1. Categorical Terms and their Meaning
2. Propositions, Axioms, Lemmas, Proofs
3. Manipulating Terms and Propositions
4. Arguments and Syllogisms
Categorical Terms and their Meaning

1. Origins and Goals
2. Form, not Content
3. Categorical Terms
4. Meaning through models

Propositions, Axioms, Lemmas, Proofs

Manipulating Terms and Propositions

Arguments and Syllogisms
Traditional Logic

Origins

Greek philosopher Aristotle (384–322 BCE) wrote treatise *Prior Analytics*; considered the earliest study in formal logic; widely accepted as the definite approach to deductive reasoning until the 19th century.
Traditional Logic

Origins
Greek philosopher Aristotle (384–322 BCE) wrote treatise *Prior Analytics*; considered the earliest study in formal logic; widely accepted as the definite approach to deductive reasoning until the 19th century.

Goal
Express relationships between sets; allow reasoning about set membership
Example 1

All humans are mortal.
All Greeks are humans.
Therefore, all Greeks are mortal.
Example 1

All humans are mortal.
All Greeks are humans.
Therefore, all Greeks are mortal.

Makes “sense”, right?
Example 1

All humans are mortal.
All Greeks are humans.
Therefore, all Greeks are mortal.

Makes “sense”, right?

Why?
Example 2

All cats are predators.
Some animals are cats.
Therefore, all animals are predators.
Example 2

*All cats are predators.*
*Some animals are cats.*
*Therefore, all animals are predators.*

Does not make sense!
Example 2

All cats are predators.
Some animals are cats.
Therefore, all animals are predators.

Does not make sense!

Why not?
**Example 3**

All slack track systems are caterpillar systems. All Christie suspension systems are slack track systems. Therefore, all Christie suspension systems are caterpillar systems.
Example 3

All slack track systems are caterpillar systems.
All Christie suspension systems are slack track systems.
Therefore, all Christie suspension systems are caterpillar systems.

Makes sense, even if you do not know anything about suspension systems.
Example 3

All slack track systems are caterpillar systems. All Christie suspension systems are slack track systems. Therefore, all Christie suspension systems are caterpillar systems.

Makes sense, even if you do not know anything about suspension systems.

Form, not content

In logic, we are interested in the form of valid arguments, irrespective of any particular domain of discourse.
Categorical Terms

Terms refer to sets

Term *animals* refers to the set of animals, term *brave* refers to the set of brave persons, etc.
Categorical Terms

Terms refer to sets

Term \textit{animals} refers to the set of animals, term \textit{brave} refers to the set of brave persons, etc.

The set \textit{Terms} contains all terms under consideration.
Categorical Terms

Terms refer to sets

Term *animals* refers to the set of animals, term *brave* refers to the set of brave persons, etc.

The set *Terms* contains all terms under consideration

Examples

animals ∈ Terms

brave ∈ Terms
Models

Meaning

A model $M$ fixes what elements we are interested in, and what we mean by each term.
A model $\mathcal{M}$ fixes what elements we are interested in, and what we mean by each term.

For a particular $\mathcal{M}$, the universe $U^\mathcal{M}$ contains all elements that we are interested in.
<table>
<thead>
<tr>
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<th></th>
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<td>For a particular $\mathcal{M}$, the universe $U^\mathcal{M}$ contains all elements that we are interested in.</td>
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<tr>
<td>Meaning of terms</td>
<td>For a particular $\mathcal{M}$ and a particular term $t$, the meaning of $t$ in $\mathcal{M}$, denoted $t^\mathcal{M}$, is a particular subset of $U^\mathcal{M}$.</td>
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Example 1A

For our examples, we have

\[ \text{Term} = \{ \text{cats, humans, Greeks, ...} \} \].
Example 1A

For our examples, we have

$$\text{Term} = \{\text{cats, humans, Greeks, ...} \}.$$

First meaning $\mathcal{M}$

- $U^\mathcal{M}$: the set of all living beings,
Example 1A

For our examples, we have

\[ \text{Term} = \{ \text{cats, humans, Greeks, \ldots} \}. \]

**First meaning** \( \mathcal{M} \)

- \( U^\mathcal{M} \): the set of all living beings,
- \( \text{cat}^\mathcal{M} \): the set of all cats,
Example 1A

For our examples, we have

\[ \text{Term} = \{\text{cats, humans, Greeks, \ldots}\}. \]

First meaning \( \mathcal{M} \)

- \( U^\mathcal{M} \): the set of all living beings,
- \( \text{cat}^\mathcal{M} \) the set of all cats,
- \( \text{humans}^\mathcal{M} \) the set of all humans,
- \( \ldots \)
Consider the same Term = \{cats, humans, Greeks, \ldots\}.
Consider the same Term = \{cats, humans, Greeks, ...\}.

Second meaning $\mathcal{M}'$

- $U^\mathcal{M}$: A set of 100 playing cards, *depicting* living beings,
Example 1B

Consider the same $\text{Term} = \{\text{cats, humans, Greeks, ...}\}$.

Second meaning $\mathcal{M}'$

- $U^\mathcal{M}$: A set of 100 playing cards, *depicting* living beings,
- $\text{cat}^\mathcal{M}$: all cards that show cats,
Example 1B

Consider the same \( \text{Term} = \{ \text{cats}, \text{humans}, \text{Greeks}, \ldots \} \).

**Second meaning \( \mathcal{M}' \)**

- \( U^\mathcal{M} \): A set of 100 playing cards, *depicting* living beings,
- \( \text{cat}^\mathcal{M} \): all cards that show cats,
- \( \text{humans}^\mathcal{M} \): all cards that show humans,
- \( \ldots \)
Example 2A

Consider the following set of terms:

Term = \{even, odd, belowfour\}
Consider the following set of terms:
\[ \text{Term} = \{ \text{even}, \text{odd}, \text{belowfour} \} \]

**First meaning \( \mathcal{M}_1 \)**

\[ U^{\mathcal{M}_1} = \mathbb{N}, \]

\( \mathbb{N} \) represents the set of natural numbers.
Example 2A

Consider the following set of terms:
Term = \{ even, odd, belowfour \}

First meaning \( M_1 \)

- \( U^{M_1} = \mathbb{N} \),
- \( \text{even}^{M_1} = \{ 0, 2, 4, \ldots \} \),
Consider the following set of terms:

\[ \text{Term} = \{ \text{even}, \text{odd}, \text{belowfour} \} \]

**First meaning \( M_1 \)**

- \( \mathcal{U}^{M_1} = \mathbb{N} \),
- \( \text{even}^{M_1} = \{0, 2, 4, \ldots \} \),
- \( \text{odd}^{M_1} = \{1, 3, 5, \ldots \} \), and
Example 2A

Consider the following set of terms:
Term = \{even, odd, belowfour\}

First meaning \(M_1\)
- \(U^{M_1} = \mathbb{N}\),
- \(even^{M_1} = \{0, 2, 4, \ldots\}\),
- \(odd^{M_1} = \{1, 3, 5, \ldots\}\), and
- \(belowfour^{M_1} = \{0, 1, 2, 3\}\).
Example 2B

Consider the same \( \text{Term} = \{ \text{even, odd, belowfour} \} \)
Consider the same $\text{Term} = \{\text{even, odd, belowfour}\}$

**Second meaning $\mathcal{M}_2$**

- $U^{\mathcal{M}_2} = \{a, b, c, \ldots, z\}$,
Example 2B

Consider the same \( \text{Term} = \{\text{even, odd, belowfour}\} \)

Second meaning \( \mathcal{M}_2 \)

- \( U^{\mathcal{M}_2} = \{a, b, c, \ldots, z\} \)
- \( \text{even}^{\mathcal{M}_2} = \{a, e, i, o, u\} \)
Consider the same \( \text{Term} = \{ \text{even, odd, belowfour} \} \)

**Second meaning \( \mathcal{M}_2 \)**

- \( U^{\mathcal{M}_2} = \{ a, b, c, \ldots, z \} \),
- \( \text{even}^{\mathcal{M}_2} = \{ a, e, i, o, u \} \),
- \( \text{odd}^{\mathcal{M}_2} = \{ b, c, d, \ldots \} \), and
Consider the same \( \text{Term} = \{ \text{even, odd, belowfour} \} \)

**Second meaning** \( \mathcal{M}_2 \)

- \( U^{\mathcal{M}_2} = \{ a, b, c, \ldots, z \} \),
- \( \text{even}^{\mathcal{M}_2} = \{ a, e, i, o, u \} \),
- \( \text{odd}^{\mathcal{M}_2} = \{ b, c, d, \ldots \} \), and
- \( \text{belowfour}^{\mathcal{M}_2} = \emptyset \).
Categorical Terms and their Meaning

Propositions, Axioms, Lemmas, Proofs

Manipulating Terms and Propositions

Arguments and Syllogisms

Categorical Propositions

Semantics of Propositions

Axioms, Lemmas and Proofs

CS 3234: Logic and Formal Systems

02—Traditional Logic
Categorical Propositions

All cats are predators

expresses a relationship between the terms cats (subject) and predators (object).
All cats are predators

expresses a relationship between the terms cats (subject) and predators (object).

**Intended meaning**

Every thing that is included in the class represented by cats is also included in the class represented by predators.
### Four Kinds of Categorical Propositions

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- **Example:** Some cats are not brave is a particular, negative proposition.
Four Kinds of Categorical Propositions

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Example

Some cats are not brave is a *particular, negative* proposition.
Meaning of Universal Affirmative Propositions

In a particular model $\mathcal{M}$, All Greeks are mortal means that $\text{Greeks}^\mathcal{M}$ is a subset of $\text{mortal}^\mathcal{M}$.
In a particular model $\mathcal{M}$, *All Greeks are mortal* means that $\text{Greeks}^\mathcal{M}$ is a subset of $\text{mortal}^\mathcal{M}$.
More formally...

\[
(\text{All subject are object})^M = \begin{cases} 
T & \text{if } subject^M \subseteq object^M, \\
F & \text{otherwise}
\end{cases}
\]

Here \(T\) and \(F\) represent the logical truth values \textit{true} and \textit{false}, respectively.
In a particular model $\mathcal{M}$, \textbf{No Greeks are cats} means that the intersection of $\text{Greeks}^\mathcal{M}$ and of $\text{cats}^\mathcal{M}$ is empty.
Meaning of Universal Negative Propositions

In a particular model $\mathcal{M}$, "No Greeks are cats" means that the intersection of $\text{Greeks}^\mathcal{M}$ and $\text{cats}^\mathcal{M}$ is empty.
More formally...

\[(\text{No subject are object})^M = \begin{cases} 
T & \text{if } \text{subject}^M \cap \text{object}^M = \emptyset, \\
F & \text{otherwise}
\end{cases}\]
Meaning of Particular Affirmative Propositions

In a particular model $\mathcal{M}$, Some humans are Greeks means that the intersection of $\text{humans}^\mathcal{M}$ and of $\text{Greeks}^\mathcal{M}$ is not empty.
In a particular model $\mathcal{M}$, *Some humans are Greeks* means that the intersection of $\text{humans}^\mathcal{M}$ and of $\text{Greeks}^\mathcal{M}$ is not empty.
More formally...

$$(\text{Some } \text{subject are object})^M = \begin{cases} 
T & \text{if } \text{subject}^M \cap \text{object}^M \neq \emptyset, \\
F & \text{otherwise}
\end{cases}$$
In a particular model $\mathcal{M}$, Some Greeks are not vegetarians means that the difference of $\text{Greeks}^\mathcal{M}$ and $\text{vegetarians}^\mathcal{M}$ is not empty.
In a particular model $\mathcal{M}$, Some Greeks are not vegetarians means that the difference of $\text{Greeks}^\mathcal{M}$ and $\text{vegetarians}^\mathcal{M}$ is not empty.
More formally...

\[
(\text{Some } \text{subject} \text{ are not } \text{object})^M = \begin{cases} 
T & \text{if } \text{subject}^M/\text{object}^M \neq \emptyset, \\
F & \text{otherwise}
\end{cases}
\]
**Axioms** are propositions that are assumed to hold.
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**Axiom (HM)**

The proposition *All humans are mortal* holds.
Axioms are propositions that are assumed to hold.

**Axiom (HM)**

The proposition *All humans are mortal* holds.

**Axiom (GH)**

The proposition *All Greeks are humans* holds.
**Graphical Notation**

```
HumansMortality
```

All humans are mortal
Lemmas are affirmations that follow from all known facts.
Lemmas are affirmations that follow from all known facts.

**Proof obligation**
A lemma must be followed by a proof that demonstrates how it follows from known facts.
Lemma

The proposition *All humans are mortal* holds.
Lemma

The proposition *All humans are mortal* holds.

Proof.

\[ \text{All humans are mortal} \]
We can choose any model for our terms, also “unusual” ones.
We can choose any model for our terms, also “unusual” ones.

Example

\[U^M = \{0, 1\}, \text{humans}^M = \{0\}, \text{mortal}^M = \{1\}\]
We can choose any model for our terms, also “unusual” ones.

Example

\[ U^M = \{0, 1\}, \text{humans}^M = \{0\}, \text{mortal}^M = \{1\} \]

Here

All humans are mortal

does not hold.
Purpose of axioms

By asserting an axiom $A$, we are focusing our attention to only those models $\mathcal{M}$ for which $A^\mathcal{M} = T$. 
Asserting Axioms

Purpose of axioms

By asserting an axiom $A$, we are focusing our attention to only those models $\mathcal{M}$ for which $A^\mathcal{M} = T$.

Consequence

The lemmas that we prove while utilizing an axiom only hold in the models in which the axiom holds.
Asserting Axioms

Purpose of axioms
By asserting an axiom $A$, we are focusing our attention to only those models $M$ for which $A^M = T$.

Consequence
The lemmas that we prove while utilizing an axiom only hold in the models in which the axiom holds.

Validity
A proposition is called *valid*, if it holds in all models.
1. Categorical Terms and their Meaning

2. Propositions, Axioms, Lemmas, Proofs

3. Manipulating Terms and Propositions
   - Complement
   - Conversion
   - Contraposition
   - Obversion
   - Combinations

4. Arguments and Syllogisms
We allow ourselves to put $\textit{non}$ in front of a term.
We allow ourselves to put non in front of a term.

**Meaning of complement**

In a model $M$, the meaning of non $t$ is the complement of the meaning of $t$. 
We allow ourselves to put \textit{non} in front of a term.

**Meaning of complement**

In a model $\mathcal{M}$, the meaning of \textit{non} $t$ is the complement of the meaning of $t$.

**More formally**

In a model $\mathcal{M}$, $(\text{non } t)^\mathcal{M} = U^\mathcal{M} / t^\mathcal{M}$.
Double Complement

Axiom (NonNon)

For any term $t$, the term $\text{non} \ \text{non} \ t$ is considered equal to $t$. 
Categorical Terms and their Meaning
Propositions, Axioms, Lemmas, Proofs
Manipulating Terms and Propositions
Arguments and Syllogisms

Double Complement

Axiom (NonNon)

For any term \( t \), the term \( \text{non non} \ t \) is considered equal to \( t \).

\[
\begin{align*}
\ldots t \ldots \\
\downarrow \text{[NNI]} \\
\ldots \text{non non} \ t \ldots \\
\downarrow \text{[NNE]} \\
\ldots t \ldots
\end{align*}
\]
Rule Schema

\[ \ldots t \ldots \]
\[ \begin{array}{c}
\non \non t
\end{array} \]
\[ \text{[NNI]} \]
\[ \ldots \non \non t \ldots \]

is a rule schema. An instance is:

Some \( t_1 \) are \( t_2 \)

\[ \begin{array}{c}
\non \non t_1 \non \non t_2
\end{array} \]

Some non non \( t_1 \) are \( t_2 \)
Definitions

We allow ourselves to state definitions that may be convenient. Definitions are similar to axioms; they fix the properties of a particular item for the purpose of a discussion.
We allow ourselves to state definitions that may be convenient. Definitions are similar to axioms; they fix the properties of a particular item for the purpose of a discussion.

**Definition (ImmDef)**

The term *immortal* is considered equal to the term *non mortal*. 
Writing a Proof Graphically

Lemma

*The proposition* All humans are non immortal *holds.*
Lemma

The proposition *All humans are non immortal* holds.

Proof.

\[\begin{align*}
\text{All humans are mortal} & \quad [\text{HM}] \\
\text{All humans are non non mortal} & \quad [\text{NNI}] \\
\text{All humans are non immortal} & \quad [\text{ImmDef}]
\end{align*}\]
Lemma

The proposition *All humans are non immortal* holds.
Writing a Text-based Proof

Lemma

The proposition *All humans are non immortal* holds.

Proof.

1. All humans are mortal  \( \text{HM} \)
2. All humans are non non mortal  \( \text{NNI 1} \)
3. All humans are non immortal  \( \text{ImmDef 2} \)
Conversion switches subject and object

Definition (ConvDef)

For all terms $t_1$ and $t_2$, we define

- $\text{convert}(\text{All } t_1 \text{ are } t_2) = \text{All } t_2 \text{ are } t_1$
- $\text{convert}(\text{Some } t_1 \text{ are } t_2) = \text{Some } t_2 \text{ are } t_1$
- $\text{convert}(\text{No } t_1 \text{ are } t_2) = \text{No } t_2 \text{ are } t_1$
- $\text{convert}(\text{Some } t_1 \text{ are not } t_2) = \text{Some } t_2 \text{ are not } t_1$
Which Conversions Hold?

If

All Greeks are humans

holds in a model, then does

All humans are Greeks

hold?
Axiom (ConvE1)

If, for some terms $t_1$ and $t_2$, the proposition

$$\text{convert}(\text{Some } t_1 \text{ are } t_2)$$

holds, then the proposition

$$\text{Some } t_1 \text{ are } t_2$$

also holds.
Valid Conversions

Axiom (ConvE2)

If, for some terms $t_1$ and $t_2$, the proposition

$$\text{convert}(\text{No } t_1 \text{ are } t_2)$$

holds, then the proposition

$$\text{No } t_1 \text{ are } t_2$$

also holds.
In Graphical Notation

In graphical notation, two rules correspond to the two cases.

\[ \text{convert}(\text{Some } t_1 \text{ are } t_2) \]

\[ \text{Some } t_1 \text{ are } t_2 \]

\[
\text{convert}(\text{No } t_1 \text{ are } t_2) \]

\[ \text{No } t_1 \text{ are } t_2 \]
Example

Axiom (AC)

The proposition *Some animals are cats* holds.
Axiom (AC)

*The proposition* Some animals are cats holds.

Lemma

*The proposition* Some cats are animals holds.
Proof

\[\text{Some animals are cats} \quad [\text{AC}]\]

\[\text{convert}(\text{Some cats are animals}) \quad [\text{ConvDef}]\]

\[\text{Some cats are animals} \quad [\text{ConvE}_{1}]\]
Example (text-based proof)

Proof.

1. Some animals are cats  \[
\text{AC}
\]
2. convert(Some cats are animals)  \[
\text{ConvDef 1}
\]
3. Some cats are animals  \[
\text{ConvE}_1 2
\]
Contraposition switches and complements

Definition (ContrDef)
For all terms $t_1$ and $t_2$, we define

contrapose(All $t_1$ are $t_2$) = All non $t_2$ are non $t_1$
contrapose(Some $t_1$ are $t_2$) = Some non $t_2$ are non $t_1$
contrapose(No $t_1$ are $t_2$) = No non $t_2$ are non $t_1$
contrapose(Some $t_1$ are not $t_2$) = Some non $t_2$ are not non $t_1$
For which propositions is contraposition valid?

\[
\text{contrapose}(\text{All } t_1 \text{ are } t_2) \\
\hline \\
\text{All } t_1 \text{ are } t_2
\]
For which propositions is contraposition valid?

\[ \text{contrapose}(\text{All } t_1 \text{ are } t_2) \]

\[ \text{[ContrE}_1\text{]} \]

All \( t_1 \) are \( t_2 \)

\[ \text{contrapose}(\text{Some } t_1 \text{ are not } t_2) \]

\[ \text{[ContrE}_2\text{]} \]

Some \( t_1 \) are not \( t_2 \)
Obversion switches quality and complements object

**Definition (ObvDef)**

For all terms \( t_1 \) and \( t_2 \), we define

\[
\begin{align*}
\text{obvert}(\text{All } t_1 \text{ are } t_2) & = \text{No } t_1 \text{ are non } t_2 \\
\text{obvert}(\text{Some } t_1 \text{ are } t_2) & = \text{Some } t_1 \text{ are not non } t_2 \\
\text{obvert}(\text{No } t_1 \text{ are } t_2) & = \text{All } t_1 \text{ are non } t_2 \\
\text{obvert}(\text{Some } t_1 \text{ are not } t_2) & = \text{Some } t_1 \text{ are non } t_2
\end{align*}
\]
Examples

Obversion switches quality and complements object
Obversion switches quality and complements object

**Example 1**

\( \text{obvert}(\text{All Greeks are humans}) = \text{No Greeks are non humans} \)
Obversion switches quality and complements object

**Example 1**

\[
\text{obvert}(\text{All Greeks are humans}) = \text{No Greeks are non humans}
\]

**Example 2**

\[
\text{obvert}(\text{Some animals are cats}) = \text{Some animals are not non cats}
\]
Validity of Obversion

Obversion is valid for all kinds of propositions.

Axiom (ObvE)

If, for some proposition \( p \)

\[ \text{obvert}(p) \]

holds, then the proposition \( p \) also holds.
Validity of Obversion

Obversion is valid for all kinds of propositions.

Axiom (ObvE)

If, for some proposition \( p \)

\[
\text{obvert}(p)
\]

holds, then the proposition \( p \) also holds.

\[
\text{obvert}(p) \quad \text{[ObvE]} \quad p
\]
Example

Axiom (SHV)

The proposition *Some humans are vegans* holds.
Example

Axiom (SHV)

The proposition *Some humans are vegans* holds.

Lemma (NNVeg)

The proposition *Some humans are not non vegans* holds.
Proof

\[
\begin{align*}
\text{Some humans are vegans} & \quad \text{[SHV]} \\
\text{Some humans are non non vegans} & \quad \text{[NNI]} \\
\text{obvert(Some humans are not non vegans)} & \quad \text{[ObvDef]} \\
\text{Some humans are not non vegans} & \quad \text{[ObvDef]}
\end{align*}
\]
Proof.

1. Some humans are vegans \(\text{SHV}\)
2. Some humans are non non vegans \(\text{NNI 1}\)
3. obvert(Some humans are not non vegans) \(\text{ObvDef 2}\)
4. Some humans are not non vegans \(\text{ObvE 3}\)
Lemma (SomeNon)

For all terms $t_1$ and $t_2$, if the proposition $\text{Some non } t_1$ are non $t_2$ holds, then the proposition $\text{Some non } t_2$ are not $t_1$ also holds.
Another Lemma

Lemma (SomeNon)

For all terms $t_1$ and $t_2$, if the proposition $\text{Some non } t_1$ are non $t_2$ holds, then the proposition $\text{Some non } t_2$ are not $t_1$ also holds.

A lemma of the form “If $p_1$ then $p_2$” is valid, if in every model in which the proposition $p_1$ holds, the proposition $p_2$ also holds.
Lemma (SomeNon)

For all terms \( t_1 \) and \( t_2 \), if the proposition \( \text{Some non } t_1 \text{ are non } t_2 \) holds, then the proposition \( \text{Some non } t_2 \text{ are not } t_1 \) also holds.
Lemma (SomeNon)

For all terms $t_1$ and $t_2$, if the proposition $\text{Some non } t_1 \text{ are non } t_2$ holds, then the proposition $\text{Some non } t_2 \text{ are not } t_1$ also holds.

Proof.

1. $\text{Some non } t_1 \text{ are non } t_2$  \hspace{1cm} \text{premise}
2. $\text{convert}(\text{Some non } t_2 \text{ are non } t_1)$  \hspace{1cm} \text{ConvDef 1}
3. $\text{Some non } t_2 \text{ are non } t_1$  \hspace{1cm} \text{ConvE 1 2}
4. $\text{obvert}(\text{Some non } t_2 \text{ are not } t_1)$  \hspace{1cm} \text{ObvDef 3}
5. $\text{Some non } t_2 \text{ are not } t_1$  \hspace{1cm} \text{ObvE 4}
“iff” means “if and only if”

Lemma (AllNonNon)

For any terms $t_1$ and $t_2$, the proposition $\text{All non } t_1 \text{ are non } t_2$ holds iff the proposition $\text{All } t_2 \text{ are } t_1$ holds.
Lemma (AllNonNon)

For any terms \( t_1 \) and \( t_2 \), the proposition \( \text{All non } t_1 \text{ are non } t_2 \) holds iff the proposition \( \text{All } t_2 \text{ are } t_1 \) holds.

\[
\begin{align*}
\text{All non } t_1 \text{ are non } t_2 & \\
\text{All } t_2 \text{ are } t_1 & \\
\text{All } t_2 \text{ are } t_1 & \\
\text{All non } t_1 \text{ are non } t_2 &
\end{align*}
\]
Categorical Terms and their Meaning
Propositions, Axioms, Lemmas, Proofs
Manipulating Terms and Propositions
Arguments and Syllogisms

1. Categorical Terms and their Meaning
2. Propositions, Axioms, Lemmas, Proofs
3. Manipulating Terms and Propositions
4. Arguments and Syllogisms
   - Arguments
   - Syllogisms
   - Barbara
   - Fun With Barbara
An argument has the form

If *premises* then *conclusion*
Argument

An argument has the form

If *premises* then *conclusion*

Sometimes also

*premises* therefore *conclusion*
Argument

An argument has the form

If *premises* then *conclusion*

Sometimes also

*premises* therefore *conclusion*

Example:

Lemma (SomeNon)

*For all terms* $t_1$ *and* $t_2$, *if the proposition* Some non $t_1$ are non $t_2$ holds, *then the proposition* Some non $t_2$ are not $t_1$ also holds.
Syllogisms

A syllogism is an argument with two premises, in which three different terms occur, and in which every term occurs twice, but never twice in the same proposition.

Example

All cats are predators.
Some animals are cats.
Therefore, all animals are predators.
Axiom (B)

For all terms minor, middle, and major, if All middle are major holds, and All minor are middle holds, then All minor are major also holds.

All middle are major      All minor are middle

---------------------------------[B]

All minor are major
Why is Barbara valid?

If the first premise holds, then areas 1 and 4 are empty, and if the second premise holds, then areas 2 and 3 are empty. The conclusion simply states that areas 1 and 2 are empty, which clearly follows from the premises, regardless what other properties the model under consideration has.
Lemma

The proposition *All Greeks are mortal* holds.
Example

Lemma

The proposition *All Greeks are mortal* holds.

Proof.

1. All Greeks are humans \(GH\)
2. All humans are mortal \(HM\)
3. All Greeks are mortal \(B\ 1,2\)
Premises

- No ducks waltz.
- No officers ever decline to waltz.
- All my poultry are ducks.
Officers as Poultry?

Premises
- No ducks waltz.
- No officers ever decline to waltz.
- All my poultry are ducks.

Conclusion
- No officers are my poultry.
Lemma (No-Officers-Are-My-Poutry)

If

- No ducks are things-that-waltz holds,
- No officers are non things-that-waltz holds,
  and
- All my-poultry are ducks holds,

then No officers are my-poultry also holds.
Proof

1. No officers are non things-that-waltz
   premise
2. obvert (All officers are things-that-waltz)  ObvDef 1
3. All officers are things-that-waltz  ObvE 2
4. No ducks are things-that-waltz  premise
5. convert (No things-that-waltz are ducks)  ConvDef 4
6. No things-that-waltz are ducks  ConvE₂ 5
<table>
<thead>
<tr>
<th></th>
<th>Proof (continued)</th>
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<tbody>
<tr>
<td>7</td>
<td>No things-that-waltz are non non ducks</td>
</tr>
<tr>
<td>8</td>
<td>obvert(All things-that-waltz are non ducks)</td>
</tr>
<tr>
<td>9</td>
<td>All things-that-waltz are non ducks</td>
</tr>
<tr>
<td>10</td>
<td>All my-poultry are ducks</td>
</tr>
<tr>
<td>11</td>
<td>All my-poultry are non non ducks</td>
</tr>
<tr>
<td>12</td>
<td>All non non my-poultry are non non ducks</td>
</tr>
<tr>
<td>Step</td>
<td>Statement</td>
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<tr>
<td>13</td>
<td>( \text{Contrapose (All non ducks are non my-poultry)} )</td>
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<tr>
<td>14</td>
<td>( \text{All non ducks are non my-poultry} )</td>
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<tr>
<td>15</td>
<td>( \text{All things-that-waltz are non my-poultry} )</td>
</tr>
<tr>
<td>16</td>
<td>( \text{All officers are non my-poultry} )</td>
</tr>
<tr>
<td>17</td>
<td>( \text{Obvert (No officers are my-poultry)} )</td>
</tr>
<tr>
<td>18</td>
<td>( \text{No officers are my-poultry} )</td>
</tr>
</tbody>
</table>
Assignment 1: out on module homepage; due 26/8, 11:00am
Coq Homework 1: out on module homepage; due 27/8, 9:30pm
Monday, Wednesday: Office hours
Tuesday: Tutorials (clarification of assignment)
Wednesday: Labs (Coq Homework 1; start earlier!)