CS3233
Competitive Programming

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Week 10 – String Processing

Outline

- Mini Contest #8 + Discussion + Break + Admins
- Covered *briefly* in class but *indirectly* examinable:
  - Basic String Processing Skills
- Skipped this semester (use this skill to solve more UVas):
  - Ad Hoc String Problems
  - String Matching (Knuth-Morris-Pratt's Algorithm)
- Today, focus on:
  - Suffix Trie/Tree/Array
- Note: DP on String has been discussed in Week 04-05

Section 6.2

BASIC STRING PROCESSING SKILLS

Basics of String Processing (01)

Data Structure

C (top)/C++ (bottom)

- C: **null-terminated character array**
  - We have to know the string length (or at least the upperbound) beforehand

```
char str[10000];
```

- C++: **string** class

```
#include <string>
using namespace std;
string str;
```

Java

- **String class**

```
String str;
```

Basics of String Processing (02)

Reading a String (**a word**)

C (top)/C++ (bottom)

```
#include <stdio.h>

scanf("%s", &str);
  // & optional
```

```
#include <iostream>
using namespace std;

cin >> str;
```

Java

```
import java.util.*;

Scanner sc = new
  Scanner(System.in);
str = sc.next();
```

Basics of String Processing (03)

Reading a **Line** of String

C (top)/C++ (bottom)

```
gets(str);
// alternative/safer version
// fgets(str, 10000, stdin);
// but you will read extra
// '\0' at the back
// PS: Mooshak prefer fgets
```

```
getline(cin, str);
```

Java

```
str = sc.nextLine();
```

Basics of String Processing (04)
Printing and Formatting String Output

C (top)/C++ (bottom)

- Preferred method ☺

```
printf("s = %s, l = %d\n",
  str, (int)strlen(str));
```

- C++ version is harder ☹

```
cout << "s = " << str <<
  ", l = " << str.length()
  << endl;
```

Java

- We can use **`System.out.print`** or **`System.out.println`**, but the best is to use C-style **`System.out.printf`**

```
System.out.printf(
  "s = %s, l = %d\n",
  str, str.length());
```

Basics of String Processing (05)
Comparing Two Strings

C (top)/C++ (bottom)

```
printf(strcmp(str, "test") ?
  "different\n" :
  "same\n" );
```

```
cout << str == "test" ?
  "same" :
  "different" << endl;
```

Java

```
System.out.println(
  str.equals("test"));
```

Basics of String Processing (06)
Combining Two Strings

C (top)/C++ (bottom)

```c
strcpy(str, "hello");
strcat(str, " world");
printf("%s\n", str);
// output: "hello world"
```

```cpp
str = "hello";
str.append(" world");
cout << str << endl;
// output: "hello world"
```

Java

```java
str = "hello";
str += " world";
System.out.println(str);
// output: "hello world"
```

Basics of String Processing (07)

String Tokenizer: Splitting Str into Tokens

C (top)/C++ (bottom)

```
#include <string.h>
for (char *p=strtok(str, " ");
  p;
  p = strtok(NULL, " "))
    printf("%s\n", p);
```

```
#include <sstream>
stringstream p(str);
while (!p.eof()) {
  string token;
  p >> token;
  cout << token << endl;
}
```

Java

```
import java.util.*;

StringTokenizer st = new
  StringTokenizer(str, " ");
while (st.hasMoreTokens())
  System.out.println(
    st.nextToken());
```

Basics of String Processing (08)

String Matching: Finding a Substr in a Str

C (top)/C++ (bottom)

```
char *p=strstr(str, substr);
if (p)
  printf("%d\n", p-str-1);
```

```
int pos = str.find(substr);
if (pos != string::npos)
  cout << pos - 1 << endl;
```

Java

```
int pos=str.indexOf(substr);
if (pos != -1)
  System.out.println(pos);
```

Basics of String Processing (09)
Editing/Examining Characters of a String

Both C & C++

```
#include <ctype.h>

for (int i = 0; str[i]; i++)
  str[i] = toupper(str[i]);
  // or tolower(ch)
  // isalpha(ch),isdigit(ch)
```

Java

- Characters of a Java `String` can be accessed with `str.charAt(i)`, but Java `String` is immutable (cannot be changed)
- You may have to create new String or use Java `StringBuffer`

Basics of String Processing (10)

Sorting Characters of a String

Both C & C++

```
#include <algorithm>

// if using C-style string
sort(s, s + (int)strlen(s));

// if using C++ string class
sort(s.begin(), s.end());
```

Java

- Java `String` is immutable (cannot be changed)
- You have to break the string `toCharArray()` and then sort the character array

Basics of String Processing (11)

Sorting Array/Vector of Strings

Preferably C++

```cpp
#include <algorithm>
#include <string>
#include <vector>

vector<string> S;
// assume that S has items
sort(S.begin(), S.end());
// Swill be sorted now
```

Java

```java
Vector<String> S =
  new Vector<String>();
// assume that S has items
Collections.sort(S);
// S will be sorted now
```

List of (simple) problems solvable with basic string processing skills

Section 6.3

Just a splash and dash for this semester

(do a few programming exercises on your own)

AD HOC STRING PROBLEMS

Ad Hoc String Problems (1)

- Cipher (Encode-Encrypt/Decode-Decrypt)
  - Transform string given a coding/decoding mechanism
  - Usually, we need to follow problem description
  - Sometimes, we have to guess the pattern
  - UVa 10878 – Decode the Tape
- Frequency Counting
  - Check how many times certain characters (or words) appear in the string
  - Use efficient data structure (or hashing technique)
  - UVa 902 – Password Search

Ad Hoc String Problems (2)

- Input Parsing
  – Given a grammar (in Backus Naur Form or in other form), check if a given string is valid according to the grammar, and evaluate it if possible
  – Use **recursive** parser, Java **Pattern (RegEx)** class
  – UVa 622 – Grammar Evaluation
- Output Formatting
  – The problematic part of the problem is in formatting the output using certain rule
  – UVa 10894 – Save Hridoy

Ad Hoc String Problems (3)

- String Comparison
  - Given two strings, are they similar with some criteria?
  - Case sensitive? Compare substring only? Modified criteria?
  - UVa 11233 – Deli Deli
- Others, not one of the above
  - But still solvable with just basic string processing skills
- Note:
  - None of these are likely appear in IOI other than as the bonus problem per contest day (no longer true in 2011)
  - In ICPC, one of these can be the bonus problem

Knuth-Morris-Pratt’s Algorithm

Section 6.4

Skipped this semester (please use Suffix Array for (long) String Matching)

STRING MATCHING

String Matching

- Given a pattern string P,
  can it be found in the longer string T?
  - Do not code naïve solution
  - Easiest solution: Use string library
    - C++: string.find
    - C: strstr
    - Java: String.indexOf
  - In CP2.9 book: KMP algorithm
  - Or later/after this: Suffix Array

The earlier form of this teaching material is credited to
A/P Sung Wing Kin, Ken from SoC, NUS

CP2.9 Section 6.6

SUFFIX TRIE, TREE, AND ARRAY

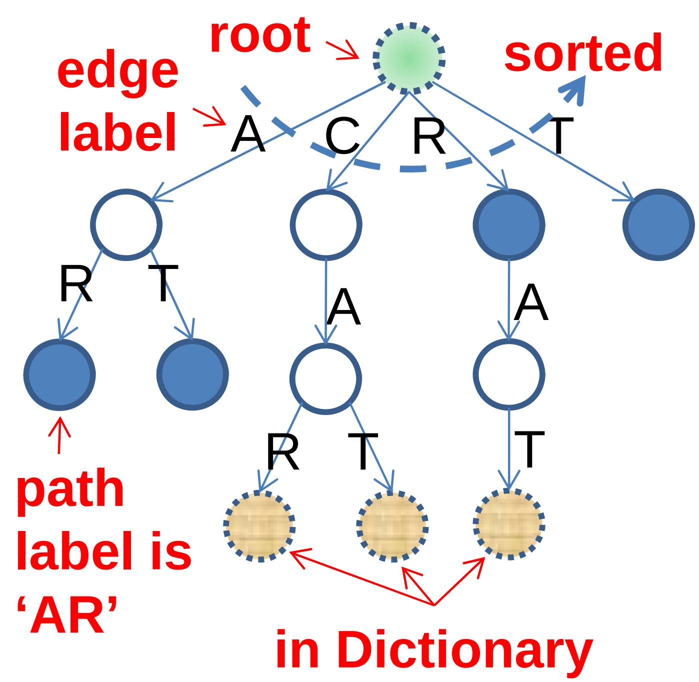
Suffix Trie ('CAR', 'CAT', 'RAT')

All Suffixes:

1. CAR
2. AR
3. R
4. CAT
5. AT
6. T
7. RAT
8. AT
9. T

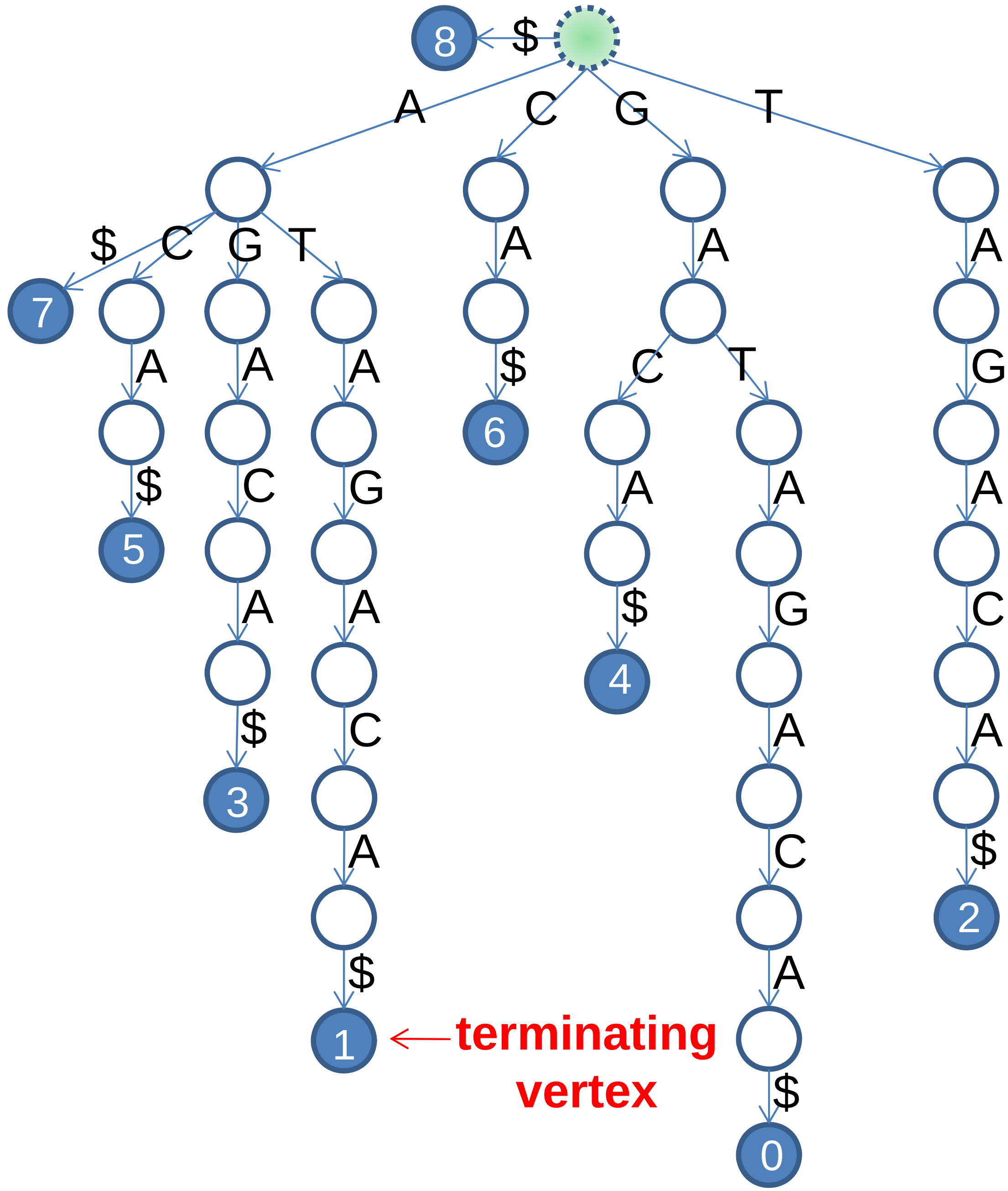
Sorted Unique Suffixes:

1. AR
2. AT
3. CAR
4. CAT
5. R
6. RAT
7. T



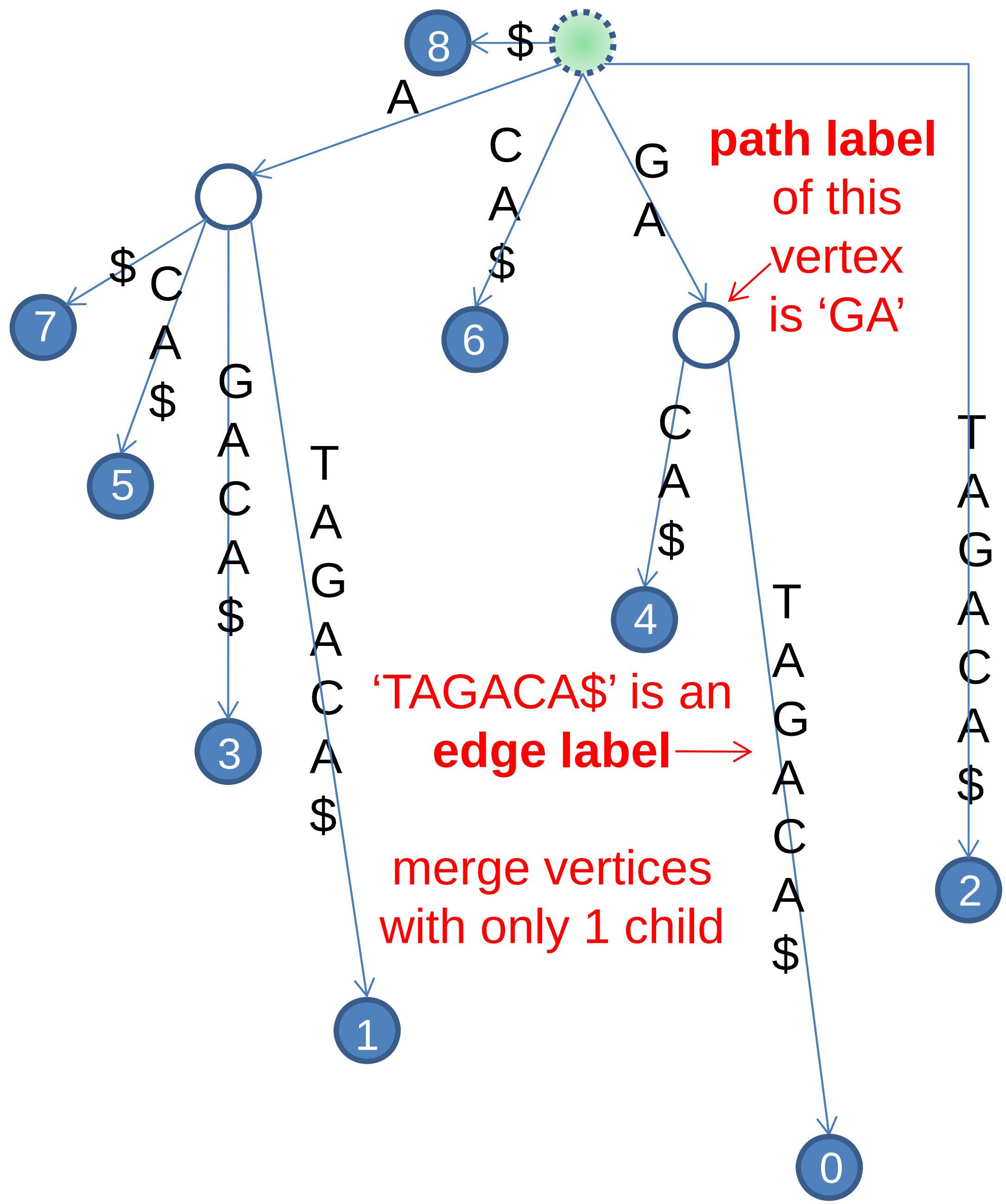
Suffix Trie (T = ‘GATAGACA$’)

i	Suffix
0	GATAGACA$
1	ATAGACA$
2	TAGACA$
3	AGACA$
4	GACA$
5	ACA$
6	CA$
7	A$
8	$



Suffix Tree (T = 'GATAGACA$')

i	Suffix
0	GATAGACA$
1	ATAGACA$
2	TAGACA$
3	AGACA$
4	GACA$
5	ACA$
6	CA$
7	A$
8	$

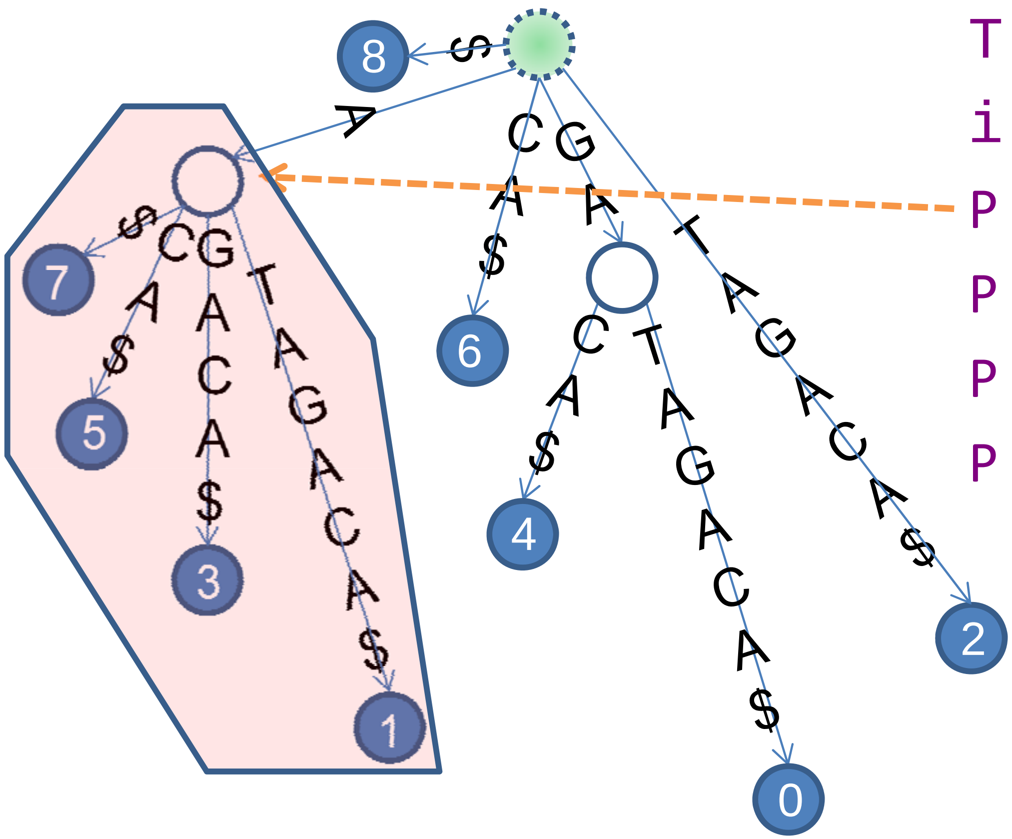


What can we do with this specialized string data structure?

APPLICATIONS OF SUFFIX TREE

String Matching

- To find all occurrences of **P** (of length m) in **T** (of length n)
  - Search for the vertex **x** in the Suffix Tree which represents **P**
  - All the leaves in the subtree rooted at x are the occurrences
- Time: O(**m** + occ) where occ is the total no. of occurrences



`T = 'GATAGACA$'`

`i = '012345678'`

P = 'A' → Occurrences: 7, 5, 3, 1

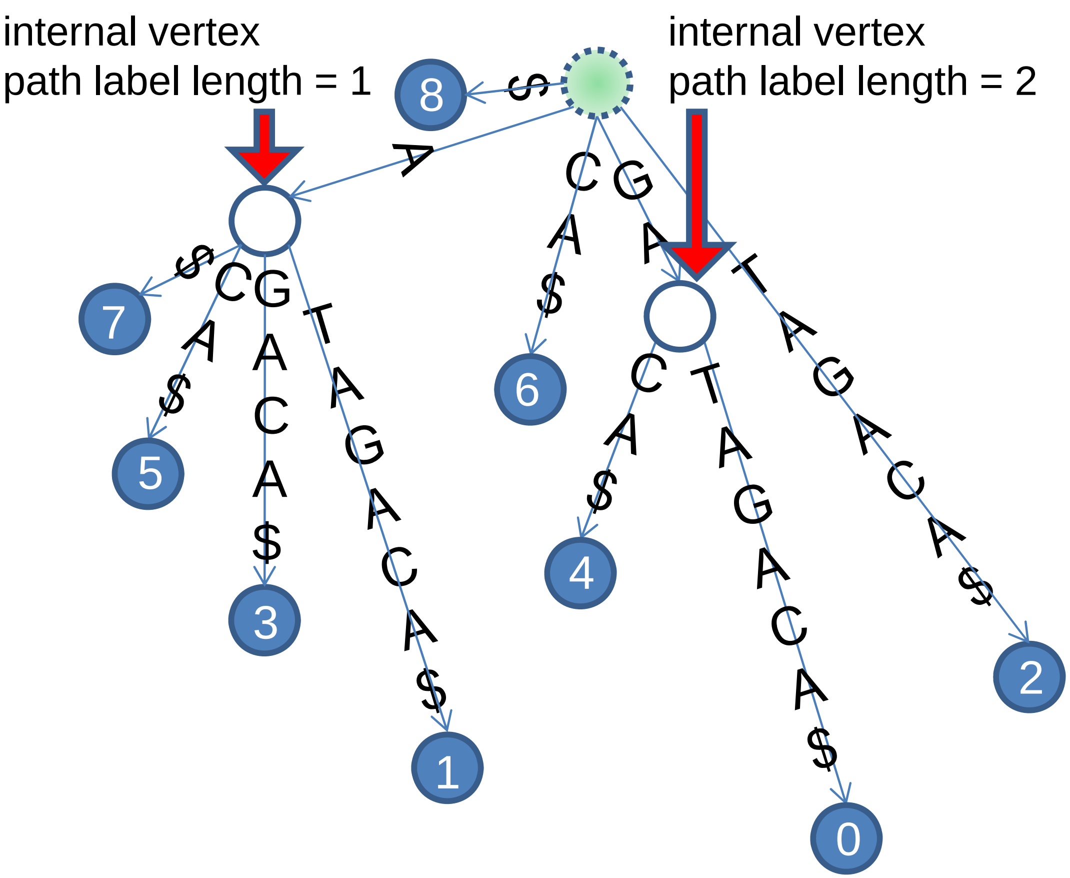
P = 'GA' → Occurrences: 4, 0

P = 'T' → Occurrences: 2

P = 'Z' → Not Found

Longest Repeated Substring

- To find the longest repeated substring in T
  - Find the deepest internal node
- Time: O(n)



e.g. T = ‘GATAGACA$’

The longest repeated substring is ‘GA’ with path label length = 2

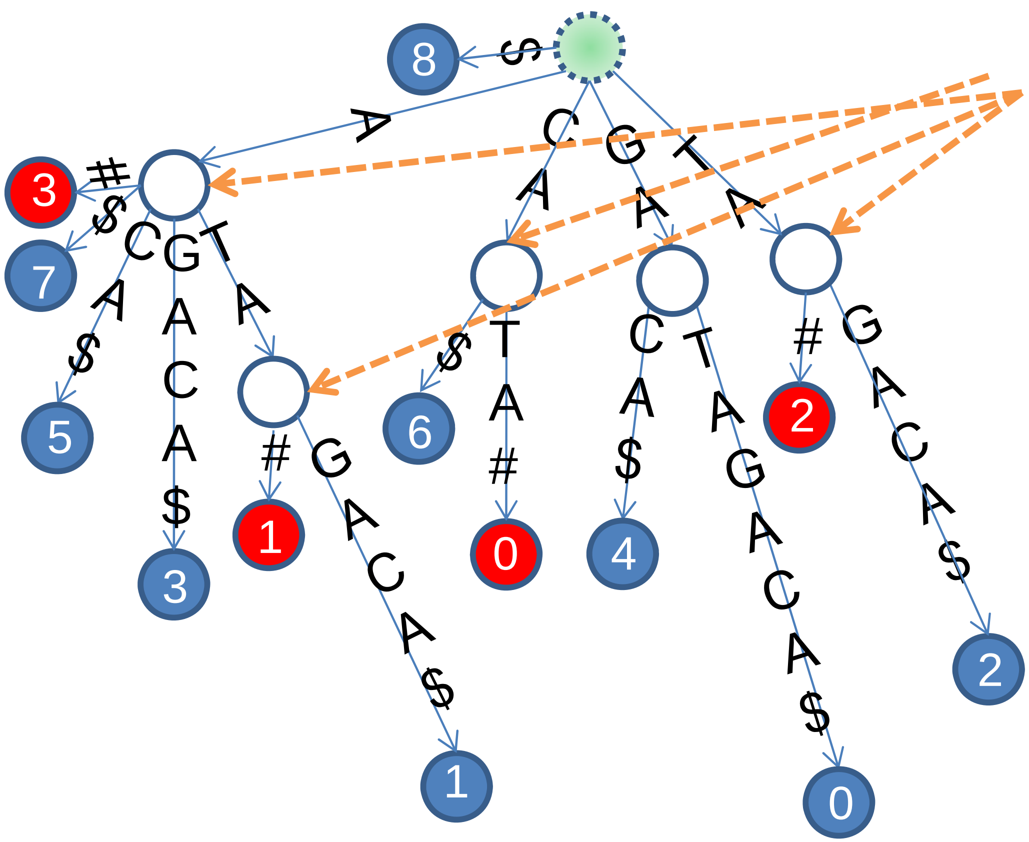
The other repeated substring is ‘A’, but its path label length = 1

Longest Common Substring

- To find the longest common substring of two or more strings
  - Note: In 1970, Donald Knuth conjectured that a linear time algorithm for this problem is impossible
  - Now, we know that it can be solved in linear time
  - E.g. consider two string T1 and T2,
    - Build a **generalized** Suffix Tree for T1 and T2
      - i.e. a Suffix Tree that combines both the Suffix Tree of T1 and T2
    - Mark internal vertices with leaves representing suffixes of both T1 and T2
    - Report the deepest marked vertex

Example of LC Substring

- T1 = ‘GATAGACA$’ (end vertices labeled with blue)
  T2 = ‘CATA#’ (end vertices labeled with red)
  - Their longest common substring is ‘ATA’ with length 3



These are the internal vertices representing suffixes from <u>both strings</u>

The deepest one has path label ‘ATA’

How to build Suffix Tree?

For programming contests, we use Suffix Array instead…

SUFFIX ARRAY

Disadvantage of Suffix Tree

- Suffix Tree is space inefficient
  - It requires $O(n|\Sigma|\log n)$ bits
    - N nodes, each node has $|\Sigma|$ branches, each pointer needs $O(\log n)$ bits
- Actual reason for programming contests
  - It is harder to construct Suffix Tree
- Manber and Myers (SIAM J. Comp 1993) proposes a new (in 1993) data structure, called the Suffix Array, which has a similar functionality as Suffix Tree
  - Moreover, it only requires $O(n \log n)$ bits
- And it is much easier to implement

Suffix Array (1)

- Suffix Array (SA) is an array that stores:
  - A permutation of **n** indices of sorted suffixes
  - Each integer takes O(log n) bits, so SA takes O(n log n) bits
- e.g. consider T = 'GATAGACA$'

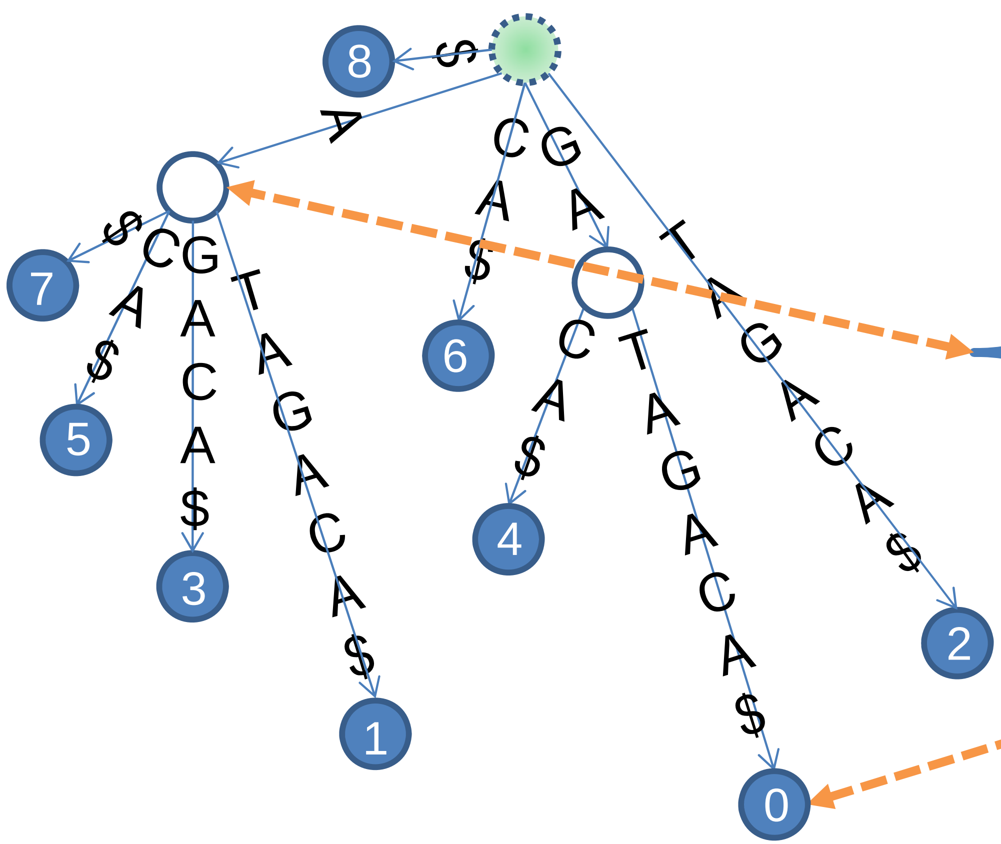
i	Suffix
0	GATAGACA$
1	ATAGACA$
2	TAGACA$
3	AGACA$
4	GACA$
5	ACA$
6	CA$
7	A$
8	$

Sort →

i	SA[i]	Suffix
0	8	$
1	7	A$
2	5	ACA$
3	3	AGACA$
4	1	ATAGACA$
5	6	CA$
6	4	GACA$
7	0	GATAGACA$
8	2	TAGACA$

Suffix Array (2)

- Preorder traversal of the Suffix Tree visits the terminating vertices in Suffix Array order
- **Internal vertex** in ST is a **range** in SA
  - **Each terminating vertex** in ST is an **individual index** in SA = a suffix



i	SA[i]	Suffix
0	8	$
1	7	A$
2	5	ACA$
3	3	AGACA$
4	1	ATAGACA$
5	6	CA$
6	4	GACA$
7	0	GATAGACA$
8	2	TAGACA$

Easy/Slow Suffix Array Construction

```
#include <algorithm>
#include <cstdio>
#include <cstring>
using namespace std;

char T[MAX_N]; int SA[MAX_N];

bool cmp(int a, int b) { return strcmp(T + a, T + b) < 0; }

int main() {
  int n = (int)strlen(gets(T));
  for (int i = 0; i < n; i++) SA[i] = i;
  sort(SA, SA + n, cmp);
}
```

This is O(N)

What is the time complexity?
Can we do better?

Overall $O(N^2 \log N)$

Most (if not all) applications related to Suffix Tree can be solved using Suffix Array

With some increase in time complexity

APPLICATIONS OF SUFFIX ARRAY

String Matching

- Given a Suffix Array SA of the string T
- Find occurrences of the pattern string P
- Example
  - T = ‘GATAGACA$’
  - P = ‘GA’
- Solution:
  - Use Binary Search twice
    - One to get lower bound
    - One to get upper bound

String Matching Animation

Finding P = 'GA'

Finding lower bound

i	SA[i]	Suffix
0	8	$
1	7	A$
2	5	ACA$
3	3	AGACA$
4	1	ATAGACA$
5	6	CA$
6	4	GACA$
7	0	GATAGACA$
8	2	TAGACA$

Finding upper bound

i	SA[i]	Suffix
0	8	$
1	7	A$
2	5	ACA$
3	3	AGACA$
4	1	ATAGACA$
5	6	CA$
6	4	GACA$
7	0	GATAGACA$
8	2	TAGACA$

Time Analysis

- Binary search runs at most O(log n) comparisons
- Each comparison takes at most O(m) time
- We run binary search twice
- In the worst case, O(2m log n) = O(m log n)

Longest Repeated Substring

- Simply find the highest entry in LCP array
  - O(n)

i	SA[i]	LCP[i]	Suffix
0	8	0	$
1	7	0	A$
2	5	1	<u>A</u>CA$
3	3	1	<u>A</u>GACA$
4	1	1	<u>A</u>TAGACA$
5	6	0	CA$
6	4	0	GACA$
7	**0**	**2**	**<u>GA</u>TAGACA$**
8	2	0	TAGACA$

Recall:
LCP = Longest Common Prefix between two successive suffices

Longest Common Substring

- T1 = ‘GATAGACA$’
- T2 = ‘CATA#’
- T = ‘GATAGACA$CATA#’
- Find the highest number in LCP array provided that it comes from two suffices with different owner
  – Owner: Is this suffix belong to string 1 or string 2?
- O(n)

i	SA[i]	LCP[i]	Owner	Suffix
0	13	0	2	#
1	8	0	1	$CATA#
2	12	0	2	A#
3	7	1	1	A$CATA#
4	5	1	1	ACA$CATA#
5	3	1	1	AGACA$CATA#
6	10	1	2	ATA#
7	**1**	**3**	**1**	**ATAGACA$CATA#**
8	6	0	1	CA$CATA#
9	9	2	2	CATA#
10	4	0	1	GACA$CATA#
11	0	2	1	GATAGACA$CATA#
12	11	0	2	TA#
13	2	2	1	TAGACA$CATA#

Summary

- In this lecture, you have seen:
  - Various string related tricks
  - Focus on Suffix Tree and Suffix Array
- But… you need to practice using them!
  - Especially, scrutinize my Suffix Array code
  - Solve at least one UVa problem involving SA
  - We will have SA-contest next week ☺
    - 2 SA problems in A/B/C

References

- CP2.9, Chapter 6
- Introduction to Algorithms, 2nd/3rd ed, Chapter 32