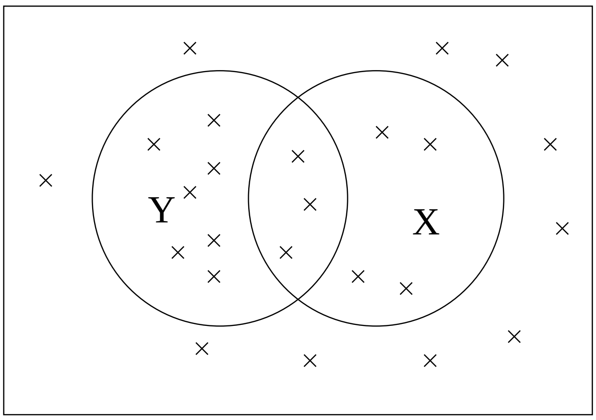
National University of Singapore
School of Computing
CS3243: Introduction to Artificial Intelligence
Tutorial 8

Readings: AIMA Chapter 13 & 14 (Sections 14.1 – 14.2)

1. Based on the following Venn diagram, complete the joint probability distribution in the table on its right.



	X	$\neg X$
Y	$\frac{3}{24}$	
$\neg Y$		

Based on the joint probability distribution, find the following: $P(X)$, $P(Y)$, $P(\neg X)$, $P(\neg Y)$, $P(X|Y)$, $P(Y|X)$, $P(X|\neg Y)$, $P(\neg Y|X)$, $P(\neg X|Y)$, $P(Y|\neg X)$, $P(\neg X|\neg Y)$ and $P(\neg Y|\neg X)$. Substituting the values of these conditional probabilities, verify the following:

$$\begin{aligned} P(X|Y) &= 1 - P(\neg X|Y) \\ P(X|\neg Y) &= 1 - P(\neg X|\neg Y) \\ P(\neg Y|X) &= \frac{P(X|\neg Y)P(\neg Y)}{P(X|\neg Y)P(\neg Y) + P(X|Y)P(Y)} \end{aligned}$$

2. (Based on CS5340 lecture notes) Consider a healthcare diagnosis system where symptoms are matched to illnesses. For example, a patient that has flu is very likely to have fever, but not always. So for a patient that has fever, then it is likely that he has flu. However, if it is determined with 100% certainty that the patient does not have flu, then it is likely that he has a cold, but it is possible that he has some other illness instead.

   Discuss the limitations of first order logic that makes it unsuitable for the inference engine of this diagnosis system. Suggest a replacement method for implementing this inference engine.

3. Assume that 2% of the population in a country carry a particular virus. A test kit developed by a pharmaceutical firm is able to detect the presence of the virus from a patient's blood sample. The firm claims that the test kit has a high accuracy of detection in terms of the following conditional probabilities obtained from their quality control testing:

   P(the kit shows positive | the patient is a carrier) = 0.998
   P(the kit shows negative | the patient is not a carrier) = 0.996

   If a patient is tested to be positive using this kit, what is the likelihood of a false positive (i.e., that he actually is not a carrier but the kit shows positive)?

4. (Question 13.1 from AIMA) Show from first principles that

$$P(a|a \wedge b) = 1$$

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5. (Question 13.12 from AIMA) Show that the three forms of independence below:

   (a) $P(a|b) = P(a)$

   (b) $P(b|a) = P(b)$

   (c) $P(a \wedge b) = P(a)P(b)$

   are equivalent.

6. Assume that the following conditional probabilities are available:

P(Wet_Grass | Sprinkler ∧ Rain)	0.95
P(Wet_Grass | Sprinkler ∧ ¬ Rain)	0.9
P(Wet_Grass | ¬ Sprinkler ∧ Rain)	0.8
P(Wet_Grass | ¬ Sprinkler ∧ ¬ Rain)	0.1
P(Sprinkler | Rainy_Season)	0.0
P(Sprinkler | ¬ Rainy_Season)	1.0
P(Rain | Rainy_Season)	0.9
P(Rain | ¬ Rainy_Season)	0.1
P(Rainy_Season)	0.7

   Construct a Bayesian network and determine the probability

$$P(\text{Wet\_Grass} \wedge \text{Rainy\_Season} \wedge \neg\ \text{Rain} \wedge \neg\ \text{Sprinkler}).$$

7. An expert system called PROSPECTOR for use in geological exploration makes use of an inference mechanism similar to a Bayesian network. The following are two modified versions of its rule patterns:

   If E1
   Then H1 (P(H1 | E1), P(H1 | ¬E1))

   If E2
   and E3
   Then H2 (P(H2 | E2∧ E3), P(H2 | E2∧ ¬E3), P(H2 | ¬E2∧ E3), P(H2 | ¬E2 ∧ ¬E3))

   The following is a hypothetical set of PROSPECTOR's rules (where we also use two letters to represent propositions for your easy working later)

   If the igneous rocks in the region have a fine to medium grain size (Gr)
   Then they have a porphyritic texture (Tx) (0.6, 0.2)

   If the igneous rocks in the region have a fine to medium grain size (Gr)
   and they have a porphyritic texture (Tx)

Then the region is a hypabyssal environment (Hy) (0.88, 0.76, 0.52, 0.02)

If the region is a hypabyssal environment (Hy)
Then the region has a favourable level of erosion (Er) (0.75, 0.12)

If the region has a favourable level of erosion (Er)
Then the region is favourable for copper deposits (Cu) (0.92, 0.03)

Construct a Bayesian network based on the above rules. Assume that a geologist could only ascertain with probability 0.15 that a region's igneous rocks have a fine to medium grain size. What is the probability that this region is favourable for copper deposits and has a favourable level of erosion, given that the region (1) has large grain size igneous rocks, (2) has non-porphyritic texture rocks, and (3) is a hypabyssal environment.