National University of Singapore
School of Computing
CS3243: Introduction to Artificial Intelligence
Tutorial 5

Readings: AIMA Chapter 7

1. Determine using a truth table whether the following sentence is valid, satisfiable, or unsatisfiable:
   (a) $(P \wedge Q) \vee \neg Q$
   (b) $((P \wedge Q) \Rightarrow R) \Leftrightarrow ((P \Rightarrow R) \vee (Q \Rightarrow R))$
2. Assume that a knowledge base KB contains the following rules:

$$\begin{aligned} poor &\Rightarrow \neg worried \\ rich &\Rightarrow scared \\ \neg rich &\Rightarrow poor \end{aligned}$$

   (a) Show that $KB \models (worried \Rightarrow scared)$, using the model checking approach.
   (b) Use resolution to prove $KB \models (worried \Rightarrow scared)$.
3. Someone says: "On either Saturday or Sunday, if I am free, I will go to the concert". Using propositional logic, the statement is represented as:

$$(saturday \vee sunday) \Rightarrow (free \Rightarrow concert)$$

   Convert the above sentence into conjunctive normal form, and then into Horn form, by using the logical equivalences shown in Table 1.

Table 1: Table of logical equivalences

$(\alpha \wedge \beta)$	$\equiv$	$(\beta \wedge \alpha)$	commutativity of $\wedge$
$(\alpha \vee \beta)$	$\equiv$	$(\beta \vee \alpha)$	commutativity of $\vee$
$((\alpha \wedge \beta) \wedge \gamma)$	$\equiv$	$(\alpha \wedge (\beta \wedge \gamma))$	associativity of $\wedge$
$((\alpha \vee \beta) \vee \gamma)$	$\equiv$	$(\alpha \vee (\beta \vee \gamma))$	associativity of $\vee$
$\neg(\neg\alpha)$	$\equiv$	α	double-negation elimination
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta)$	$\equiv$	$(\neg\alpha \vee \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta)$	$\equiv$	$((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \wedge \beta)$	$\equiv$	$(\neg\alpha \vee \neg\beta)$	De Morgan
$\neg(\alpha \vee \beta)$	$\equiv$	$(\neg\alpha \wedge \neg\beta)$	De Morgan
$(\alpha \wedge (\beta \vee \gamma))$	$\equiv$	$((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$	distributivity of $\wedge$ over $\vee$
$(\alpha \vee (\beta \wedge \gamma))$	$\equiv$	$((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$	distributivity of $\vee$ over $\wedge$

4. (Question 7.2 from AIMA) (Adapted from Barwise and Etchemendy (1993).) Given the following, can you prove that the unicorn is mythical? How about magical? Horned?

If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.

5. (Question 7.8 from AIMA) We have defined four different binary logical connectives (namely $\wedge, \vee, \Rightarrow, \Leftrightarrow$).
   (a) Are there any others that might be useful?
   (b) How many binary connectives can there be?
   (c) Why are some of them not very useful?