## CS2100: Computer Organisation Tutorial \#7: Combinational Circuits

(Week 9: 18-22 March 2024)
Answers to Selected Questions

## Tutorial questions

Note that for questions on logic design, you may assume that logical constants 0 and 1 are always available. However, complemented literals are not available unless otherwise stated.

1. [Past-year's question]

You are to design a circuit to implement a function $V(A, B, C, D, E)$ that takes in input $A B C D E$ and generates output 1 if $A B C D E$ is a valid input for the circuit in question $D 3$ above, or 0 if $A B C D E$ is an invalid input. You are allowed to use only the following devices: full adder, 2-bit parallel adder, and 4bit magnitude comparator. You should use the fewest number of these approved devices, and no other devices or logic gates. The block diagrams for these devices are shown below.


Answer:
Idea: Count the number of 1's in $A B C D E$. If count $>1$, then it's a valid input.

2. [Past year's exam question]
a. You want to construct a circuit that takes in a 4-bit unsigned binary number $A B C D$ and outputs a 4-bit unsigned binary number $E F G H$ where $E F G H=(A B C D+1) / 2$. Note that the division is an integer division. For example, if $A B C D=0110$ (or 6 in decimal), then $E F G H=0011$ (or 3 in decimal). If $A B C D=1101$ (or 13 in decimal), then $E F G H=0111$ (or 7 in decimal).

Construct the above circuit using a single 4-bit parallel adder and at most one logic gate with no restriction on its fan-in.
b. The following table shows the 4221 code and 8421 code (also known as BCD code) for the ten decimal digits 0 through 9.

| Digit | 4221 code | 8421 code |
| :---: | :---: | :---: |
| 0 | 0000 | 0000 |
| 1 | 0001 | 0001 |
| 2 | 0010 | 0010 |
| 3 | 0011 | 0011 |
| 4 | 0110 | 0100 |
| 5 | 1001 | 0101 |
| 6 | 1100 | 0110 |
| 7 | 1101 | 0111 |
| 8 | 1110 | 1000 |
| 9 | 1111 | 1001 |

You want to construct a 4221-to-8421 decimal code converter, which takes in a 4-bit 4221 decimal code PQRS and generates the corresponding 4-bit 8421 decimal code $W X Y Z$.

Let's call the circuit you created in part (a) above the A1H (Add-1-then-Half) device, represented by the block diagram below. Implement your 4221-to-8421 decimal code converter using this A1H device with the fewest number of additional logic gates.


Answers:
Solution 1:


| $\boldsymbol{A}$ | $\boldsymbol{B}$ | $\boldsymbol{C}$ | $\boldsymbol{D}$ | $\boldsymbol{E}$ | $\boldsymbol{F}$ | $\boldsymbol{G}$ | $\boldsymbol{H}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |

Alternative solution for $E$ :


2(b)


$$
s \longrightarrow Z
$$

| $\boldsymbol{P}$ | $\mathbf{Q}$ | $\boldsymbol{R}$ | $\boldsymbol{S}$ | $\boldsymbol{W}$ | $\boldsymbol{X}$ | $\boldsymbol{Y}$ | $\boldsymbol{Z}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |

4. Given a 4-bit magnitude comparator as shown on the right, implement the following 4 -variable Boolean functions using only this single magnitude comparator with no other logic gates. (Note that there could be multiple answers.)
(a) $F(A, B, C, D)=\Sigma m(12-15)$.
(b) $G(A, B, C, D)=\Sigma m(0,6,9,15)$.
(c) $H(A, B, C, D)=\Sigma m(0,1,6,7,8,9,14,15)$.
(d) $Z(A, B, C, D)=\Sigma m(1,3,5,7,9,11,13)$.


Answers:
(a)

(b)


