

A Retrospective on Naturally Embedded Query Languages

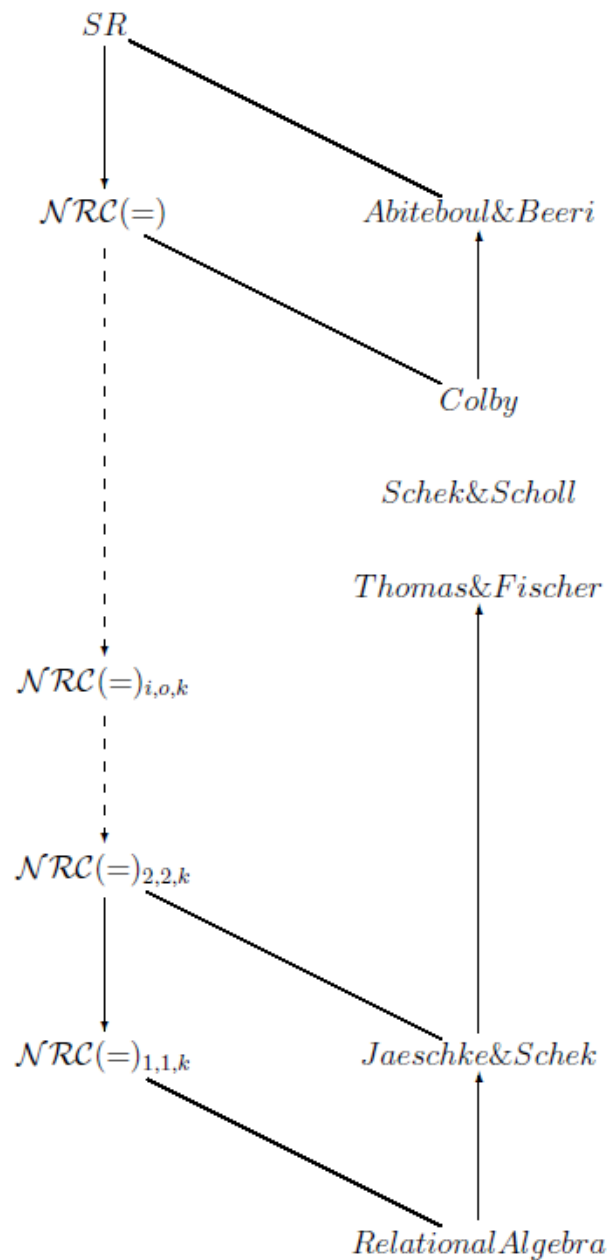
**Peter Buneman, Val Tannen,
Limsoon Wong**



Outline

- **Design of query languages**
- **Engineering data integration systems**
- **Understanding expressive power**
- **Exploring intensional expressive power**
- **Adding annotations**
- **Open problems**

DESIGN OF QUERY LANGUAGES



Two ways to
develop query
languages

Structural Recursion

- Let $u : t \times t \rightarrow t$, $f : s \rightarrow t$, and $e : t$ be such that $\langle t, u, e \rangle$ forms a commutative idempotent monoid. Then there is a unique $h : \{s\} \rightarrow t$ satisfying

$$\begin{aligned} h\{\} &= e \\ h\{x\} &= f(x) \\ h(R \cup S) &= u(h(R), h(S)) \end{aligned}$$

- Such a h is said to be defined by **structural recursion** on the union representation of sets. Denote this h by $sru(u, f, e)$

MapReduce is Structural Recursion

- $sru(u, f, e) \{o_1, \dots, o_n\} = f(o_1) u \dots u f(o_n) u e$
- The function f is “map”; it is applied (in parallel) to all elements in the input set
- The function u is “reduce”; it is applied (in parallel) to combine the results of the map

Examples

- **Structural recursion is expressive and can be used to write relatively efficient queries**

$$\text{cartprod}(R, S) \triangleq \text{sru}(\cup, \lambda x. \text{sru}(\cup, \lambda y. \{(x, y)\}, \{\})) (S), \{\})(R)$$

$$\text{map}(f) \triangleq \text{sru}(\cup, \lambda x. \{f(x)\}, \{\})$$

$$\text{flatten} \triangleq \text{sru}(\cup, \lambda x. x, \{\})$$

$$\text{powset} \triangleq \text{sru}(\text{flatten} \circ \text{map}(\cup) \circ \text{cartprod}, \lambda x. \{\{\}, \{x\}\}, \{\{\}\})$$

- But $\langle t, u, e \rangle$ has to be a commutative idempotent monoid in order for $sru(u, f, e)$ to be well defined on sets. E.g., $sru(+, \lambda x.1, 0)$ is not well defined
- \Rightarrow Restrict use of structural recursion to $sru(\cup, f, \{\})$, which is always well defined

More considerations in (Tannen, Subrahmanyam, ICALP91)

Nested Relational Calculus (NRC)

- Types

$$s, t ::= \mid \text{bool} \mid b \mid s \times t \mid \{s\}$$

- Expressions

$$\begin{array}{c}
 \frac{}{x^s : s} \quad \frac{e_1 : s \quad e_2 : t}{(e_1, e_2) : s \times t} \quad \frac{e : s \times t}{\pi_1 e : s \quad \pi_2 e : t} \\
 \\
 \frac{}{\text{true} : \text{bool}} \quad \frac{}{\text{false} : \text{bool}} \quad \frac{e_1 : \text{bool} \quad e_2 : s \quad e_3 : s}{\text{if } e_1 \text{ then } e_2 \text{ else } e_3 : s} \\
 \\
 \frac{}{\{ \}^s : \{s\}} \quad \frac{e : s}{\{e\} : \{s\}} \quad \frac{e_1 : \{s\} \quad e_2 : \{s\}}{e_1 \cup e_2 : \{s\}} \\
 \\
 \frac{e_1 : \{s\} \quad e_2 : \{t\}}{\cup \{e_1 \mid x^t \in e_2\} : \{s\}} \quad \frac{e : \{s\}}{\text{empty } e : \text{bool}} \quad \frac{e_1 : s \quad e_2 : s}{e_1 = e_2 : \text{bool}}
 \end{array}$$

where $\cup \{e_1 \mid x \in e_2\} = \text{sru}(\cup, \lambda x. e_1, \{ \})(e_2)$

NRC is equivalent to ...

- These operations are expressible in NRC: Project, Join, Union, Select, Difference, Intersect, Unnest, Nest. E.g.:

- Relational projection

$$\Pi_2(R) := \cup\{\{\pi_2 x\} \mid x \in R\}$$

- Relational selection

$$\sigma(p)(R) := \cup\{\text{if } p(x) \text{ then } \{x\} \text{ else } \{\} \mid x \in R\}$$

- Cartesian product

$$\otimes(R, S) := \cup\{\cup\{\{(x, y)\} \mid x \in R\} \mid y \in S\}$$

- Theorem 1 (Tannen, Buneman, Wong, ICDT92)

NRC has the same expressive power as the algebras of Schek&Scholl, Thomas&Fischer, etc.

Comprehension Syntax

- Translating into comprehension syntax

$$\cup\{e_1 \mid x \in e_2\} = \{y \mid x \in e_2, y \in e_1\}$$

- Translating from comprehension syntax

$$\{e_1 \mid x \in e_2, \Delta\} = \cup\{\{e_1 \mid \Delta\} \mid x \in e_2\}$$

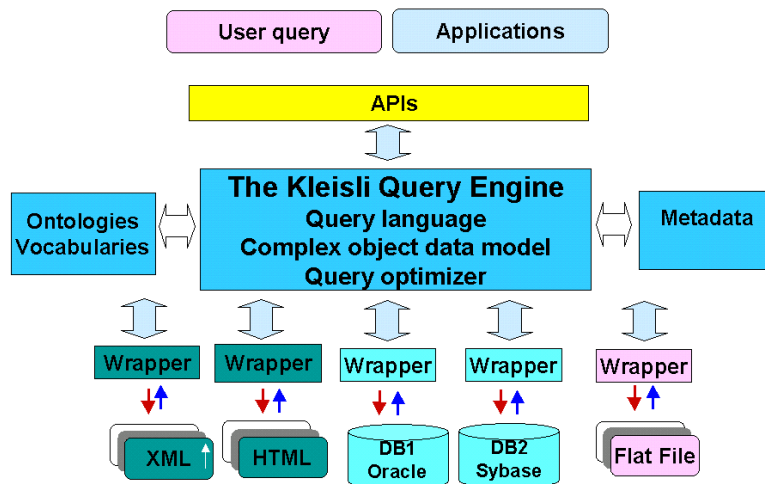
$$\{e_1 \mid C, \Delta\} = \text{if } C \text{ then } \{e_1 \mid \Delta\} \text{ else } \{\}$$

$$\{e_1 \mid \}$$

⇒ Treat comprehension as a nice syntactic sugar

ENGINEERING DATA INTEGRATION SYSTEMS

Kleisli Query System



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Buneman, Davidson, Hart, Overton, Wong, VLDB95
 Wong, ICFP00

- Nested set/bag/list model
- Self-describing data exchange format
- Lots of thin wrappers
- High-level query language with type inference
- Powerful query optimizer
- Nested set/bag/list store

US DOE “Impossible Query”, 1993

- For each gene on a given cytogenetic band, find its non-human homologs

source	type	location	remarks
GDB	Sybase	Baltimore	Flat tables SQL joins Location info
Entrez	ASN.1	Bethesda	Nested tables Keywords Homolog info

Solution in Kleisli

- **Using Kleisli:**
 - Clear
 - Succinct
 - Efficient
- **Handles**
 - Heterogeneity
 - Complexity

```

sybase-add (#name:"GDB", ...);
create view L from locus_cyto_location using GDB;
create view E from object_genbank_eref using GDB;
select
    #accn: g.#genbank_ref, #nonhuman-homologs: H
from
    L as c, E as g,
    {select u
     from g.#genbank_ref.na-get-homolog-summary as u
     where not(u.#title string-islike "%Human%") &
           not(u.#title string-islike "%H.sapien%")} as H
where
    c.#chrom_num = "22" &
    g.#object_id = c.#locus_id &
    not (H = { });
  
```

UNDERSTANDING EXPRESSIVE POWER

Conservative Extension Property

A language \mathcal{L} has **conservative extension property** if

for every function f definable in \mathcal{L} ,

there is an implementation f^* of f in \mathcal{L} such that

for any input i and corresponding output o ,

each intermediate data item created
in the course of executing f^* on i to
produce o has set nesting complexity
no more than that of i and o

Expressive Power of NRC

- Theorem 2 (Wong, PODS93)

NRC has the conservative extension property

- Corollary 3

Every function from flat relations to flat relations expressible in NRC is expressible in relational algebra

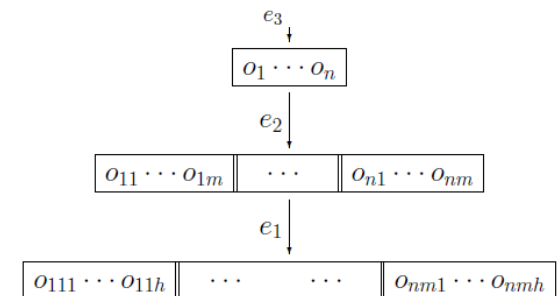
Proof Idea

• Strongly normalizing rewrite system

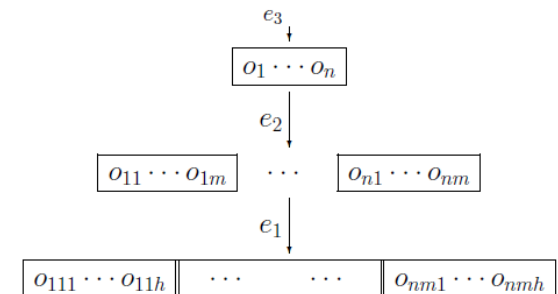
- $(\lambda x.e)(e') \rightsquigarrow e[e'/x]$
- $\pi_i(e_1, e_2) \rightsquigarrow e_i$
- $e \text{ (if } e_1 \text{ then } e_2 \text{ else } e_3) \rightsquigarrow \text{if } e_1 \text{ then } e_2 \text{ else } e_3$
- $\text{if true then } e_2 \text{ else } e_3 \rightsquigarrow e_2$
- $\text{if false then } e_2 \text{ else } e_3 \rightsquigarrow e_3$
- $\bigcup\{e \mid x \in e_1 \cup e_2\} \rightsquigarrow \bigcup\{e \mid x \in e_1\} \cup \bigcup\{e \mid x \in e_2\}$
- $\bigcup\{e \mid x \in \{\}\} \rightsquigarrow \{\}$
- $\bigcup\{e \mid x \in \{e'\}\} \rightsquigarrow e[e'/x]$
- $\bigcup\{e \mid x \in \text{if } e_1 \text{ then } e_2 \text{ else } e_3\}$
 $\rightsquigarrow \text{if } e_1 \text{ then } \bigcup\{e \mid x \in e_2\} \text{ else } \bigcup\{e \mid x \in e_3\}$
- $\bigcup\{e_1 \mid x \in \bigcup\{e_2 \mid y \in e_3\}\} \rightsquigarrow \bigcup\{\bigcup\{e_1 \mid x \in e_2\} \mid y \in e_3\}$

• Vertical loop fusion

$$\bigcup\{e_1 \mid x \in \bigcup\{e_2 \mid y \in e_3\}\}$$



$$\rightsquigarrow \bigcup\{\bigcup\{e_1 \mid x \in e_2\} \mid y \in e_3\}$$



Theoretical Reconstruction of SQL

- Expressions of $\text{NRC}(\mathbb{Q}, +, \cdot, -, \div, \Sigma, =, \leq^{\mathbb{Q}})$ are those of NRC plus the followings

$$\begin{array}{ccc}
 \frac{e_1 : \mathbb{Q} \quad e_2 : \mathbb{Q}}{e_1 + e_2 : \mathbb{Q}} & \frac{e_1 : \mathbb{Q} \quad e_2 : \mathbb{Q}}{e_1 \cdot e_2 : \mathbb{Q}} & \frac{e_1 : \mathbb{Q} \quad e_2 : \mathbb{Q}}{e_1 \div e_2 : \mathbb{Q}} \\
 \\
 \frac{e_1 : \mathbb{Q} \quad e_2 : \mathbb{Q}}{e_1 - e_2 : \mathbb{Q}} & \frac{e_1 : \mathbb{Q} \quad e_2 : \{s\}}{\Sigma\{e_1 \mid x^s \in e_2\} : \mathbb{Q}} & \frac{e_1 : \mathbb{Q} \quad e_2 : \mathbb{Q}}{e_1 \leq e_2 : \text{bool}}
 \end{array}$$

- Here $\Sigma \{e_1 \mid x \in e_2\} = f(o_1) + \dots + f(o_n)$, where f is the function $f(x) = e_1$ and $\{o_1, \dots, o_n\}$ is the set e_2

Example Aggregate Functions

- Count the number of records

$$\mathit{count}(R) := \Sigma\{ | 1 \mid x \in R \}$$

- Total the first column

$$\mathit{total}_1(R) := \Sigma\{ | \pi_1 x \mid x \in R \}$$

- Average of the first column

$$\mathit{ave}_1(R) := \mathit{total}_1(R) \div \mathit{count}(R)$$

- A totally generic query expressible in SQL but inexpressible in FO(=)

$$\mathit{eqcard}(R,S) := \mathit{count}(R) = \mathit{count}(S)$$



Expressive Power of $\text{NRC}(\mathbb{Q}, +, \cdot, -, \div, \Sigma, =, \leq^{\mathbb{Q}})$

- Theorem 4 (Libkin, Wong, DBPL93)

$\text{NRC}(\mathbb{Q}, +, \cdot, -, \div, \Sigma, =, \leq^{\mathbb{Q}})$ has the conservative extension property

- Corollary 5

Every function from flat relations to flat relations is expressible in $\text{NRC}(\mathbb{Q}, +, \cdot, -, \div, \Sigma, =, \leq^{\mathbb{Q}})$ iff it is also expressible in “entry-level” SQL

Finite/Co-finite Property I

- Theorem 6 (Libkin, Wong, DBPL93)
Let $P : \mathbb{Q} \rightarrow \mathbb{B}$ be a predicate definable in $\text{NRC}(\mathbb{Q}, +, \cdot, -, \div, \Sigma, =, \leq^{\mathbb{Q}})$. Then either P holds for finitely many natural numbers or P fails for finitely many natural numbers
- Corollary 7
 $\text{NRC}(\mathbb{Q}, +, \cdot, -, \div, \Sigma, =, \leq^{\mathbb{Q}})$ cannot test whether a natural number is even or odd

Proof Idea

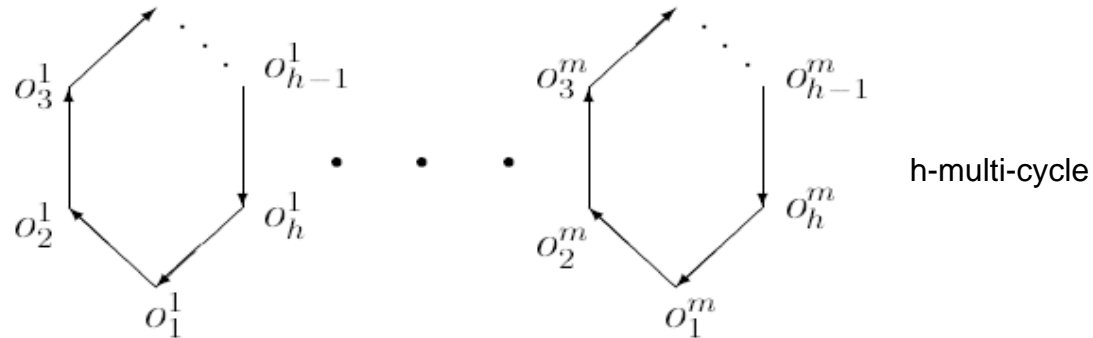
- $P : Q \rightarrow B$ has height 0. By conservative extension property on $\text{NRC}(Q, +, \cdot, -, \div, \Sigma, =, \leq^Q)$, any implementation of it in $\text{NRC}(Q, +, \cdot, -, \div, \Sigma, =, \leq^Q)$ is equivalent to one that does not use sets. Such an implementation must be equivalent to something like

$$\lambda n. (\bigvee \bigwedge p_i(n) = 0) \vee (\bigvee \bigwedge p_i(n) \neq 0)$$

where $p_i(n)$ are polynomials in n .

- Finite/co-finiteness then follows from the fact that polynomials have finite number of roots

Finite/Co-finite Property II



- Theorem 8 (Libkin, Wong, PODS94)

Let $P : \{b \times b\} \rightarrow B$ be a predicate definable in $\text{NRC}(\mathbb{Q}, +, \cdot, -, \div, \Sigma, =, \leq^{\mathbb{Q}})$. Then there is a h such that either P holds for all h -multi-cycles or P fails for all h -multi-cycles

- Corollary 9

$\text{NRC}(\mathbb{Q}, +, \cdot, -, \div, \Sigma, =, \leq^{\mathbb{Q}})$ cannot test the parity of a set and cannot express transitive closure

Locality Property

A language \mathcal{L} has **locality property** if the result of every flat relational query q definable in \mathcal{L} is determined by a small neighbourhood of its input

I.e., for all flat relational query expression $e[R]$ in \mathcal{L} , there is a finite number r such that, for all $\mathcal{A} = \langle A, O \rangle$ in $STRUCT[R]$, for all two m -ary vectors a and b of elements in A , $N_r^{\mathcal{A}}(a) \approx N_r^{\mathcal{A}}(b)$ implies $a \in e[O/R]$ if and only if $b \in e[O/R]$

Notations: $N_r^{\mathcal{A}}(b)$ means the neighbourhood of b in \mathcal{A} , up to a radius r .

Bounded Degree Property

A language \mathcal{L} has **bounded degree property** if

for every function f , on graphs, definable in \mathcal{L} , and
 for any number k ,

there is a number c such that

for any graph G with $\deg(G) \in \{0, 1, \dots, k\}$,
 it is the case that $c \geq \text{card}(\text{deg}(f(G)))$

That is, \mathcal{L} cannot define a function that produces complex
 graphs from simple graphs



Expressive Power of $\text{NRC}(Q, +, \cdot, -, \div, \Sigma, =, \leq^Q)$

- Theorem 10 (Dong, Libkin, Wong, ICDT97)
 $\text{NRC}(Q, +, \cdot, -, \div, \Sigma, =, \leq^Q)$ has the locality property, when restricted to flat relational queries on input structures of degree less than some fixed k
- Theorem 11 (Dong, Libkin, Wong, ICDT97)
Every language that has the locality property also has the bounded degree property
- Theorem 12 (Dong, Libkin, Wong, ICDT97)
 $\text{NRC}(Q, +, \cdot, -, \div, \Sigma, =, \leq^Q)$ has the bounded degree property

EXPLORING INTENSIONAL EXPRESSIVE POWER

What is intensional expressive power?

- Saying a function with linear complexity is expressible in a given query language *is not* the same as saying its implementation in that query language has linear complexity

I.e., we are looking at

- What the *algorithms* expressible in a query language are,
- Rather than what the *functions* expressible in a query language are

NRC(poweraset)

$$\frac{}{c : b} \quad \frac{}{x^s : s} \quad \frac{e_1 : s_1 \quad \dots \quad e_n : s_n}{(e_1, \dots, e_n) : s_1 \times \dots \times s_n} \quad \frac{e : s_1 \times \dots \times s_n}{\pi_i e : s_i} \quad 1 \leq i \leq n$$

$$\frac{}{\{\}^s : \{s\}} \quad \frac{e : s}{\{e\} : \{s\}} \quad \frac{e_1 : \{s\} \quad e_2 : \{s\}}{e_1 \cup e_2 : \{s\}} \quad \frac{e_1 : \{s\} \quad e_2 : \{t\}}{\cup\{e_1 \mid x^t \in e_2\} : \{s\}}$$

$$\frac{}{true : bool} \quad \frac{}{false : bool} \quad \frac{e_1 : bool \quad e_2 : s \quad e_3 : s}{if \ e_1 \ then \ e_2 \ else \ e_3 : s}$$

$$\frac{e_1 : b \quad e_2 : b}{e_1 = e_2 : bool} \quad \frac{e : \{b \times \dots \times b\}}{isempty \ e : bool}$$

Powerset Operator in $\mathcal{NRC}(\text{poweraset})$

$$\frac{e : \{b \times \dots \times b\}}{powerset \ e : \{\{b \times \dots \times b\}\}}$$

NRC cannot express recursive queries. Adding a powerset operation enables this.

$$\frac{}{c \Downarrow c} \quad \frac{e_1 \Downarrow C_1 \quad \dots \quad e_n \Downarrow C_n}{(e_1, \dots, e_n) \Downarrow (C_1, \dots, C_n)} \quad \frac{e \Downarrow (C_1, \dots, C_n)}{\pi_i e \Downarrow C_i} \quad 1 \leq i \leq n$$

$$\frac{}{\{\} \Downarrow \{\}} \quad \frac{e \Downarrow C}{\{e\} \Downarrow \{C\}} \quad \frac{e_1 \Downarrow C_1 \quad e_2 \Downarrow C_2}{e_1 \cup e_2 \Downarrow C_1 \cup C_2}$$

$$\frac{e_2 \Downarrow \{C_1, \dots, C_n\} \quad e_1[C_1/x] \Downarrow C'_1 \quad \dots \quad e_1[C_n/x] \Downarrow C'_n}{\cup\{e_1 \mid x \in e_2\} \Downarrow C'_1 \cup \dots \cup C'_n}$$

$$\frac{}{true \Downarrow true} \quad \frac{}{false \Downarrow false}$$

$$\frac{e_1 \Downarrow true \quad e_2 \Downarrow C}{if \ e_1 \ then \ e_2 \ else \ e_3 \ \Downarrow \ C} \quad \frac{e_1 \Downarrow false \quad e_3 \Downarrow C}{if \ e_1 \ then \ e_2 \ else \ e_3 \ \Downarrow \ C}$$

$$\frac{e_1 \Downarrow C_1 \quad e_2 \Downarrow C_2}{e_1 = e_2 \Downarrow true} C_1 = C_2 \quad \frac{e_1 \Downarrow C_1 \quad e_2 \Downarrow C_2}{e_1 = e_2 \Downarrow false} C_1 \neq C_2$$

$$\frac{e \Downarrow C}{isempty \ e \ \Downarrow \ true} C = \{\} \quad \frac{e \Downarrow C}{isempty \ e \ \Downarrow \ false} C \neq \{\}$$

$$\frac{e \Downarrow \{C_1, \dots, C_n\}}{powerset \ e \ \Downarrow \ \{C'_1, \dots, C'_{2^n}\}}$$

where C'_1, \dots, C'_{2^n} are the subsets of $\{C_1, \dots, C_n\}$

Operational Semantics

Recursive queries are costly in NRC(power set)

- Theorem 13 (Suciu, Paredaens, PODS94)
Any implementation of transitive closure in NRC(power set) must use exponential space
- Theorem 14 (Van den Bussche, TCS01)
Every flat relational query on unary schemas in NRC(power set) is either already expressible in NRC w/o using the power set operation or must use exponential space
- Theorem 15 (Biskup, Paredaens, Schwentick, Van den Bussche, SIAM J Comput 04)
Any implementation of set parity in the “Equation Algebra” must use exponential space

- **These intensional expressive power results are quite query specific, and their proofs are not easily “portable” to other queries**

Motifs and Bounded Structures

- Given a signature τ . A “motif” of radius r is a first-order formula $\rho(u)$ with a single free variable u and has locality index r on all τ structures
- A τ structure \mathcal{A} is “bounded” by a motif $\rho(u)$ at a threshold g if there are at most rg elements in the universe of \mathcal{A} that make $\rho(u)$ true, where r is the radius of $\rho(u)$
- A class \mathcal{C} of τ structures is “bounded” by a motif $\rho(u)$ at a threshold g if $\rho(u)$ bounds all structures in \mathcal{C} at the threshold g . On the other hand, \mathcal{C} is said to be “unbounded” by $\rho(u)$ if for each $g > 0$, there is $\mathcal{A} \in \mathcal{C}$ that is not bounded by $\rho(u)$ at threshold g

Dichotomous Structures

- A class \mathcal{C} of τ structures is “dichotomous” at threshold g iff
 - (i) \mathcal{C} is unbounded at threshold g by some motifs, and
 - (ii) \mathcal{C} is bounded by all other motifs at threshold g
 - A dichotomous class \mathcal{C} is “deep” if it is unbounded by some motifs of radius r at every r
 - A dichotomous class \mathcal{C} is “severe” if for every motif $\rho(u)$ that unbounds \mathcal{C} , there is a sequence of structures $\mathcal{A}_1, \mathcal{A}_2, \dots$, in \mathcal{C} having universe of increasing size, and the ratio $|\{a \in \mathcal{A}_i \mid \mathcal{A}_i, [a/u] \models \rho(u)\}|/|\mathcal{A}_i|$ tends to 1 as i tends to infinity.
- Deep severely dichotomous structures include long chains, long circles, deep trees, etc. Non-deep severely dichotomous structures include large sets of points, fat trees, etc.

Dichotomy Theorem

- Theorem 16 (Wong, PODS13)

Let f be a flat relational query in $\text{NRC}(\mathbb{Q}, +, \cdot, -, \div, \Sigma, =, \leq^{\mathbb{Q}}, \text{powerset})$ on structures from a class \mathcal{C} where (i) \mathcal{C} is severely dichotomous and (ii) its structures have degree $\leq k$. Then either f is already expressible in $\text{NRC}(\mathbb{Q}, +, \cdot, -, \div, \Sigma, =, \leq^{\mathbb{Q}})$ or must use exponential space

- Corollary 17

All implementations of transitive closure, set parity, etc. in $\text{NRC}(\mathbb{Q}, +, \cdot, -, \div, \Sigma, =, \leq^{\mathbb{Q}}, \text{powerset})$ must use exponential space

This theorem generalizes earlier intensional expressive power results

- **Works for all queries on “severely dichotomous” structures**
- **Works for a more powerful query language**
- **Uses a proof technique that is “portable”**

Another form of structural recursion mentioned in our ICDT92 paper

$$\frac{i : s \times t \rightarrow t \quad e : t}{sri(i, e) : \{s\} \rightarrow t}$$

- Semantics**

$$\begin{aligned} sri(i, e)(\{\}) &= e \\ sri(i, e)(\{o\} \cup O) &= i(o, sri(i, e)(O)) \end{aligned}$$

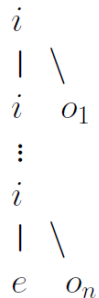
- In short...**

$$sri(i, e)\{o_1, \dots, o_n\} = i(o_1, \dots, i(o_n, e) \dots)$$

Equivalence

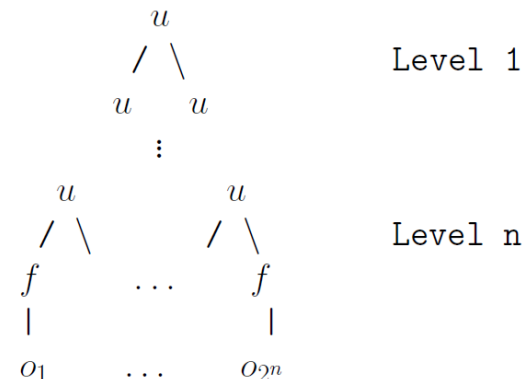
- Proposition 18 (Suciu, Wong, ICDT95)
There are uniform translations between NRC(sru) and NRC(sri). So for any set of external functions Σ , we have $\text{NRC}(\text{sru}, \Sigma) = \text{NRC}(\text{sri}, \Sigma)$
- **Our uniform sri \rightarrow sru translation is expensive**

$\text{sri}(i, e)\{o_1, \dots, o_n\}$ evaluates like this ...



$\text{sru}(u, f, e)\{o_1, \dots, o_{2^n}\}$ evaluates like this ...

$\text{sru}(u, f, e)\{o_1, \dots, o_{2^n}\}$ evaluates like this ...



Some sri queries cannot be parallelized

- Theorem 19 (Suciu, Wong, ICDT95)
Any uniform translation of NRC(sri) queries to NRC(sru) / NRC(hom) must map some PTIME queries into EXPSPACE ones
- In fact, in the presence of certain external functions, there is a PTIME NRC(sri) query for which every equivalent NRC(sru) / NRC(hom) query requires EXPSPACE

NC is strictly in PTIME?

- Theorem 20 (Tannen, Suci, PODS94)
 - $\text{NRC}^1(\text{hom}, \leq)$ captures NC
 - $\text{NRC}^1(\text{sri}, \leq)$ captures PTIME
- Corollary 21 (Suci, Wong, ICDT95)

There is no uniform translation of a language for PTIME into a language for NC

Notations: NRC^1 = the flat-types fragment of NRC

A cute result on lists

- Treat $\{\}$ as empty list, $\{e\}$ as singleton list, \cup as list concatenation. Then $\text{NRC}(\text{sru})$ and $\text{NRC}(\text{sri})$ become query languages for list
- Theorem 20
 The $\text{zip} : \{b\} \times \{b\} \rightarrow \{b \times b\}$ function cannot be implemented in $\text{NRC}(\text{sru})$ and $\text{NRC}(\text{sri})$ in $O(\min(m, n))$ time, where (m, n) are length of the two input lists to zip

Proof Idea

- Suppose *zip* can be implemented in $O(\min(m,n))$ time. Then *head* : $\{b\} \rightarrow \{b\}$ can be implemented in constant time in $\text{NRC}(\text{sri})$

$$\text{head}(L) = \text{sri}(\lambda x. \{\pi_1 x\}, \{\}) (\text{zip}(L, \{\{\}\}))$$

- But it is easy to show that *head* cannot be implemented in $\text{NRC}(\text{sri})$ in constant time

ADDING ANNOTATIONS

What are annotations

- **Data can be annotated for many reasons**
 - Confidentiality policy
 - **Public < Confidential < Secret < Top Secret < 0**
 - Provenance
 - Probability
 - Uncertainty
- **It is desirable to propagate annotations on source tuples to query results**

Example

Source and Answer as *K*-Relations:

<i>R</i>		
A	B	C
<i>a</i>	<i>b</i>	<i>c</i>
<i>d</i>	<i>b</i>	<i>e</i>
<i>f</i>	<i>g</i>	<i>e</i>

<i>S</i>	
B	C
<i>b</i>	<i>c</i>
<i>g</i>	<i>c</i>

<i>Q</i>	
A	C
<i>a</i>	<i>c</i>
<i>a</i>	<i>e</i>
<i>d</i>	<i>c</i>
<i>d</i>	<i>e</i>
<i>f</i>	<i>c</i>
<i>f</i>	<i>e</i>

$$Q = \pi_{AC}(\pi_{AB}(R) \bowtie (\pi_{BC}(R) \cup S))$$

- “Thesis” 21 (Green, Karvounarakis, Tannen, PODS07)

The propagation of a rich variety of annotations can be expressed as a semi-ring $\langle K, +, *, 0, 1 \rangle$

$$\llbracket l \rrbracket_K^\rho = l \quad \llbracket x \rrbracket_K^\rho = \rho(x) \quad \llbracket \{\} \rrbracket_K^\rho(x) = 0_K$$

$$\llbracket \{e\} \rrbracket_K^\rho(x) = \text{if } x = \llbracket e \rrbracket_K^\rho \text{ then } 1_K \text{ else } 0_K$$

$$\llbracket e_1 \cup e_2 \rrbracket_K^\rho(x) = \llbracket e_1 \rrbracket_K^\rho(x) + \llbracket e_2 \rrbracket_K^\rho(x)$$

How to
propagate
annotations
for positive
NRC

$$\frac{\llbracket e_1 \rrbracket_K^\rho = s_1}{\llbracket \cup(x \in e_1) e_2 \rrbracket_K^\rho(y) = \sum_{v \in \text{dom}(s_1)} s_1(v) \cdot \llbracket e_2 \rrbracket_K^{\rho[x \leftarrow v]}(y)}$$

$$\frac{\text{if } \llbracket e_1 \rrbracket_K^\rho = \llbracket e_2 \rrbracket_K^\rho \text{ then } \llbracket e_3 \rrbracket_K^\rho \text{ else } \llbracket e_4 \rrbracket_K^\rho = v}{\llbracket \text{if } e_1 = e_2 \text{ then } e_3 \text{ else } e_4 \rrbracket_K^\rho = v}$$

$$\frac{\llbracket e_1 \rrbracket_K^\rho = v_1 \quad \llbracket e_2 \rrbracket_K^\rho = v_2}{\llbracket (e_1, e_2) \rrbracket_K^\rho = (v_1, v_2)} \quad \frac{\llbracket e \rrbracket_K^\rho = (v_1, v_2) \quad i \in \{1, 2\}}{\llbracket \pi_i(e) \rrbracket_K^\rho = v_i}$$

- Theorem 22 (Foster, Green, Tannen, PODS08)

If $h : K1 \rightarrow K2$ is a homomorphism of semi-rings
then $h(e(v)) = h(e)(h(v))$

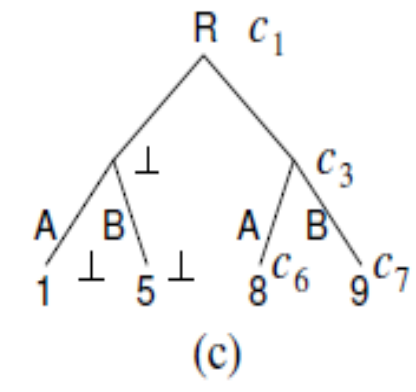
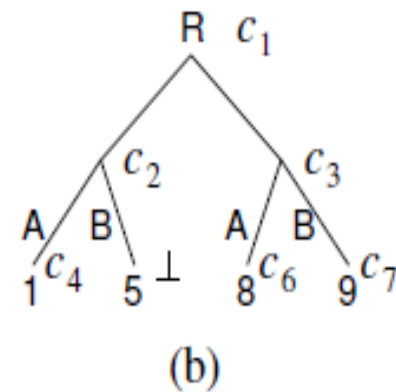
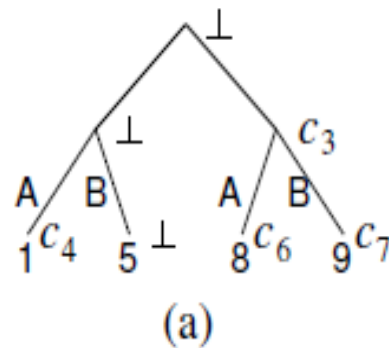
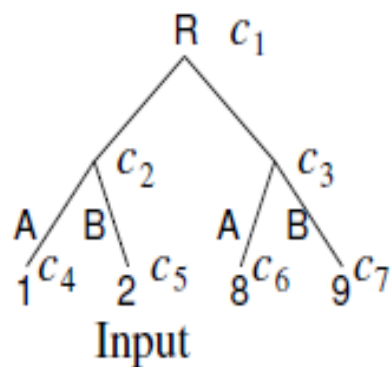
Finer Notions of Provenance

- (select * from R where A \leftrightarrow 1) union
(select A, 5 as B from R where A = 1)**
- update R set B = 5 where A = 1**
- delete from R where A = 1;
insert into R values (1, 5)**

Copying

Kind Preserving

If output item has same color as input item then they are of the same kind: both sets, both tuples, or identical atoms



Color propagation for query (a) and updates (b) and (c).

NRL(color) = NRC + NUL

- Add these NUL constructs for updates to NRC

$$\begin{array}{c}
 \frac{}{\text{skip}^s : s \rightarrow s} \qquad \frac{u_1 : r \rightarrow s \quad u_2 : s \rightarrow t}{u_1; u_2 : r \rightarrow t} \qquad \frac{e : t}{\text{repl}^s e : s \rightarrow t} \qquad \frac{u : s \rightarrow t}{[x^s] u : s \rightarrow t} \\
 \\
 \frac{e : \{s\}}{\text{insert } e : \{s\} \rightarrow \{s\}} \qquad \frac{e : \{s\}}{\text{remove } e : \{s\} \rightarrow \{s\}} \qquad \frac{u : s \rightarrow t}{\text{iter } u : \{s\} \rightarrow \{t\}} \\
 \\
 \frac{u : r \rightarrow t}{\text{updl}^s u : r \times s \rightarrow t \times s} \qquad \frac{u : s \rightarrow t}{\text{updr}^r u : r \times s \rightarrow r \times t}
 \end{array}$$

- Add a new type “color” to indicating provenance annotations
 - \perp is color to mean “newly created”
 - write \underline{s} to mean type with provenance annotations

Provenance Semantics

$$\begin{array}{ll}
 \mathcal{P}[a] := (a, \perp) & \mathcal{P}[\lambda x^s. e] := \lambda x^s. \mathcal{P}[e] \\
 \mathcal{P}[x^s] := x^s & \mathcal{P}[e_1 e_2] := \mathcal{P}[e_1] \mathcal{P}[e_2] \\
 \mathcal{P}[] := ((), \perp) & \mathcal{P}[\pi_1 e] := \pi_1 \pi_1 \mathcal{P}[e] \\
 \mathcal{P}[\pi_2 e] := \pi_2 \pi_1 \mathcal{P}[e] & \mathcal{P}[(e_1, e_2)] := ((\mathcal{P}[e_1], \mathcal{P}[e_2]), \perp) \\
 \mathcal{P}[\{\}] := (\{\}, \perp) & \mathcal{P}[e_1 \cup e_2] := ((\pi_1 \mathcal{P}[e_1] \cup \pi_1 \mathcal{P}[e_2]), \perp) \\
 \mathcal{P}[\{e\}] := (\{\mathcal{P}[e]\}, \perp) & \mathcal{P}[\bigcup\{e_2 \mid x^s \in e_1\}] := (\bigcup\{\pi_1 \mathcal{P}[e_2] \mid x^s \in \pi_1 \mathcal{P}[e_1]\}, \perp) \\
 \mathcal{P}[\text{if } e_1 = e_2 \text{ then } e_3 \text{ else } e_4] := \text{if } \text{val}(\mathcal{P}[e_1]) = \text{val}(\mathcal{P}[e_2]) \text{ then } \mathcal{P}[e_3] \text{ else } \mathcal{P}[e_4] & \\
 \\
 \mathcal{P}[\text{skip}^s] := \text{skip}^s & \mathcal{P}[u; u'] := \mathcal{P}[u]; \mathcal{P}[u'] \\
 \mathcal{P}[\text{repl}^s e] := \text{repl}^s \mathcal{P}[e] & \mathcal{P}[[x^s] u] := [x^s] \mathcal{P}[u] \\
 \mathcal{P}[\text{insert } e] := \text{updl insert } (\pi_1 \mathcal{P}[e]) & \mathcal{P}[\text{iter } u] := \text{updl iter } \mathcal{P}[u] \\
 \mathcal{P}[\text{updl } u] := \text{updl updl } \mathcal{P}[u] & \mathcal{P}[\text{updr } u] := \text{updl updr } \mathcal{P}[u] \\
 \mathcal{P}[\text{remove } e] := \text{updl } ([x] \text{ remove } \{y \mid y \in x, \text{val}(y) \in \text{val}(\mathcal{P}[e])\}) &
 \end{array}$$

Provenance-Aware DB Operations

- $f : \underline{s} \rightarrow \underline{t}$ is **color propagating** if f does not let input colors influence the uncolored part of the output and f is insensitive to actual colors used
- $f : \underline{s} \rightarrow \underline{t}$ is **bounded inventing** if f does not create many new values
- A **provenance-aware db operation (pado)** is a color-propagating and bounded-inventing function $f : \underline{s} \rightarrow \underline{t}$

Soundness and Completeness

- **Theorem 23** (Buneman, Cheney, VanSummeren, ICDT07)
Every function is in CP if and only if it is in PNRC
- **Theorem 24** (Buneman, Cheney, VanSummeren, ICDT07)
Every function is in KP if and only if it is in PNUL

$CP := \{ f \mid f: \underline{s} \rightarrow \underline{t} \text{ in } \mathcal{NRL}(color) \text{ defines a copying pado } \},$
 $KP := \{ f \mid f: \underline{s} \rightarrow \underline{t} \text{ in } \mathcal{NRL}(color) \text{ defines a kind-preserving pado } \}.$

$\mathcal{PNRC} := \{ \mathcal{P}[e] \mid e \text{ expression in } \mathcal{NRC} \}.$

$\mathcal{PNUL} := \{ \mathcal{P}[u] \mid u \text{ expression in } \mathcal{NUL} \}.$

OPEN PROBLEMS

Maybe you know the answer ...

- In the presence of an order on base types,
 - Locality theorem becomes useless
 - Bounded degree property fails
 - Dichotomy theorem fails

Can this be fixed?

- Is there a PTIME query in $\text{NRC}(\text{sri})$ that has no PTIME equivalent in $\text{NRC}(\text{sru})$ in the absence of external functions? Is transitive closure such a query?