CS2100: Computer Organisation

Tutorial #6: Boolean Algebra, Logic Gates and Simplification

(Week 8: 11 – 15 March 2024)

Discussion Questions:

D1. (a) One common mistake that students make is the following: $A \cdot B + A' \cdot B' = 1$ … (equation 1)

This seems to be erroneously "derived" from the following rule: $X + X' = 1$.
Explain why the rule is wrongly applied here.

(b) Is the following equation correct? Why? $A \cdot B + (A \cdot B)' = 1$ … (equation 2)

D2. Given the following two 3-variable Boolean functions:

$$F(A,B,C) = \Sigma\, m(0, 2, 4, 6, 7)$$
$$G(A,B,C) = \Sigma\, m(1, 2, 3, 6)$$

(a) Write the product-of-maxterms expressions in ΠM notation for F and G.

(b) If $X = F + G$, write the sum-of-minterms expressions in Σm notation for X.

(c) If $Y = F \cdot G$, write the sum-of-minterms expressions in Σm notation for Y.

(d) If $Z = F \oplus G$, write the sum-of-minterms expressions in Σm notation for Z.

Do you know how to generalise the above for any arbitrary Boolean functions F and G?

[If it is not convenient to type symbols like Σ and Π in the forums, you may use Sum-m to mean Σm and Prod-M to mean ΠM. Example: Sum-m(0, 2, 4, 6, 7), Prod-M(2, 3, 5).]

D3. For each part below, how many prime implicants (PIs) and essential prime implicants (EPIs) are there in the K-map? What is/are the simplified **SOP expressions**? List out all alternative answers. [d(…) and D(…) denote don't-cares.]

(a) $F1(A,B,C,D) = \Sigma\, m(5, 8, 10, 12, 13, 14)$

(b) $F2(W,X,Y,Z) = \Pi\, M(0, 1, 2, 8, 9, 10)$

(c) $F3(K,L,M,N) = \Sigma\, m(1, 7, 10, 13, 14) + d(0, 5, 8, 15)$

(d) $F4(A,B,C,D) = \Pi\, M(4, 8, 9, 11, 12) \cdot D(2, 3, 6, 7, 10, 14)$

D4. For each of the functions in D3 above, find the simplified **POS expression**. List out all alternative answers, if any.

You are encouraged to do the above discussion questions and discuss them on Canvas or QnA. These are fundamental concepts that you must know, before you attempt the tutorial questions below.

Reminder:
Do not omit the · operator when writing product terms such as $A \cdot B$. Omitting the · operator, like AB, means that AB is a 2-bit value. Marks will be deducted.

Tutorial Questions:

Note: By default, we assume that complemented literals are NOT available, unless otherwise stated.

1. The **consensus theorem** is given as

$$x \cdot y + x' \cdot z + y \cdot z = x \cdot y + x' \cdot z$$

Can you prove this using the laws and theorems of Boolean algebra given in class?
[Hint: Draw the K-map which will give you an idea which laws/theorems should be used.]

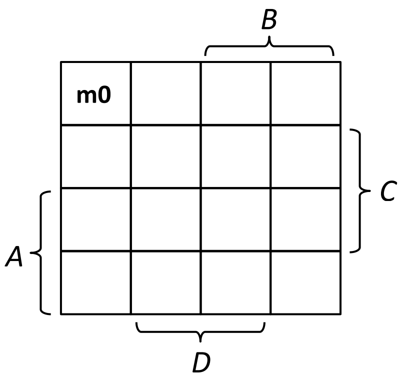
2. Using Boolean algebra, simplify each of the following expressions into simplified **sum-of-products (SOP) expression**. Indicate the law/theorem used at every step.

   (a) $F(x,y,z) = (x + y \cdot z') \cdot (y' + y) + x' \cdot (y \cdot z' + y)$

   (b) $G(p,q,r,s) = \Pi\, M(5, 9, 13)$

   [Tip: For (b), it is easier to start with the given expression and get done in about 5 steps, rather than to expand it into sum-of-products/sum-of-minterms expression first.]

3. (a) The following K-map layout is used for a 4-variable Boolean function $T(A,B,C,D)$. Fill in the minterm positions m1 to m15 into the respective cells. m0 has been filled for you.



   (b) Given the following 4-variable Boolean function:

$$T(A,B,C,D) = \Pi M(3,7,8,10,12,13) \cdot X(6,11,14,15)$$

   where X's are the don't-cares, write out the Σm notation for $T(A,B,C,D)$.

   (c) Draw the K-map for T using the layout above.

   (d) How many PIs (prime implicants) are there in the K-map? List out all the PIs.

   (e) How many EPIs (essential prime implicants) are there? List out all the EPIs.

   (f) What is the simplified SOP expression for T? List out all alternative solutions.

   (g) What is the simplified POS expression for T? List out all alternative solutions.

   (h) Implement the simplified SOP expression for T using a 2-level AND-OR circuit and a 2-level NAND only circuit.

4. A circuit takes in four inputs *K,L,M,N* and generates 3 outputs *X,Y,Z* as follow:

$X(K,L,M,N) = 1$ if $KL = MN$, or 0 otherwise,
where *KL* and *MN* are 2-bit unsigned integers.

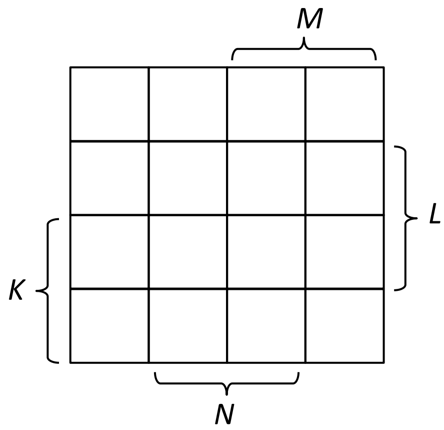
$Y(K,L,M,N) = 1$ if $KL \leq MN$, or 0 otherwise,
where *KL* and *MN* are 2-bit unsigned integers.

$Z(K,L,M,N) = 1$ if $KLM < LMN$, or 0 otherwise,
where *KLM* and *LMN* are 3-bit unsigned integers.

For parts (a) – (c) below, you may assume that the input 0000 will not occur.

(a) Fill in the truth table for the circuit. Write 'd' for don't cares.

(b) Fill in the K-maps of *X*, *Y* and *Z* using the layout given below.



(c) Write out the simplified SOP expressions of *X*, *Y* and Z.

(d) After designing the circuit according to the simplified SOP expressions in (c), if you feed the input 0000 into it, what will be the outputs?