CS3234 Logic and Formal Systems

Assignment 06:
Modal Logic II

Submission on A-4 paper (use as many sheets as you want), to the office COM1, 03-28. Staple or tie your sheets together and write your name and matriculation number on the top of the front page.
Latest submission: Monday, 25/10/10, 5:00pm.

1. (5 marks) Prove the following sequent in basic modal logic K, using natural deduction:
$$\Box(p \to q) \vdash \Box p \to \Box q$$

2. (10 marks) Prove the following sequent in modal logic KT5, using natural deduction:
$$\Diamond\Box p \to \Box p$$

3. (5 marks) Assume that G_1 and G_2 are sets of agents. Does the following implication hold in every Kripke model of $KT45^n$?
$$E_{G_1}E_{G_2}p \wedge E_{G_2}E_{G_1}p \to E_{G_1\cup G_2}E_{G_1\cup G_2}p$$
Justify your answer with at most two sentences.

4. (6 marks) Consider the following modified scenario of the Wise Women's Puzzle:

   The following scenario is common knowledge among the wise women:

   > There are three red hats and two white hats. The queen puts a hat on each wise woman so that they are not able to see their own hat, but each sees both of the other women's hats. The queen asks each one in turn whether they know the colour of the hat on their head. **The second woman is deaf and thus does not come to know the first woman's answer. Everyone knows that she is deaf.**

   All three women answer "no".

   - (3 marks) Now the king asks the first woman again if she knows the colour of her hat. What is his answer? Why?

- (3 marks) The formula set Γ in KT45^n that describes the scenario before the women answer remains unchanged:

$$\begin{aligned}\Gamma \quad = \{ \quad & C(p_1 \vee p_2 \vee p_3), \\ & C(p_1 \to K_2 p_1), C(\neg p_1 \to K_2 \neg p_1), \\ & C(p_1 \to K_3 p_1), C(\neg p_1 \to K_3 \neg p_1), \\ & C(p_2 \to K_1 p_2), C(\neg p_2 \to K_1 \neg p_2), \\ & C(p_2 \to K_3 p_2), C(\neg p_2 \to K_3 \neg p_2), \\ & C(p_3 \to K_1 p_3), C(\neg p_2 \to K_1 \neg p_3), \\ & C(p_3 \to K_2 p_3), C(\neg p_2 \to K_2 \neg p_3)\}\end{aligned}$$

  What formula can we add to Γ to represent the first woman's answer "no"?

5. (6 marks)

   - Consider the following variant of the Wise Women's Puzzle: The first woman answers "no", like in the original puzzle. However, the second women answers "yes" instead of "no". Can we infer the color of her hat? Give a short explanation.
   - After the first woman answers "no", and the second woman answers "yes", what will be the answer of the third woman? Give a short explanation.
   - Consider the original version, where the first and second women answer "no".
     - (1 mark) Assume that the first woman is blind, and everyone knows she is blind. Can we predict the answer of the third woman? (no eplanation needed)
     - (1 mark) Assume that the second woman is blind, and everyone knows she is blind. Can we predict the answer of the third woman? (no eplanation needed)
     - (1 mark) Assume that the third woman is blind, and everyone knows she is blind. Can we predict the answer of the third woman? (no eplanation needed)
     - (3 marks) Write a formula in KT45^n that expresses that everyone in the group of three women knows that the first woman is blind (using the encoding of the puzzle shown in the lecture).

6. (8 marks) Consider the following scenario: There are 7 children playing in the garden. A child does not notice when he gets mud on his forehead. However, all children can see the mud on each other's forehead. All children are rational agents in the sense of KT45^n. Four children have mud on their forehead. At the beginning, the children's father announces: "At least three of you have mud on their forehead." Then, he repeatedly asks

his question Q: “Does any of you know whether you have mud on your own forehead?”

- What are the answers the first time around? (give a brief explanation)
- What are the answers the second time around? (give a brief explanation)
- What are the answers the third time around? (give a brief explanation)
- What are the answers the fourth time around? (give a brief explanation)