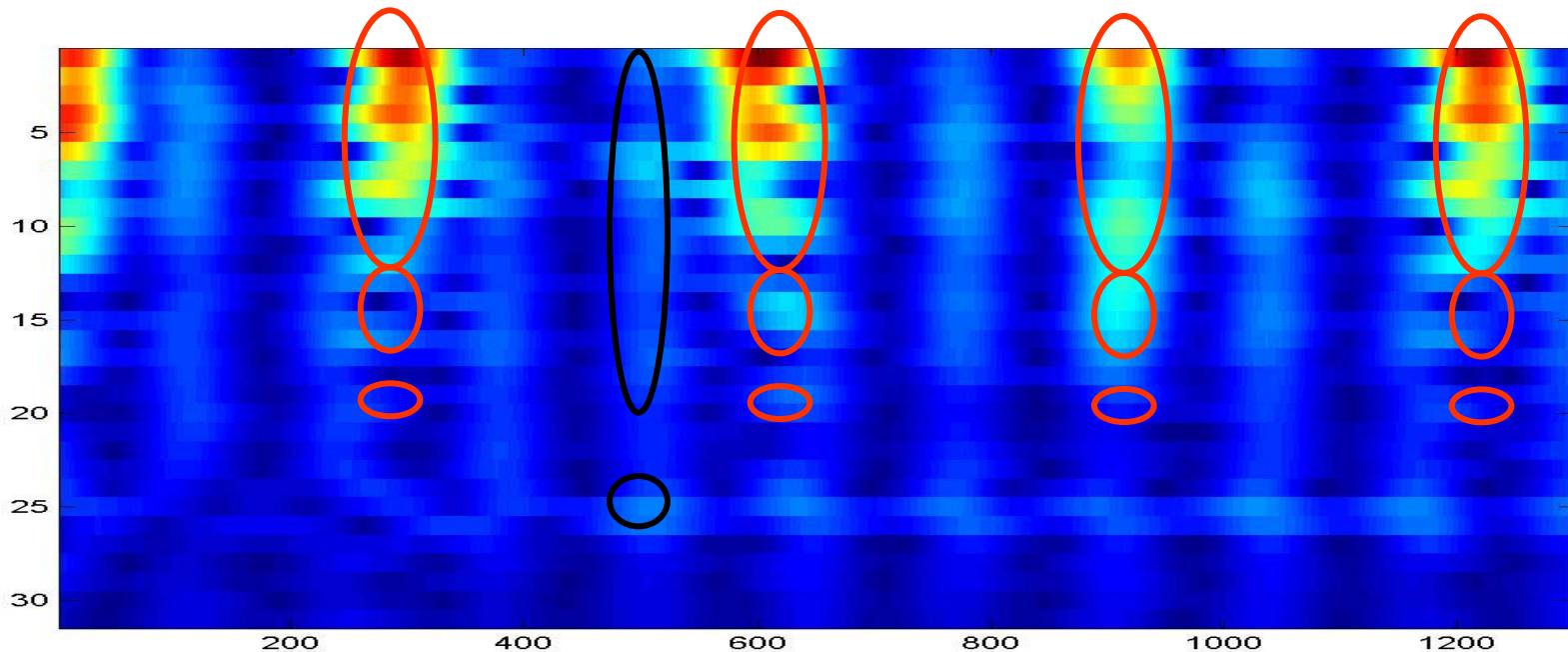
$$f(s)$$



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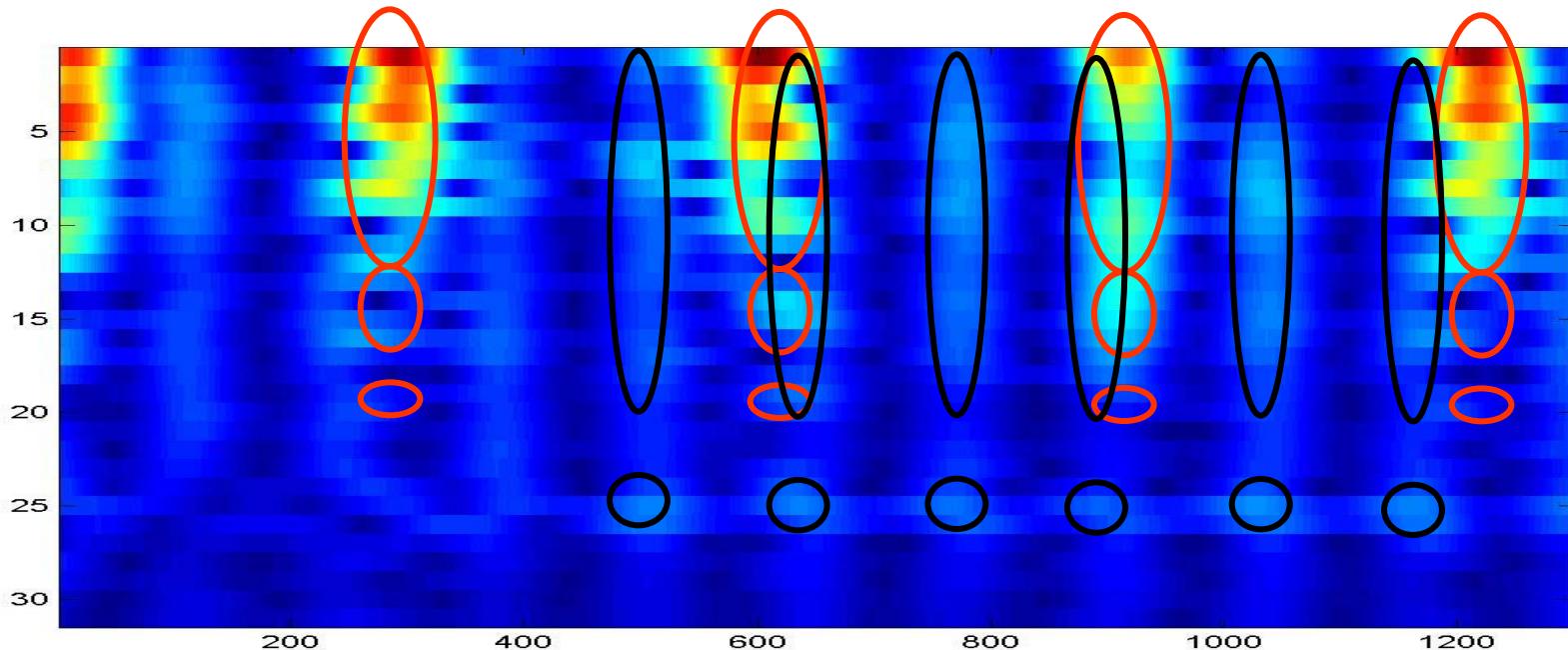
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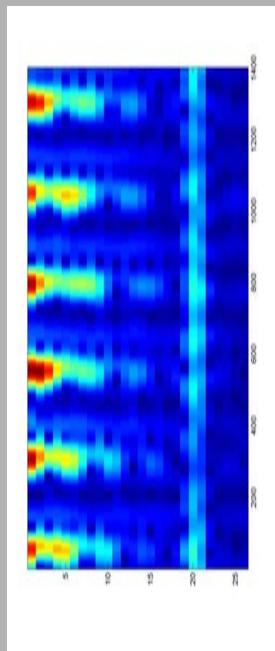
- Identify the heart beat in the spectrogram.
- Observation: with the right window size used in the spectrogram, a ECG complex in the spectrogram can be viewed as a separable function.

$$f(s,t) = f(s) f(t)$$



we want to find the “best”  $U_m$ ,  $V_m$ ,  $U_f$  and  $V_f$  s.t.

$$S = U_m V_m + U_f V_f$$



$$= \begin{matrix} \text{blue vertical bar} \\ \text{blue horizontal bar} \end{matrix} * \begin{matrix} \text{blue vertical bar} \\ \text{blue horizontal bar} \end{matrix} + \begin{matrix} \text{orange vertical bar} \\ \text{orange horizontal bar} \end{matrix} * \begin{matrix} \text{orange vertical bar} \\ \text{orange horizontal bar} \end{matrix}$$

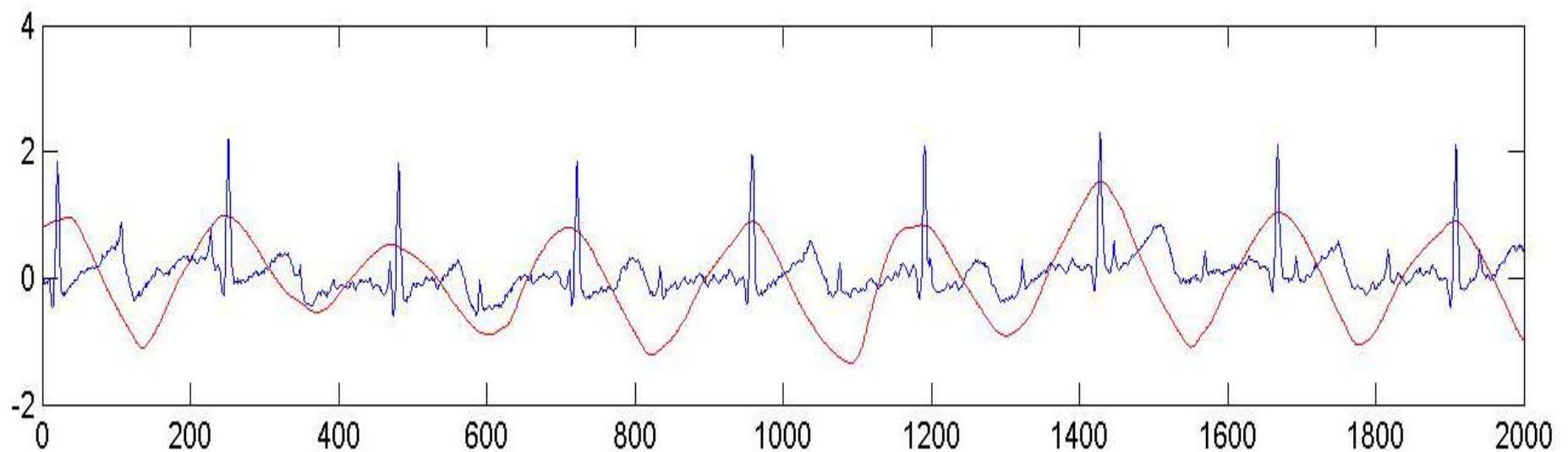
Suppose we want to find the solutions that minimized the noise in the sense that  $\| N \|^2$  is minimized,

$$S = U_m V_m + U_f V_f + N$$

then, we can employ the well-known SVD. However, experimental studies show that it gives unsatisfactory results.

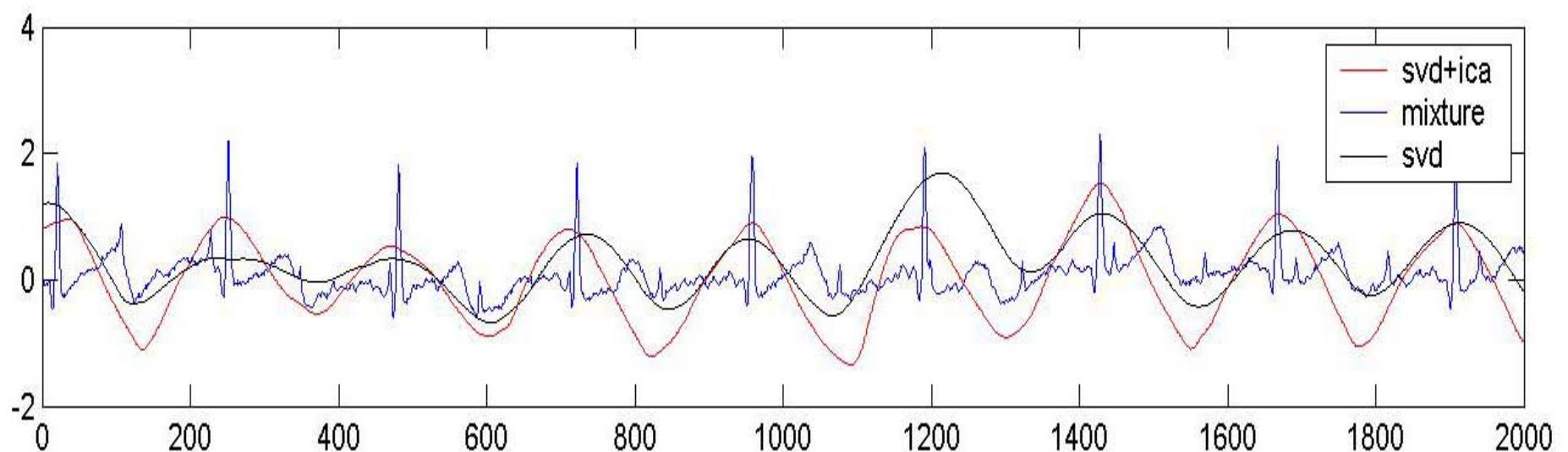
We borrow idea of ICA  
(Independent Component Analysis).  
In the proposed algorithm, we attempt to find the solutions that are “statistically independent” and non-Gaussian like.

# Experiments

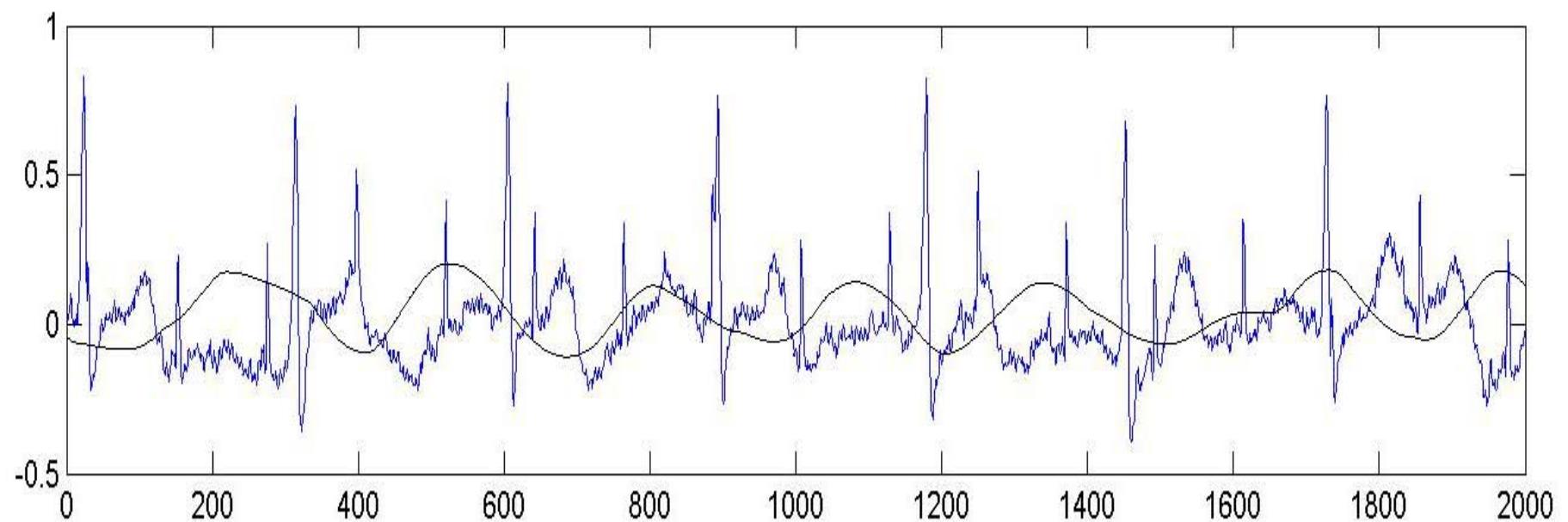


Maternal heartbeat trend  $U_m$  using ica+svd

# Experiments



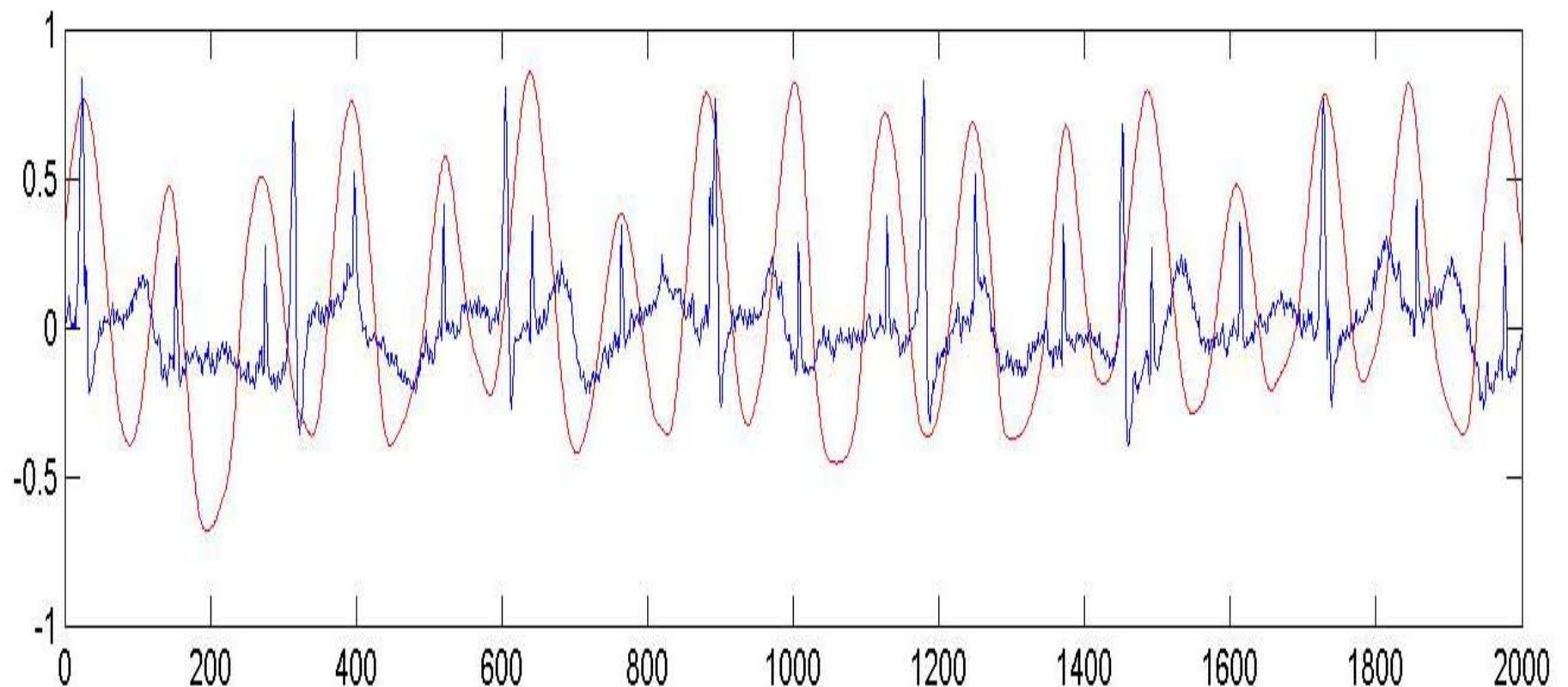
# Experiments



Fetal heartbeat trend  $U_v$  using SVD

108.raw

# Experiments



Fetal heartbeat trend  $U_v$  using svd+ica

# Independent Components Analysis

$$x_1(t) = a_{11} s_1(t) + a_{12} s_2(t)$$

$$x_2(t) = a_{21} s_1(t) + a_{22} s_2(t)$$

unknowns > equations, under constrained.

We want to find the  $s_1$  and  $s_2$  which has maximum independence and minimum nongaussianity.



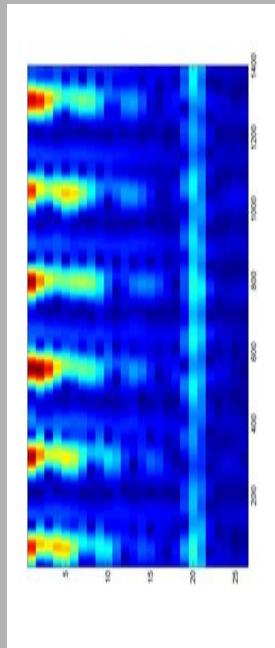
$$\sum s_1(t) s_2(t) = \sum s_1(t) \sum s_2(t)$$

Negentropy

In our experiment, we use FastICA

we want to find the “independent”  $U_m$ ,  $V_m$ ,  $U_f$  and  $V_f$  s.t.

$$S = U_m V_m + U_f V_f$$



=



\*



+



\*



## Proposed method

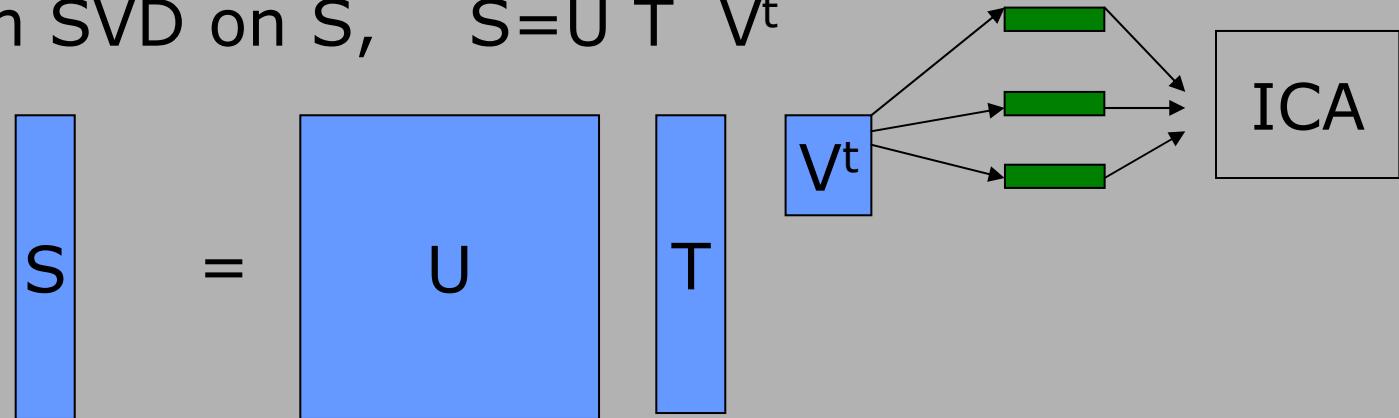
1. Compute Spectrogram  $S$
2. Perform SVD on  $S$ ,  $S=U \Sigma V^t$

$$\begin{matrix} S \\ \hline \end{matrix} = \begin{matrix} U \\ \Sigma \\ \hline V^t \end{matrix}$$

3. Apply ICA on the  $k$  most significant spectral components, i.e. on  $V_1, V_2, V_3, \dots, V_k$ .
4. Update the  $U$  using the mixture obtained in step 3.
5. Apply ICA on the  $k$  most significant spectral of the updated  $U$ .
6. Choose two ``best'' components in  $U$  and call them  $u_m$  and  $u_f$ .

# Proposed method

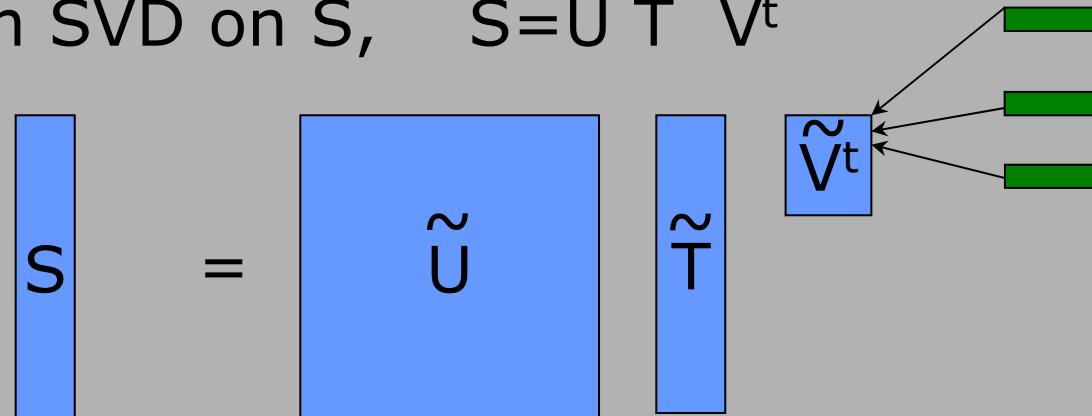
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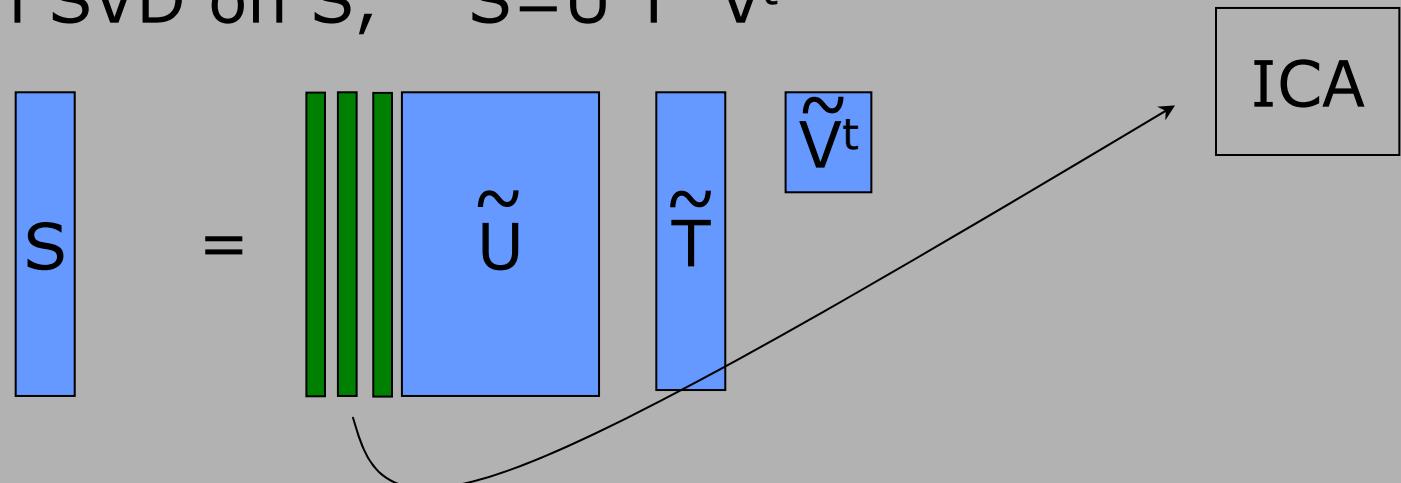
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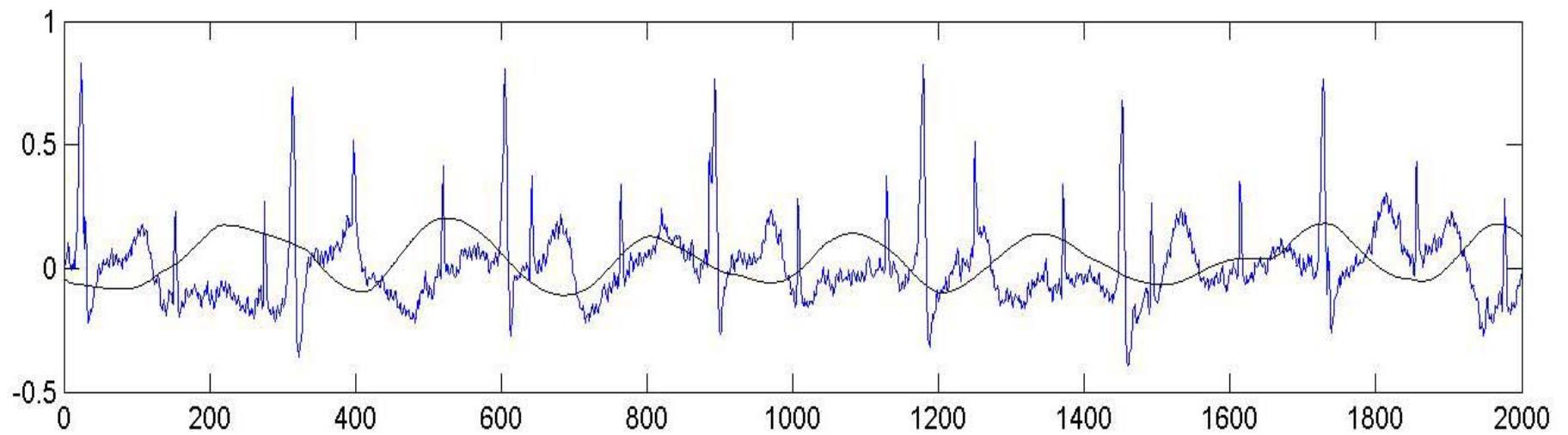
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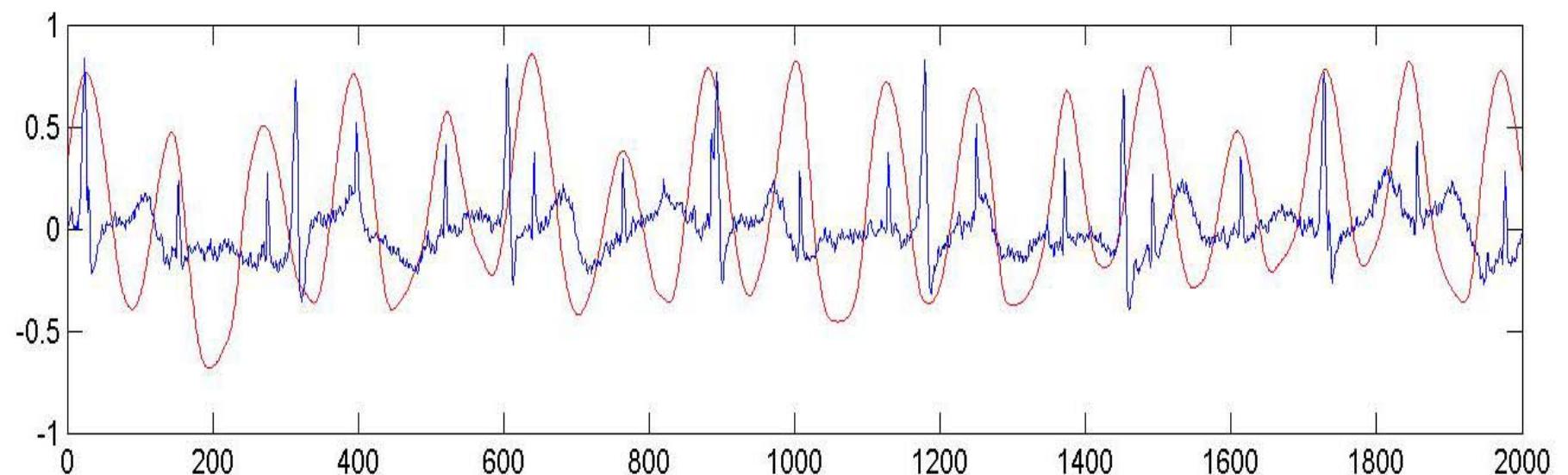
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Components obtained after SVD



Components obtained after SVD + ICA

## Remarks & Future works

1. The proposed method does not find the intended  $u_f$  and  $u_m$  as stated in the formulation. It is an approximation.
2. Extend the method to other applications.
3. More analysis.

# Acknowledgement

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Prof Ho Ting Fei, Department of Physiology,  
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