

This course material is now made available for public usage.
Special acknowledgement to School of Computing, National University of Singapore
for allowing Steven to prepare and distribute these teaching materials.



CS3233

Competitive Programming

Dr. Steven Halim

Week 03 – Problem Solving Paradigms
(Focus on Complete Search)

Outline

- Mini Contest #2 + Break + Discussion + Admins
- Complete Search
 - **Iterative: (Nested) Loops, Permutations, Subsets**
 - **Recursive Backtracking (N Queens), from easy to (very) hard**
 - **State-Space Search**
 - **Meet in the Middle (Bidirectional Search)**
- Read at home (will not be tested in mini contest A/B):
 - Some tips to speed up your solution
 - Greedy algorithms
 - Divide and Conquer (D&C) algorithms
 - Especially Binary Search the Answer technique

Iterative: (Nested) Loops, Permutations, Subsets

Recursive Backtracking (N Queens), from easy to (very) hard

State-Space Search

Meet in the Middle (Bidirectional Search)

COMPLETE SEARCH

Iterative Complete Search

Loops (1)

- UVa 725 – Division
 - Find two 5-digits number s.t. $\rightarrow \mathbf{abcde} / \mathbf{fghij} = N$
 - $\mathbf{abcdefg hij}$ must be all different, $2 \leq N \leq 79$
- Iterative Complete Search Solution (Nested Loops):
 - Try all possible \mathbf{fghij} (one loop)
 - Obtain \mathbf{abcde} from $\mathbf{fghij} * N$
 - Check if $\mathbf{abcdefg hij}$ are all different (*another* loop)

Iterative Complete Search Loops (2)

- More challenging variants:
 - 2-3-4-...-K nested loops
 - Some pruning are possible,
e.g. using “continue”, “break”, or if-statements

Iterative Complete Search

Permutations

- UVa 11742 – Social Constraints
 - There are $0 < n \leq 8$ movie goers
 - They will sit in the front row with n consecutive open seats
 - There are $0 \leq m \leq 20$ seating constraints among them, i.e. a and b must be at most (or at least) c seats apart
 - How many possible seating arrangements are there?
- Iterative Complete Search Solution (Permutations):
 - Set counter = 0 and then try all possible $n!$ **permutations**
 - Increase counter if a permutation satisfies all m constraints
 - Output the final value of counter

Iterative Complete Search Subsets

- UVa 12346 – Water Gate Management
 - A dam has $1 \leq n \leq 20$ water gates to let out water when necessary, each water gate has **flow rate** and **damage cost**
 - Your task is to manage the opening of the water gates in order to get rid of *at least* the specified **total flow rate** condition that the **total damage cost** is minimized!
- Iterative Complete Search Solution (Subsets):
 - Try all possible 2^n subsets of water gates to be opened
 - For each subset, check if it has sufficient flow rate
 - If it is, check if the total damage cost of this subset is smaller than the overall minimum damage cost so far
 - If it is, update the overall minimum damage cost so far
 - Output the minimum damage cost

Iterative: (Nested) Loops, Permutations, Subsets

Recursive Backtracking (N Queens), from easy to (very) hard

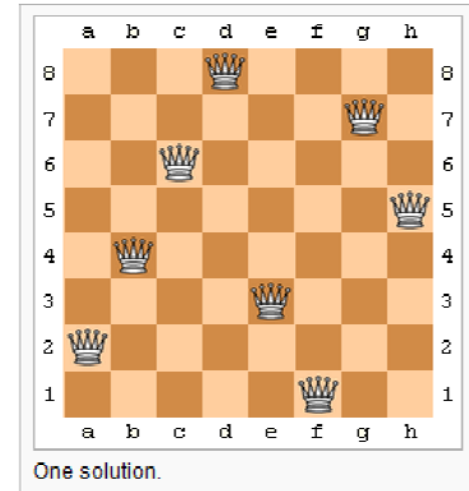
State-Space Search

Meet in the Middle (Bidirectional Search)

COMPLETE SEARCH

Recursive Backtracking (1)

- UVa 750 – 8 Queens Chess Problem
 - Put 8 queens in 8x8 Chessboard
 - No queen can attack other queens
- Naïve ways (Time Limit Exceeded)
 - Choose 8 out of 64 cells...
 - ${}_{64}C_8 = 4$ Billion possibilities... 😞
 - Insight 1: Put one queen in each column...
 - $8^8 = 17$ Million possibilities... :O



Recursive Backtracking (2)

- Better way, recursive backtracking
 - Insight 2: **all-different constraint** for the rows too
 - We put one queen in each column **AND each row**
 - Finding a valid permutation out of $8!$ possible permutations...
 - Search space goes down from $8^8 = 17\text{M}$ to $8! = 40\text{K}$!
 - Insight 3: main diagonal and secondary diagonal check
 - Another way to prune the search space
 - Queen A (i, j) attacks Queen B (k, l) iff
$$\text{abs}(i - k) == \text{abs}(j - l)$$
- Scrutinize the sample code of recursive backtracking!

Important Code (3)

```
int rw[8], TC, a, b, lineCounter;           // ok to use global variables

bool place(int r, int c) {
    for (int prev = 0; prev < c; prev++)    // check previously placed queens
        if (rw[prev] == r || (abs(rw[prev] - r) == abs(prev - c)))
            return false;                   // share same row or same diagonal -> infeasible
    return true; }

void backtrack(int c) {
    if (c == 8 && rw[b] == a) {             // candidate sol, (a, b) has 1 queen
        printf("%2d      %d", ++lineCounter, rw[0] + 1);
        for (int j = 1; j < 8; j++) printf(" %d", rw[j] + 1);
        printf("\n"); }
    for (int r = 0; r < 8; r++)             // try all possible row
        if (place(r, c)) {                  // if can place a queen at this col and row
            rw[c] = r; backtrack(c + 1);    // put this queen here and recurse
        } }
```

Is that the best n-Queens solution?

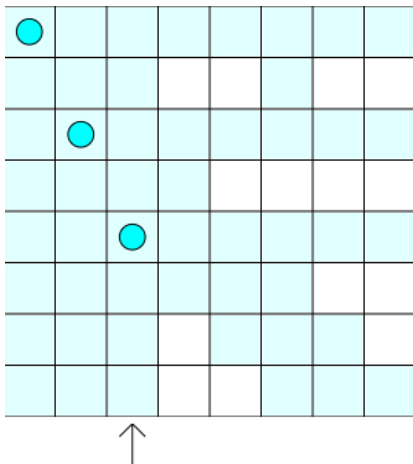
- Maybe not
 - See [UVa 11195 – Another n-Queen Problem](#)
- Several cells are forbidden
 - Do this helps?
- n can now be as large as n=14 :O??
 - How to run 14! algorithm in a few seconds?

Speeding Up Diagonal Checks

- This check is slow:

```
bool place(int r, int c) {  
    for (int prev = 0; prev < c; prev++)    // check previously placed queens  
        if (rw[prev] == r || (abs(rw[prev] - r) == abs(prev - c)))  
            return false;                // share same row or same diagonal -> infeasible  
    return true; }  
}
```

- We can speed up this part by using $2*n-1$ boolean arrays (or bitset) to test if a certain left/right diagonal can be used



<http://www.comp.nus.edu.sg/~stevenha/visualization/recursion.html>

Is that enough?

- Unfortunately no
- But fortunately there is a better way of using diagonal checks 😊



Iterative: (Nested) Loops, Permutations, Subsets

Recursive Backtracking (N Queens), from easy to (very) hard

State-Space Search

Meet in the Middle (Bidirectional Search)

COMPLETE SEARCH

UVa 11212 – Editing a Book

Rujia Liu's Problem

- Given n equal-length paragraphs numbered from 1 to n
- Arrange them in the order of 1, 2, ..., n
- With the help of a clipboard, you can press Ctrl-X (cut) and Ctrl-V (paste) several times
 - You cannot cut twice before pasting, but you can cut several contiguous paragraphs at the same time - they'll be pasted in order
- The question: What is the minimum number of steps required?
- Example 1: In order to make {2, 4, (1), 5, 3, 6} sorted, you can cut 1 and paste it before 2 \rightarrow {1, 2, 4, 5, (3), 6} then cut 3 and paste it before 4 \rightarrow {1, 2, 3, 4, 5, 6} \rightarrow done \checkmark
- Example 2: In order to make {(3, 4, 5), 1, 2} sorted, you can cut {3, 4, 5} and paste it after {1, 2} \rightarrow {1, 2, 3, 4, 5} \checkmark or cut {1, 2} and paste it before {3, 4, 5} \rightarrow {1, 2, 3, 4, 5} \checkmark

Loose Upper Bound

- Answer: $k-1$
 - Where k is the number of paragraphs initially the wrong positions
- Trivial but wrong algorithm:
 - Cut a paragraph that is in the wrong position
 - Paste that paragraph in the correct position
 - After $k-1$ such cut-paste, we will have a sorted paragraph
 - The last wrong position will be in the correct position at this stage
 - But this may not be the shortest way
- Examples:
 - $\{(3), 2, 1\} \rightarrow \{(2), 1, \underline{3}\} \rightarrow \{1, \underline{2}, 3\} \rightarrow 2$ steps
 - $\{(5), 4, 3, 2, 1\} \rightarrow \{(4), 3, 2, 1, \underline{5}\} \rightarrow \{(3), 2, 1, \underline{4}, 5\} \rightarrow \{(2), 1, \underline{3}, 4, 5\} \rightarrow \{1, \underline{2}, 3, 4, 5\} \rightarrow 4$ steps

The Actual Answers

- {3, 2, 1}
 - Answer: 2 steps, e.g.
 - {(3), 2, 1} → {(2), 1, **3**} → {1, **2**, 3}, or
 - {3, 2, (1)} → {**1**, (3), 2,} → {1, 2, **3**}
- {5, 4, 3, 2, 1}
 - Answer: Only **3** steps, e.g.
 - {5, 4, (3, 2), 1} → {**3**, (**2**, 5), 4, 1} → {3, 4, (1, **2**), **5**} → {**1**, **2**, 3, 4, 5}
- How about {5, 4, 9, 8, 7, 3, 2, 1, 6}?
 - Answer: 4, but very hard to compute manually
- How about {9, 8, 7, 6, 5, 4, 3, 2, 1}?
 - Answer: 5, but very hard to compute manually

Some Analysis

- There are at most $n!$ permutations of paragraphs
 - With maximum $n = 9$, this is $9!$ or 362880
 - The number of vertices is not that big actually
- Given a permutation of length n (a vertex)
 - There are $\binom{n}{2}$ possible cutting points (index $i, j \in [1..n]$)
 - There are n possible pasting points (index $k \in [1..(n-(j-i+1))]$)
 - Therefore, for each vertex, there are about $O(n^3)$ branches
- The worst case behavior if we run a single BFS on this State-Space graph is: $O(V+E) = O(n! + n! * n^3) = O(n! * n^3)$
 - With $n = 9$, this is $9! * 9^3 = 264539520 \sim 265 \text{ M}$, TLE (or maybe MLE...)

Iterative: (Nested) Loops, Permutations, Subsets

Recursive Backtracking (N Queens), from easy to (very) hard

State-Space Search

Meet in the Middle (Bidirectional Search)

COMPLETE SEARCH



More Search Algorithms...

- Depth Limited Search (DLS) + Iterative DLS
- A* / Iterative Deepening A* (IDA*) / Memory Bounded A*
- Branch and Bound (BnB)
- Maybe in Week12 😊 or ...
 - We will not test any of these in mini contests problem A/B

Summary

- We have seen some Complete Search techniques...
 - There are (several) others...
 - We still need lots of practice though 😊
- We “skipped” Greedy and Divide & Conquer this time
 - Read Section 3.3-3.4 by yourself
 - We will not test these on mini contests problem A/B
- Next week, we will see (revisit) the fourth paradigm:
 - Dynamic Programming

References

- **Competitive Programming 2.9**, Section 3.2 + 8.2
 - Steven, Felix 😊