

# BLIND SEPARATION OF FETAL ECG FROM SINGLE MIXTURE USING SVD AND ICA

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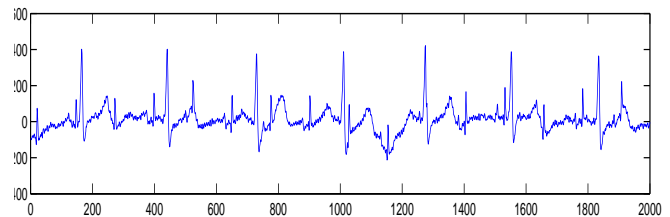
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## ABSTRACT

In this paper, we propose a novel blind-source separation method to extract fetal ECG from a single-channel signal measured on the abdomen of the mother. The signal is a mixture of the fetal ECG, the maternal ECG and noise. The key idea is to project the signal into higher dimensions, and then use an assumption of statistical independence between the components to separate them from the mixtures. This is achieved by applying Singular Value Decomposition (SVD) on the spectrogram, followed by an iterated application of Independent Component Analysis (ICA) on the principle components. The SVD contributes to the separability of each component and the ICA contributes to the independence of the two components. We further refine and adapt the above general idea to ECG by exploiting a-prior knowledge of the maternal ECG frequency distribution and other characteristics of ECG. Experimental studies show that the proposed method is more accurate than using SVD only. Because our method does not exploit extensive domain knowledge of the ECGs, the idea of combining SVD and ICA in this way can be applied to other blind separation problems.

## 1. INTRODUCTION

A fetal ECG contains important indications about the health conditions of fetus, and could become a diagnostic tool of particular importance. One way to obtain the fetal ECG is by recording it on the mother's abdomen. Fig. 1 shows an example of the recorded signals. The recording is complicated by the existence of a strong maternal ECG and random contaminations due to noncardiac sources. In Fig. 1, the prominent repeating peaks are from the maternal ECG, while the less visible peaks are from the fetus. Like for adults, the fetal ECG complex and the heart rate variability are two important measures that can be gleaned from the ECG(Fig. 2(a) and (b) are the maternal and fetal ECG complex respectively).

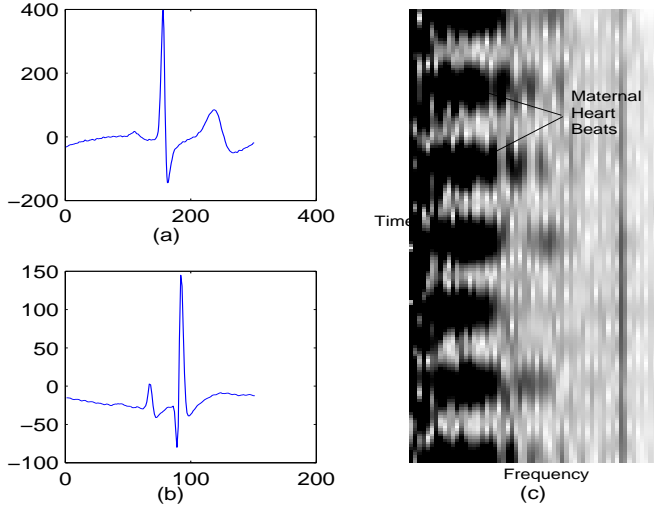


**Fig. 1.** Original abdominal signal. The label of x-axis is the sample number. The sampling period is (1/300) second. In our problem, the actual scaling factor for the y-axis (signal amplitude) is not important.

Since 1960, many different methods[1, 3, 5, 6] have been developed for detecting the fetal ECG. Most of these methods focus on multi-channel mixtures of signals [6]. One direct method subtracts a thoracic maternal ECG from the abdominal composite ECG. A more recent approach[6] employs Independent Component Analysis (ICA), which extracts the sources from their mixtures by assuming the sources are statistically independent. Relatively few works address the problem separating ECG signals recorded on a single-channel. Kanjilal et. al. [3] developed a method for single-channel signals by first detecting both the maternal and fetal heart beats. Next, the signal is “cut” into pieces. These pieces are aligned (to form a matrix) and SVD is then performed to obtain the ECG complex.

In this paper, we consider a single-channel abdominal signal. The main challenge is the detection of the occurrence of fetal heartbeats. If the beat locations can be detected, then the fetal complex can be obtained by averaging, SVD ([3]), or even ICA. Hence, we will focus our discussion on the fetal heartbeats detection.

By projecting the single-channel signal into a higher dimension, the multi-channel technique can be employed. We propose a blind-source separation method using a SVD of the spectrogram, followed by an iterative application of



**Fig. 2.** (a) Maternal ECG complex. (b). Fetal ECG complex. (c). Spectrogram of Fig. 1.

ICA on both the spectral and a temporal representations of the ECG signals. To define the decomposition problem, we make both an assumption of statistical independence of the heart-beat occurrences, and a similar assumption on the basis vectors we will use to represent the spectral slices of the individual signals. This is a reasonable assumption in our domain because each signal is a repeating ECG complex. The method we develop is likely to be applicable to other problem domains defined by the need to separate nearly periodic sources from noisy mixtures.

## 2. BACKGROUND ON ICA

Independent Component Analysis[4] is a method for finding “hidden factors” in multivariate data sets. If we assume that the data are linear combinations of basis vectors, we can write:

$$x_1(k) = a_{11}s_1(k) + a_{12}s_2(k) \quad (1)$$

$$x_2(k) = a_{21}s_1(k) + a_{22}s_2(k) \quad (2)$$

or written more compactly and for possibly larger dimension:

$$\mathbf{x}(k) = \mathbf{A}\mathbf{s}(k) \quad (3)$$

where  $\mathbf{x}$  is our observed data,  $\mathbf{A}$  is a “mixing matrix” with column oriented “basis vectors”,  $\mathbf{s}$  are the “hidden” sources, and  $k$  is the (typically time) index of the data vectors. The assumption of statistical independence of the signals  $\mathbf{s}$  is enough to allow us to estimate both unknowns,  $\mathbf{s}$  and matrix  $\mathbf{A}$  (except for scale factors and column permutations).

The independence of the resultant hidden factors is useful in many applications such as segregating mixtures of multiple sources. In the simplest formulation of ICA, we need  $n$  different non-Gaussian sources.

We use the FastICA algorithm[2] which is an iterative fixed point algorithm that with each step, increases a measure of independence between the hypothesized sources.

We will use ICA on two different representations of our signals; one on a set of spectral basis vectors that are linearly combined to represent individual spectral slices of a spectrogram, the other on a time domain representation of signal mixtures and sources.

## 3. PROBLEM FORMULATION AND MOTIVATION

Given the source signal  $x(t)$ . Let  $S$  be its spectrogram. We treat  $S$  as a matrix, with each row corresponding to the spectrum at a particular time(Fig. 2(c)). We assume that  $S$  is the mixture of the column vector  $\mathbf{u}_m, \mathbf{v}_m$  and  $\mathbf{u}_f, \mathbf{v}_f$  in the following way,

$$S = \mathbf{u}_m \mathbf{v}_m^t + \mathbf{u}_f \mathbf{v}_f^t + \mathbf{n}, \quad (4)$$

where  $\mathbf{n}$  is the noise. We call the vector  $\mathbf{u}_m$  and  $\mathbf{u}_f$  the maternal and fetal *heartbeat trend* respectively.

Consider a signal which consists of a repeating ECG complex. Its spectrogram also consists of repeating patterns. This can be seen in Fig. 2(c). By carefully choosing the right window width for the spectrogram, the spectrogram of a ECG complex could be separable. In this case, we would expect the heartbeat trends  $\mathbf{u}_m$  and  $\mathbf{u}_f$  to be approximately sinusoidal with each cycle corresponding to a heart beat, and expect  $\mathbf{v}_m$  and  $\mathbf{v}_f$  to approximate the spectrum of the ECG complex. Therefore, an accurate estimation of  $\mathbf{u}_f$  is sufficient to determine the heartbeat, which in-turn can be used to obtain the ECG complex.

Now, given  $S$ , our problem is to estimate  $\mathbf{u}_m, \mathbf{v}_m, \mathbf{u}_f$ , and  $\mathbf{v}_f$ . If we attempt to minimize the energy of  $\mathbf{n}$ , then this amounts to finding the two best separable functions whose sum approximates  $S$ , which can be obtained using SVD. However, numerical experiment on the synthetic signal (Fig. 3) gives disappointing results.

Alternatively, we can borrow the idea of ICA. Besides minimizing the noise, we propose finding the components such that  $\mathbf{u}_m$  and  $\mathbf{v}_m$  are respectively statistically independent from  $\mathbf{u}_f$  and  $\mathbf{v}_f$ . In next section, we describe a method that attempts to find such components.

## 4. PROPOSED METHOD

Given the source signal  $x(t)$ , we first compute its spectrogram,  $S$  (the choice of window width will be discussed in Section 6.1).

1. Perform SVD on  $S$ . Let  $S = U\Sigma V^t$ .

Here,  $S$  is the spectrogram with rows representing time slices.  $\Sigma$  is a square diagonal matrix with weights corresponding to the significance of the related spectral vector in  $V$ ,  $U$  is oriented the same way as the spectrogram with columns that are orthonormal time-indexed weights associated with a given spectral vector from  $V$  which sum to create spectral slices of  $S$ .

2. Based on the property of SVD, the first  $k$  columns of  $U$ , and  $V$  are the  $k$  most significant components,  $S$  then could be approximated as:

$$S \approx U_k \Sigma_k V_k^t$$

where  $\Sigma_k$  is the diagonal matrix whose elements are the first  $k$  singular values of  $S$ . Here  $k > 2$  is a fixed constant.

3. Apply ICA on the  $k$  most significant spectral components  $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k$  (columns in  $V_k$ ). Let the independent components be  $\tilde{\mathbf{v}}_1, \dots, \tilde{\mathbf{v}}_k$ , and let  $\tilde{V}_k$  be the matrix composed using the independent components as column vectors. Let the ‘‘mixing’’ matrix for  $V_k$  be  $M$ , that is:

$$V_k^t = M\tilde{V}_k^t$$

4. Update the time vectors to recover the one-to-one correspondence between  $U$  time vectors and  $V$  spectral vectors. That is, compute  $[\tilde{\mathbf{u}}_1, \tilde{\mathbf{u}}_2, \dots, \tilde{\mathbf{u}}_k]$  by

$$[\tilde{\mathbf{u}}_1, \tilde{\mathbf{u}}_2, \dots, \tilde{\mathbf{u}}_k] = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_k] \Sigma_k M,$$

where  $\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_k$  are columns of  $U_k$ .

By doing so, the independence of  $\tilde{\mathbf{v}}_1, \dots, \tilde{\mathbf{v}}_k$  is guaranteed and the energy of  $S$  is kept constant, which are helpful for the solution stability.

5. Perform ICA on the  $\tilde{\mathbf{u}}_1, \tilde{\mathbf{u}}_2, \dots, \tilde{\mathbf{u}}_k$ . Let  $\hat{\mathbf{u}}_1, \dots, \hat{\mathbf{u}}_k$  be the independent components.

This step ensures that the time vectors are independent.

6. Select and output the two best components as  $\mathbf{u}_m$  and  $\mathbf{u}_f$  from  $\hat{\mathbf{u}}_1, \hat{\mathbf{u}}_2, \dots, \hat{\mathbf{u}}_k$ . (The selection criteria will be discussed in Section 6.2)

The above algorithm requires a parameter  $k$ , which we take it as 10 in our experiment. That is, we choose the 10 most significant components from the much larger set corresponding to the number of frequency channels in the spectrogram. The number is chosen to be large enough to retain the significant information from the original signal, but is reduced for fast computation and so that we have a reasonable number of channels for the ICA algorithm to work on.

## 5. EXPERIMENTAL RESULTS

Due to the lack of ground truth, we evaluate the performance on a few recorded signals by visual inspection. We also evaluate it on a few synthetic mixtures where ground truth are available.

The fetal heart rate can vary across the time, especially for the fetus who might move during the recording. Nevertheless, the maternal heart rate is slower, and ranges around 60 to 110 times per minute. Same as Fig. 2, the x-axis label in all figures is the sample number, and the sampling period is (1/300) second.

*Synthetic data:* In the first set of experiments, we use a synthetic mixture that is constructed from two simulated ECG complexes (Fig. 2(a) and (b)). Note that the energy of one complex is higher than the other. This is to emulate the relatively strong maternal ECG and weak fetal ECG. One period of the maternal and fetal ECG complex is 240 and 100 samples respectively.

We compare the proposed method with the method that uses SVD as described in Section 3, which finds the  $\mathbf{u}_m$ ,  $\mathbf{v}_m$ ,  $\mathbf{u}_f$ , and  $\mathbf{v}_f$ , that minimize the noise.

For maternal heart beats detection, both methods give a promising results. But when it comes to the fetal heart beats, the SVD fails while our proposed method is still working with a high accuracy.

Fig. 3 compares the fetal heartbeat trend  $\mathbf{u}_f$  computed by our method with those found using SVD. We take the local maximums of  $\mathbf{u}_f$  as the heart beats occurrences. Clearly, the proposed method of using ICA is better than using solely SVD.

*Recorded signal:* In the second set of experiments, we performed the comparison on a number of recorded signals. Each signal is about 10 minutes, with sampling rate 300Hz (roughly  $1.8 \times 10^5$  samples). We will present two in this section. The signals are obtained from two patients with a gestation period of 37 weeks. Fig. 4 shows a short part of the original signals (approximately 6.7seconds long).

Comparison results between the two methods for synthetic data and real data are quite similar.

Fig. 5 compares the detection of maternal heart-beat occurrences using SVD and our method. Both methods give good detection. However, our method is able to detect most of the occurrences where SVD fails.

Fig. 6 and 7 are comparisons for detection of fetal heart-beat occurrences between the two methods. The SVD performs poorly. It gives a heartbeat trend that is seriously influenced by the maternal’s. The proposed method gives good detection. It successfully detects all the heartbeat occurrences in both figures, but falsely detects two occurrences in Fig. 6 (the false detections can be filtered out using do-

main knowledge). Note that it succeeds in cases where the maternal and fetal heartbeat coincide.

Once the trend of the heart beats is obtained, there are many methods that can be used to get the ECG complex. Here, we adopt the time domain averaging method. Since it is not the main part of this paper, we will not go into detail about it.

Fig. 2(a) and (b) are the Maternal and Fetal ECG complex we obtained by the proposed method respectively.

## 6. REFINING FOR ECG

In the proposed method, there are two steps where special attention is needed.

### 6.1. Choice of window width of spectrogram

The choice of window width is essential to retain sufficient information in the spectrogram, and at the same time gives the nice separability property. If the window is too long, say triple the duration of one ECG complex, then the spectrogram is smooth along the time and no interesting heartbeat trend can be obtained. On the other hand, if the width is small, say only a fifth of the duration of one ECG complex, then the spectrogram capture the fine details of the non-stationary ECG complex. Due to these details, its spectrogram is no longer separable. In our experiment, we use the Blackman window with the width of a healthy maternal ECG complex.

### 6.2. Selecting the best component

ICA yields the components in arbitrary order. In order to find which component is for maternal heartbeats and which is for fetal heartbeats, we take the frequency characteristic into account. Since the ECG signal is quasi-periodic, the expected spectrum should have only one peak whose location and height can be estimated by the approximate heart rate. Therefore, the sampling frequency will be enough for us to select the correct heart beats trend. Assigning maternal and fetal labels is facilitated by the *a priori* knowledge that the fetal heartbeat frequency is higher than the maternal.

## 7. DISCUSSION AND CONCLUSION

We propose a method that incorporates SVD and ICA in obtaining the heartbeat occurrence of fetal ECG from single-channel composite signal. Based on the heartbeat occurrences, the actual fetal ECG can be separated from the composite signal using known methods. Our main idea is to project the signal into a higher dimension. ICA is then applied to obtain the heartbeat occurrences. The ambiguities

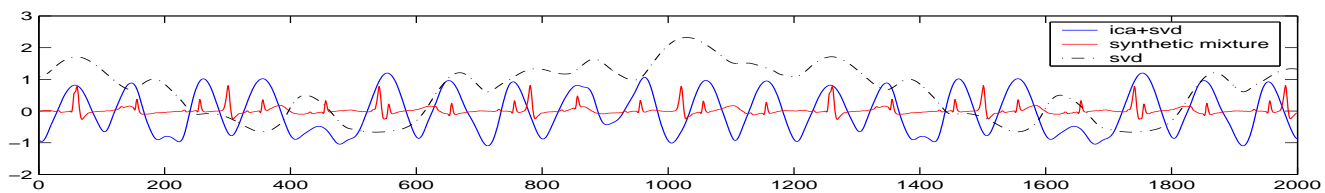
of ICA (lack of any ordering to the separated signals) is manageable with an obvious application of domain knowledge.

Most ICA algorithms, are either iterative fixed point algorithms (such as FastICA) or gradient descent algorithms, both of which optimize a solution only locally and are sensitive to initial randomization conditions that can produce quite different solutions, even for exactly the same signal. We view our technique of first separating the spectral basis vectors before submitting the remixed time domain signals to ICA as a way of setting up advantageous initial conditions that contribute to the stability of the solutions for the time domain separation.

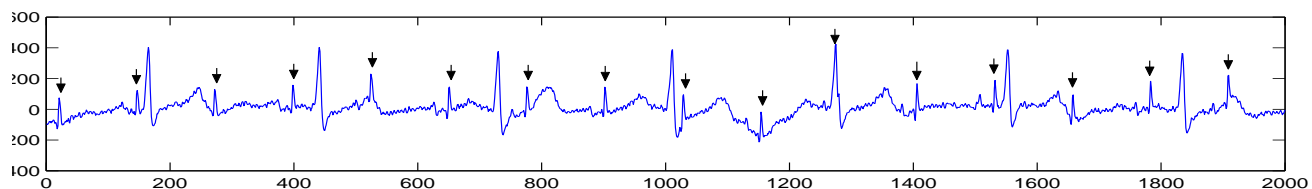
Results show that the proposed algorithm works well for extracting a fetal ECG from the composite signal. Since it only uses single-channel recording, there are no confounding issues that arise from having original signals that can differ more complicated ways than simply signal mixture levels.

## 8. REFERENCES

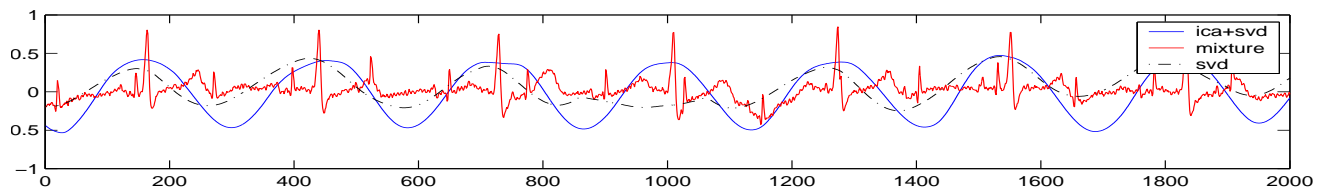
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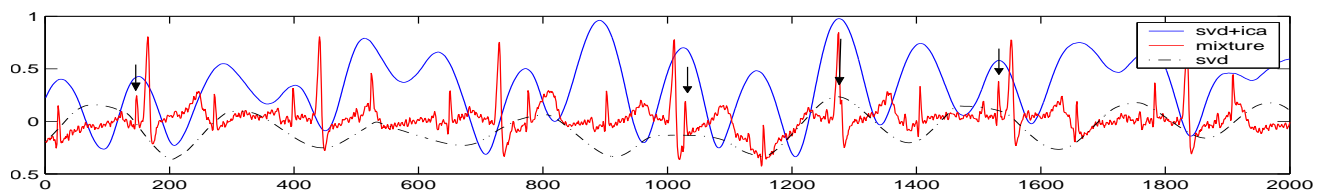
**Fig. 3.** Comparison of fetal heartbeat trend  $u_f$  computed by SVD and SVD+ICA on synthetic signal. The local maximums can be treated as the heartbeat occurrences.



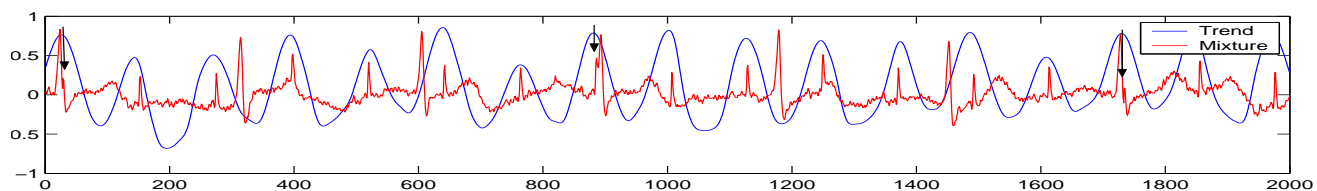
**Fig. 4.** Original signal 292.raw (the entire signal is ten-minute long, only a short segment is shown here for clarity). Arrows indicate the occurrences of fetal heart beats, which are detected by visual inspection.



**Fig. 5.** Maternal trend comparison of SVD and ICA for 292.raw.



**Fig. 6.** Fetal trend comparison of SVD and ICA for 292.raw. Arrows indicates fetal heartbeat that are difficult to detect.



**Fig. 7.** Another example: fetal heart-beat occurrences detection by SVD + ICA. Arrows indicates fetal heartbeat that are difficult to detect.