

**Question 7 [Algorithm 16 marks].**

Let  $a[0..n - 1]$  be an array of  $n$  distinct numbers. If  $i < j$  and  $a[i] > a[j]$ , then the pair  $(i, j)$  is called a *mismatch* of  $a$ . For example, the array  $a[] = [2, 8, 3]$  has one mismatch,  $(1, 2)$ , whereas  $b[] = [1, 2, 3, 4]$  does not have any mismatches.

- (a) List all the mismatches of the array  
 $a = [4, 6, 1, 2]$ .

- (b) How many arrays of size  $n$  have *more than*  
 $n(n + 1)/2$  mismatches?

- (c) Devise an algorithm to calculate the number of mismatches in an integer array  $a$  of size  $n$ . You may assume that all array elements are distinct and the first element is stored at position 0. You will be given full marks if your solution runs in  $O(n * \log n)$ . A sketch of the algorithm is sufficient.