Phase-unwrapping of SAR Interferogram with Multi-frequency or Multi-baseline

Wei Xu, Ee Chien Chang, Leong Keong Kwoh, Hock Lim, Wang Cheng Alice Heng
Centre for Remote Imaging, Sensing and Processing
National University of Singapore, Singapore 0511
T: (65) 772 3220, F: (65) 775 7717, EMail: crsxuw@nusunix.nus.sg

ABSTRACT

The use of multiple InSAR images of the same area for phase unwrapping is discussed. It is shown that by combining the information in the two InSAR images, the ambiguity interval of the phase angle can be considerably lengthened, which will facilitate unwraping. Three methods for combining multiple InSAR images are proposed and their error characteristics discussed.

TERRAIN HEIGHT AND INSAR PHASE

After removing an approximately linear, look-angle dependent term, the phase angle of an InSAR image is given in terms of terrain height h as follows:

$$\phi_a = (2\pi/\lambda^*)h + O(h^2) \tag{1}$$

where

$$\lambda^* = \frac{\lambda}{\eta}\frac{\sqrt{r^2+B^2-B[\frac{\sqrt{\alpha_0}}{R+H}\cos\omega - \frac{r^2+2RH+H^2}{R+H}\sin\omega]}}{B[\frac{\alpha_1}{4(R+H)\sqrt{\alpha_0}}\cos\omega - \frac{R}{(R+H)}\sin\omega]} \tag{2}$$

with λ being the wavelength, R the radius of the earth, H the height of the master SAR antenna, B the baseline, ω the elevation angle of the baseline, r the slant range of target from the master SAR (Fig. 1), and

$$\alpha_0 = (r^2 - H^2)[4R(R+H) - (r^2 - H^2)], \tag{3}$$
$$\alpha_1 = 4R(r^2 + 2RH + H^2). \tag{4}$$

The value of η is 1 or 2 for single pass or repeat pass interferometry respectively.

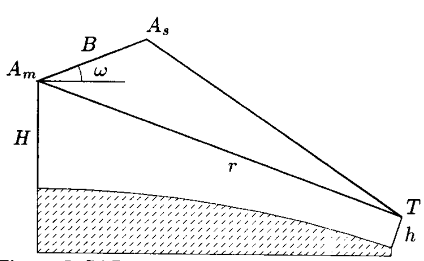


Fig. 1 InSAR geometry. A_m: Master antenna. A_s: Slave antenna. T: Target.

Measurement yields however only the wrapped phase angle ϕ_w which is given by

$$\phi_w = \phi_a - 2\pi k, \qquad \text{with} \quad 0 \le \phi_w < 2\pi, k \text{ an integer.} \tag{5}$$

The unwrapping of ϕ_w to determine ϕ_a is a critical step in terrain height measurement by interferometric SAR. In region of steep terrain (height change of $\lambda^*/2$ or more between neighbouring pixels), unwrapping by conventional methods (Zebker and Goldstein, 1986; Lin and Zebker, 1992) can be difficult, especially in the presence of noise. Madsen and Zebker (1992) and Madsen *et al.* (1993) proposed an absolute phase retrieval method based on sub-band processing. The method however still requires heavy averaging (over more than 10,000 pixels in their report) to obtain consistent unwrapped phase angles.

PHASE UNWRAPPING WITH MULTIPLE INSAR IMAGES

We shall investigate the possibility of incorporating the information in multiple InSAR images with different λ^* for phase unwrapping. The values of λ^* may differ due to a change in imaging wavelength λ, or for repeat pass of the same satellite, due to the change in the baseline (B, ω, r). For simplicity, we shall discuss the case of two InSAR images, although the idea is easily generalised to more images. For a pixel with terrain height h, we have

$$\begin{aligned} 2\pi h &= \phi_{a1}\lambda_1^* = (\phi_{w1} + k_1 2\pi)\lambda_1^* \\ &= \phi_{a2}\lambda_2^* = (\phi_{w2} + k_2 2\pi)\lambda_2^* \end{aligned} \tag{6}$$

where the subscripts denote the relevant parameters for each InSAR image. Eq. (6) shows that, as value of h increase, the graph of ϕ_{a2} versus ϕ_{a1} is a straight line passing through the origin. In terms of the wrapped phases ϕ_{w1} and ϕ_{w2}, the straight line is "folded" within a $2\pi \times 2\pi$ square (Fig. 2). When the ratio λ_1^*/λ_2^* is a rational number, say, m_1/m_2 with m_1, m_2 mutually prime, the folded line retraces itself and a total of $m_1 + m_2 - 1$ line segments will be obtained. When the ratio is irrational, the folded line covers the whole square.

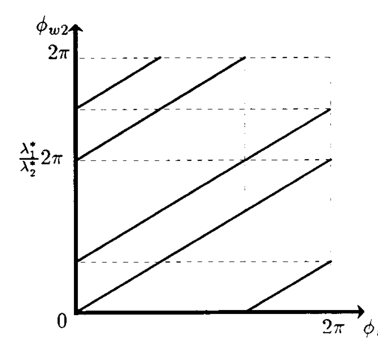


Fig. 2 Relationship between wrapped phases ϕ_{w1} and ϕ_{w2}.

For the case of rational ratio, we note that the values of ϕ_{w1} and ϕ_{w2} provide a partial unwrapping since the terrain height ambiguity interval is now lengthened to $m_2\lambda_1^*$ ($= m_1\lambda_2^*$). In the case of irrational ratio, the unwrapping is in principle complete. When implementing this idea, however, we must develop procedures which are robust to noise. A few methods we have experimented on are described below.

UNWRAPPING PROCEDURES

1. Chinese Remainder Theorem Method

When $\lambda_1^*/\lambda_2^* = m_1/m_2$ is rational, a natural way to combine the two InSAR images is through the Chinese Remainder Theorem (CRT). Let $n_i = \lfloor m_i \phi_{wi}/(2\pi) \rfloor$ and $f_i = m_i\phi_{wi}/(2\pi) - n_i$, for $i = 1$ and 2. (In the absence of noise, $f_1 = f_2$.) An integer p, $0 \le p < m_1 m_2$, can be uniquely determined by the CRT. The unwrapped phase is then $(p + f_i)\lambda_i^*/m_i$, for either $i = 1$ or 2. This method is useful when m_i are small integers. Unfortunately, this approach is not robust to noise as we shall explain later.

2. Projection Method

In the absence of noise, (ϕ_{w1}, ϕ_{w2}), considered as a point in the ϕ_{w1}-ϕ_{w2} plane, will lie on one of the line segments of Fig. 2. In general, there will be too many line segments to consider. To facilitate unwrapping, an estimate of the terrain height $\hat{h}$ for a pixel may be made, based on neighbouring pixels' values or other information. Depending on the noise level of the images to be unwrapped, we may then restrict our attention to the few line segments which correspond to values of h within an interval centred at $\hat{h}$. The presence of noise will cause the point (ϕ_{w1}, ϕ_{w2}) to be shifted away from the line segments, and we will have to assign it to the nearest line segment (among the relevant few) and unwrapped accordingly. Note that in this process, some noise-filtering has also been effected.

Another way of looking at this unwrapping process is to consider the h_1-h_2 plane, where $h_i = \lambda_i^*\phi_{ai}/(2\pi)$. The wrapped phase point (ϕ_{w1}, ϕ_{w2}) defines the point (h_1, h_2) only up to the array of crosses shown in Fig. 3:

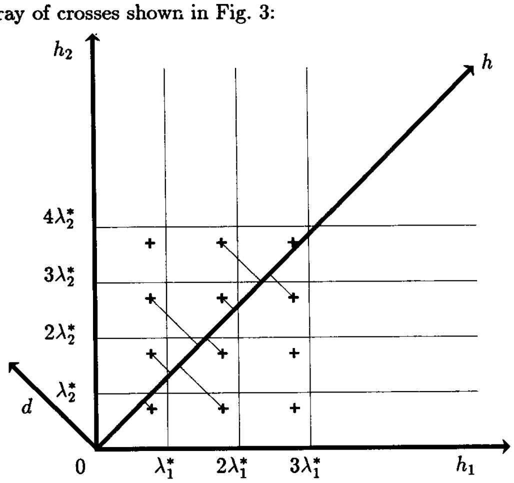
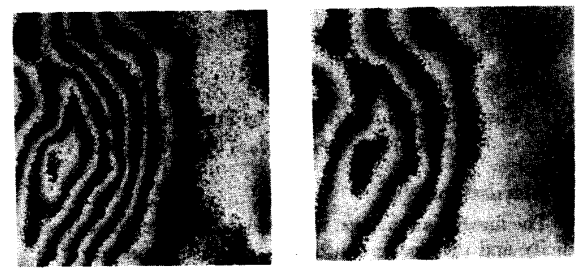


Fig. 3 Possible locations of (h_1, h_2).

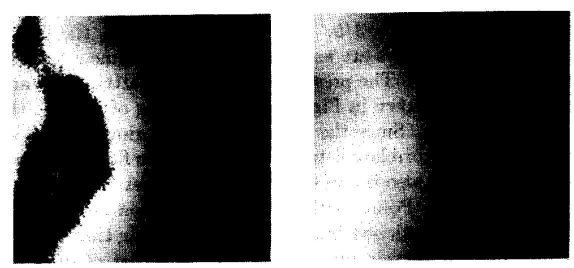
The projection method unwrapping is equivalent to selecting the crossed point which lies closest to the straight line $h_1 = h_2$. This point of view greatly facilitates implementation. By changing the coordinate system to $h = (h_2 + h_1)/2$, $d = (h_2 - h_1)/2$, we may then select the nearst point based on d, and its corresponding value of h then provides the unwrapped terrain height.

Fig. 4a and 4b show two simulated noisy interferograms with $\lambda_1^*/\lambda_2^* = 3/5$. Figs. 5a and 5b show the results of two implementations of the projection unwrapping method. In Fig. 5a, partial unwrapping up to the lengthened ambiguity interval is shown. Full unwraping as shown in Fig. 5b is easily carried out through neighbouring pixel constraints.



(a) (b)

Figs. 4a and 4b Two simulated noisy interferograms with $\lambda_1^*/\lambda_2^* = 3/5$.



(a) (b)

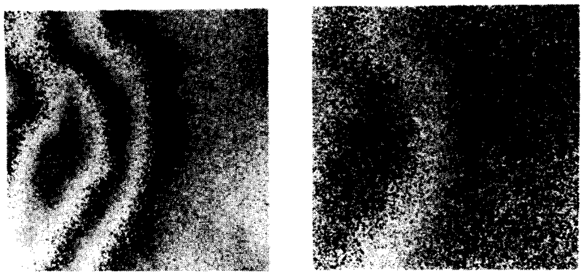
Fig. 5 (a) Partial unwrapping by projection method.
(b) Full unwrapping with neighbouring pixel constraints.

3. Linear Combination Method

We may very simply combine two InSAR images by differencing the wrapped phases and then mod 2π. Since

$$\Delta\phi_w = \phi_{w1} - \phi_{w2} = \phi_{a1} - \phi_{a2} + k2\pi = 2\pi h \frac{\lambda_2^* - \lambda_1^*}{\lambda_1^*\lambda_2^*}, \quad (7)$$

we see that the ambiguity of the difference InSAR image is now $\lambda_d^* = \lambda_1^*\lambda_2^*/|\lambda_2^* - \lambda_1^*|$ which offers a partial unwrapping. To widen the ambiguity interval, it is sometimes possible to use linear combination of the form $a\phi_{w1} + b\phi_{w2}$ with integral a, b. For instance, when $\lambda_1^*/\lambda_2^* = 3/5$, the combination $2\phi_{w2} - \phi_{w1}$ has an ambiguity interval of $5\lambda_1^*$ while $\phi_{w2} - \phi_{w1}$ has an ambiguity interval of $2.5\lambda_1^*$. The results of combining Figs. 4a and 4b by such computations are shown in Figs. 6a and 6b.



(a) (b)

Fig. 6 Partial unwrapping by linear combination method. (a) $\phi_{w2} - \phi_{w1}$, (b) $2\phi_{w2} - \phi_{w1}$.

4. Discussions

The linear combination method is attractive due to its simplicity and light computational effort. Unfortunately, the errors of the original InSAR images are compounded in the difference image. For instance, if each of the original image contains Gaussian noise of variance σ^2, the difference image will contain Gaussian noise of variance $2\sigma^2$. However, since no systematic error is introduced, the noise in the difference image is relatively easy to filter.

The CRT method is not very practical due to its peculiar error characteristics. To illustrate this point, we show Fig. 4 for the case $\lambda_1^*/\lambda_2^* = 3/5$. On the ϕ_{w1}-ϕ_{w2} plane, the points in the hatched boxes are assigned to the line segment passing through $(0, 2\pi/5)$. The presence of regions of "attraction" and "repulsion" as shown in Fig. 7 leads clustering of error in the combined image. Since the errors of neighbouring pixels tend to be strongly correlated, treatment by simple filtering is not effective. Fig. 7 also shows the error characteristics of the projection method. Points within the dotted region are assigned to the above-mentioned line segment. It is clear that this assignment is intuitively more reasonable than that of the CRT method. If the original images each contains Gaussian noise of variance σ^2, the combined image, assuming no wrong assignment of pixel, will contain Gaussian noise of variance approximately $\sigma^2/2$. The noise level is lowered due the availability of two measurements of the same quatity. However, points which are assigned to a wrong line segment will show large errors. When the noise in the original images is random, we may expect that such large errors will be easy to detect. This benign error characteristics suggests the projection method to be a promising method for further investigation.

In actual phase unwrapping implementations, the different methods may be used to supplement each other. For instance, the result of linear combination method, properly smoothed, may be used as reference data for unwrappping by the projection method.

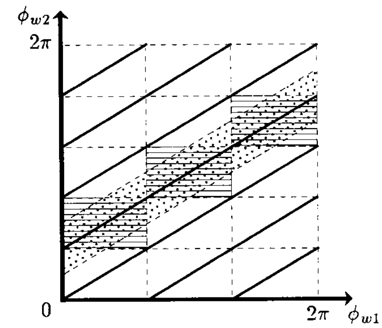


Fig. 7 Schematic illustration of the operation of the CRT method and the projection method. Points in the hatched boxes are assigned to the line segment passing through $(0, 2\pi/5)$ by the CRT method. Points in the dotted band are assigned to the line segment by the projection method.

CONCLUSIONS

Our study shows that multiple InSAR images can be combined to faciliate phase unwrapping. Three methods for combining InSAR images have been examined. They are found to have difference error properties and produce combined interferograms of distinct characteristics. The linear combination method has the advantage of requiring only very simple calculation, but the combined interferogram tends to be noisy. The projection method may be expected to produce the best combined interferogram in general.

REFERENCES

Goldstein, R.M., H.A. Zebker, H.A., and Werner, C.L. "Satellite radar interferometry: two-dimensional phase unwrapping." *Radio Science.* 23(1988):713–720.

Lin, Q., Vesecky, J.F., and Zebker, H.A. "New approaches in interferometric SAR data processing." *IEEE Transactions on Geoscience and Remote Sensing.* 30(1992):560–567.

Madsen, S.N. and Zebker, H.A. "Automated absolute phase retrieval in cross-track interferometry," in Proceedings of the International Geoscience and Remote Sensing Symposium, 1992, pp. 1582–1584.

Madsen, S.N., Zebker, H.A., and Martin, J. "Topographic mapping using radar interferometry: processing techniques." *IEEE Transactions on Geoscience and Remote Sensing.* 31(1993):246–256.