CS2100: Computer Organisation
Tutorial #2: Boolean Algebra and Logic Gates
Answers to Selected Questions

---

3. Using Boolean algebra, simplify each of the following expressions into minimal sum-of-products (SOP) expressions. Indicate the laws/theorems used.

   (a) $(b' + a'\cdot c)' + (a + c)'$

   (b) $p\cdot r + p\cdot q\cdot r + r'\cdot(p'\cdot q' + p'\cdot q + p\cdot q' + p\cdot \text{q})$

   (c) $g(A,B,C,D) = \Pi\ M(2,4,6)$

   For (c), it is simpler to start with the given product-of-sums expression, rather than to convert it into sum-of-products expression first.

   Answers:
   There might be more than one way of derivation. The derivation is not shown for part (b), but most of the steps involve absorption theorem.

   (b) $p\cdot r + p\cdot q\cdot r + r'\cdot(p'\cdot q' + p'\cdot q + p\cdot q' + p\cdot q)$
   $= \boldsymbol{p + r'}$

   For part (c), the derivation is shown. You can see that the following distributive law (Boolean Algebra, slide 10) is used:

   $(X + Y) \cdot (X + Z) = X + Y\cdot Z$

   (c) $g(A,B,C,D) = \Pi\ M(2, 4, 6)$
   $= (A + B + C' + D) \cdot \underline{(A + B' + C + D) \cdot (A + B' + C' + D)}$
   $= (A + B + C' + D) \cdot [\ (A + B' + D) + C\cdot C'\ ]$
   $= (A + B + C' + D) \cdot (A + B' + D)$ [distributive, complement]
   $= (A + D) + [(B + C') \cdot B'\ ]$ [distributive]
   $= \boldsymbol{A + D + B'\cdot C'}$ [absorption]

5. Design a divide-by-3 circuit as follows: the input is a 4-bit unsigned binary number *ABCD*, and the output is a 3-bit unsigned binary number *XYZ* which is the quotient of *ABCD* / 3. For example, if *ABCD* = 1100 (or 12 in decimal), then *XYZ* = 100 (or 4 in decimal); if *ABCD* = 0111 (or 7 in decimal), then *XYZ* = 010 (or 2 in decimal).

   (a) Draw the truth table and try to obtain the simplified SOP expressions of *X*, *Y*, and *Z* just from observation. (It is quite easy to obtain the simplified SOP expressions for *X* and *Y* just from observing the truth table, but harder for *Z*. The K-map technique is probably useful here, but we'll do that in the next tutorial.)

   (b) Verify your answers by first writing out their sum-of-minterms expressions and then simplifying the expressions from there using Boolean algebra. Write out the theorem(s) you use at each step.

(c) From the simplified SOP expressions, implement X, Y, and Z using (i) 2-level AND-OR circuits, and (ii) 2-level NAND-only circuits. Assume that primed literals are not available.

Answers:

(a)

A	*B*	*C*	*D*	*X*	*Y*	*Z*
0	0	0	0	0	0	0
0	0	0	1	0	0	0
0	0	1	0	0	0	0
0	0	1	1	0	0	1
0	1	0	0	0	0	1
0	1	0	1	0	0	1
0	1	1	0	0	1	0
0	1	1	1	0	1	0

A	*B*	*C*	*D*	*X*	*Y*	*Z*
1	0	0	0	0	1	0
1	0	0	1	0	1	1
1	0	1	0	0	1	1
1	0	1	1	0	1	1
1	1	0	0	1	0	0
1	1	0	1	1	0	0
1	1	1	0	1	0	0
1	1	1	1	1	0	1

$\boldsymbol{X = A \cdot B}$ (easy from truth table)
$\boldsymbol{Y = A \cdot B' + A' \cdot B \cdot C}$ (still easy from truth table)
$\boldsymbol{Z = A' \cdot B \cdot C' + B' \cdot C \cdot D + A \cdot C \cdot D + A \cdot B' \cdot D + A \cdot B' \cdot C}$ (hard to derive from truth table; may need other method such as Boolean Algebra)

(b)

Simplifying from their sum-of-minterms expressions:

$Z = \Sigma m(3\text{-}5, 9\text{-}11, 15)$

$= A' \cdot B' \cdot C \cdot D + \underline{A' \cdot B \cdot C' \cdot D' + A' \cdot B \cdot C' \cdot D} + A \cdot B' \cdot C' \cdot D + A \cdot B' \cdot C \cdot D' + A \cdot B' \cdot C \cdot D + A \cdot B \cdot C \cdot D$

$= \underline{A' \cdot B \cdot C' \cdot (D' + D)} + A' \cdot B' \cdot C \cdot D + A \cdot B' \cdot C' \cdot D + A \cdot B' \cdot C \cdot D' + A \cdot B' \cdot C \cdot D + A \cdot B \cdot C \cdot D$ [Distributive]

$= A' \cdot B \cdot C' + A' \cdot B' \cdot C \cdot D + A \cdot B' \cdot C' \cdot D + A \cdot B' \cdot C \cdot D' + A \cdot B' \cdot C \cdot D + A \cdot B \cdot C \cdot D$ [Complement, Identity]

(Expand $A \cdot B' \cdot C \cdot D$ into $A \cdot B' \cdot C \cdot D + A \cdot B' \cdot C \cdot D + A \cdot B' \cdot C \cdot D + A \cdot B' \cdot C \cdot D$ [Idempotency])

$= A' \cdot B \cdot C'$
$+ \underline{\boldsymbol{A \cdot B' \cdot C \cdot D} + A' \cdot B' \cdot C \cdot D} + \underline{\boldsymbol{A \cdot B' \cdot C \cdot D} + A \cdot B' \cdot C' \cdot D} + \underline{\boldsymbol{A \cdot B' \cdot C \cdot D} + A \cdot B' \cdot C \cdot D'} + \underline{\boldsymbol{A \cdot B' \cdot C \cdot D} + A \cdot B \cdot C \cdot D}$

$= \boldsymbol{A' \cdot B \cdot C' + B' \cdot C \cdot D + A \cdot B' \cdot D + A \cdot B' \cdot C + A \cdot C \cdot D}$ [Distributive, Complement, Identity]

The above derivation for Z shows that sometimes, we need to ‘expand’ a term into two or more terms before we can proceed with the simplification.