Traditional Logic: Cheat Sheet

CS 3234: Logic and Formal Systems

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Starting the Module

```
Module TraditionalLogicCheat.
```

Defining Terms

```
Parameter Term : Type.
```

Populate the type `Term` with a particular instance.

```
Parameter cats : Term.
Parameter lions : Term.
```

Propositions

```
Record Quantity : Type :=
  universal : Quantity
| particular : Quantity.
```

```
Record Quality : Type :=
  affirmative : Quality
| negative : Quality.
```

```
Record CategoricalProposition : Type :=
cp {
  quantity : Quantity;
  quality : Quality;
  subject : Term;
  object : Term
}.
```

Example:

```
Definition LionsAreCats : CategoricalProposition :=
  cp universal affirmative lions cats.


Notation "'All' subject 'are' object " :=
   (cp universal affirmative subject object) (at level 50).

Notation "'No' subject 'are' object " :=
   (cp universal negative subject object) (at level 50).

Notation "'Some' subject 'are' object " :=
   (cp particular affirmative subject object) (at level 50).

Notation "'Some' subject 'are' 'not' object " :=
   (cp particular negative subject object) (at level 50).
```

Now you can write:

```
Definition LionsAreCats2 : CategoricalProposition :=
   All lions are cats.
```

Propositions may "hold", which means they can be used in following proofs.

```
Parameter holds : CategoricalProposition -> Prop.
```

Example:

```
Axiom LionsAreCatsHolds: holds (All lions are cats).
```

Complement

```
Parameter non: Term -> Term.
```

Axiom (NonNon)**.** *For any term t, the term `non non` t is considered equal to t.*

```
Axiom NonNon: forall t, non (non t) = t.
```

Conversion

Definition (ConvDef)**.** *For all terms t_1 and t_2, we define*

$$\begin{aligned} \mathit{convert}(\mathit{All}\ t_1\ \mathit{are}\ t_2) &= \mathit{All}\ t_2\ \mathit{are}\ t_1 \\ \mathit{convert}(\mathit{Some}\ t_1\ \mathit{are}\ t_2) &= \mathit{Some}\ t_2\ \mathit{are}\ t_1 \\ \mathit{convert}(\mathit{No}\ t_1\ \mathit{are}\ t_2) &= \mathit{No}\ t_2\ \mathit{are}\ t_1 \\ \mathit{convert}(\mathit{Some}\ t_1\ \mathit{are\ not}\ t_2) &= \mathit{Some}\ t_2\ \mathit{are\ not}\ t_1 \end{aligned}$$

```
Definition convert: CategoricalProposition -> CategoricalProposition :=
  fun x : CategoricalProposition => match x with
    | cp quantity quality subject object =>
      cp quantity quality object subject
  end.
```

Axiom (ConvE1)**.** *If, for some terms t_1 and t_2, the proposition*

$$\mathit{convert}(\mathit{Some}\ t_1\ \mathit{are}\ t_2)$$

holds, then the proposition

$$\mathit{Some}\ t_1\ \mathit{are}\ t_2$$

also holds.

Axiom (ConvE2)**.** *If, for some terms t_1 and t_2, the proposition*

$$\mathit{convert}(\mathit{No}\ t_1\ \mathit{are}\ t_2)$$

holds, then the proposition

$$\mathit{No}\ t_1\ \mathit{are}\ t_2$$

also holds.

$$\frac{\mathit{convert}(\texttt{Some}\ t_1\ \texttt{are}\ t_2)}{\texttt{Some}\ t_1\ \texttt{are}\ t_2}\,[\mathrm{ConvE}_1]$$

$$\frac{\mathit{convert}(\texttt{No}\ t_1\ \texttt{are}\ t_2)}{\texttt{No}\ t_1\ \texttt{are}\ t_2}\,[\mathrm{ConvE}_2]$$

```
Axiom ConvE1:
   forall subject object,
   holds (convert (Some subject are object))
   ->
   holds (Some subject are object).

Axiom ConvE2:
   forall subject object,
   holds (convert (No subject are object))
   ->
   holds (No subject are object).
```

Custom-made tactic `eliminateConversion1` applies `ConvE1` and then unfolds `convert`. Custom-made tactic `eliminateConversion2` applies `ConvE2` and then unfolds `convert`.

Contraposition

Definition (ContrDef)**.** *For all terms t_1 and t_2, we define*

$$\begin{aligned} \mathtt{contrapose}(\mathtt{All}\ t_1\ \mathtt{are}\ t_2) &= \mathtt{All\ non}\ t_2\ \mathtt{are\ non}\ t_1 \\ \mathtt{contrapose}(\mathtt{Some}\ t_1\ \mathtt{are}\ t_2) &= \mathtt{Some\ non}\ t_2\ \mathtt{are\ non}\ t_1 \\ \mathtt{contrapose}(\mathtt{No}\ t_1\ \mathtt{are}\ t_2) &= \mathtt{No\ non}\ t_2\ \mathtt{are\ non}\ t_1 \\ \mathtt{contrapose}(\mathtt{Some}\ t_1\ \mathtt{are\ not}\ t_2) &= \mathtt{Some\ non}\ t_2\ \mathtt{are\ not\ non}\ t_1 \end{aligned}$$

```
Definition contrapose: CategoricalProposition ->
                       CategoricalProposition :=
  fun x : CategoricalProposition => match x with
    | cp quantity quality subject object =>
      cp quantity quality (non object) (non subject)
  end.
```

Axiom (ContrE1)**.** *If, for some terms t_1 and t_2, the proposition*

$$\mathtt{contrapose}(\mathtt{All}\ t_1\ \mathtt{are}\ t_2)$$

holds, then the proposition

$$\mathtt{All}\ t_1\ \mathtt{are}\ t_2$$

also holds.

Axiom (ContrE2)**.** *If, for some terms t_1 and t_2, the proposition*

$$\mathtt{contrapose}(\mathtt{Some}\ t_1\ \mathtt{are\ not}\ t_2)$$

holds, then the proposition

$$\mathtt{Some}\ t_1\ \mathtt{are\ not}\ t_2$$

also holds.

$$\frac{\textit{contrapose}(\texttt{All}\ t_1\ \texttt{are}\ t_2)}{\texttt{All}\ t_1\ \texttt{are}\ t_2}\,[\mathrm{ContrE}_1]$$

$$\frac{\textit{contrapose}(\texttt{Some}\ t_1\ \texttt{are not}\ t_2)}{\texttt{Some}\ t_1\ \texttt{are not}\ t_2}\,[\mathrm{ContrE}_2]$$

```
Axiom ContrE1:
   forall subject object,
   holds (contrapose (All subject are object))
   ->
   holds (All subject are object).

Axiom ContrE2:
   forall subject object,
   holds (contrapose (Some subject are not object))
   ->
   holds (Some subject are not object).
```

As with conversion, we have defined corresponding tactics `eliminateContraposition1` and `eliminateContraposition2` which allow us to rewrite the proof.

Custom-made tactic `eliminateContraposition1` applies `ContrE1` and then unfolds `contrapose`. Custom-made tactic `eliminateContraposition2` applies `ContrE2` and then unfolds `contrapose`.

Obversion

Definition (ObvDef)**.** *For all terms t_1 and t_2, we define*

$$\begin{aligned}
\textit{obvert}(\textit{All}\ t_1\ \textit{are}\ t_2) &= \textit{No}\ t_1\ \textit{are non}\ t_2\\
\textit{obvert}(\textit{Some}\ t_1\ \textit{are}\ t_2) &= \textit{Some}\ t_1\ \textit{are not non}\ t_2\\
\textit{obvert}(\textit{No}\ t_1\ \textit{are}\ t_2) &= \textit{All}\ t_1\ \textit{are non}\ t_2\\
\textit{obvert}(\textit{Some}\ t_1\ \textit{are not}\ t_2) &= \textit{Some}\ t_1\ \textit{are non}\ t_2
\end{aligned}$$

We first introduce a means to obtain the opposite of a quality.

```
Definition complement: Quality -> Quality :=
  fun x : Quality => match x with
    | affirmative => negative
    | negative => affirmative
  end.
```

```
Definition obvert: CategoricalProposition -> CategoricalProposition :=
  fun x : CategoricalProposition => match x with
    | cp quantity quality subject object
      => cp quantity (complement quality) subject (non object)
  end.
```

Axiom (ObvE)**.** *If, for some proposition p*

$$\texttt{obvert}(p)$$

holds, then the proposition p also holds.

```
Axiom ObvE :
   forall catprop, holds (obvert catprop) -> holds catprop.
```

$$\frac{obvert(p)}{p}\ [\text{ObvE}]$$

Custom-made tactic `eliminateObversion` applies `ObvE` and then unfolds `obvert`.

Syllogisms

Axiom (Barbara)**.** *For all terms minor, middle, and major, if* `All` *middle* `are` *major holds, and* `All` *minor* `are` *middle holds, then* `All` *minor* `are` *major also holds.*

$$\frac{\texttt{All}\ \textit{middle}\ \texttt{are}\ \textit{major} \qquad \texttt{All}\ \textit{minor}\ \texttt{are}\ \textit{middle}}{\texttt{All}\ \textit{minor}\ \texttt{are}\ \textit{major}}\ [\text{Barbara}]$$

```
Axiom Barbara : forall major minor middle,
    holds (All middle are major)
 /\ holds (All minor are middle)
 -> holds (All minor are major).
```

Axiom (Celarent)**.** *For all terms minor, middle, and major, if* `No` *middle* `are` *major holds, and* `All` *minor* `are` *middle holds, then* `No` *minor* `are` *major also holds.*

$$\frac{\texttt{No}\ \textit{middle}\ \texttt{are}\ \textit{major} \qquad \texttt{All}\ \textit{minor}\ \texttt{are}\ \textit{middle}}{\texttt{No}\ \textit{minor}\ \texttt{are}\ \textit{major}}\ [\text{Celarent}]$$

```
Axiom Celarent : forall major minor middle,
    holds (No middle are major)
 /\ holds (All minor are middle)
 -> holds (No minor are major).
```

Axiom (Darii)**.** *For all terms minor, middle, and major, if* `All` *middle* `are` *major holds, and* `Some` *minor* `are` *middle holds, then* `Some` *minor* `are` *major also holds.*

$$\frac{\texttt{All}\ \textit{middle}\ \texttt{are}\ \textit{major} \qquad \texttt{Some}\ \textit{minor}\ \texttt{are}\ \textit{middle}}{\texttt{Some}\ \textit{minor}\ \texttt{are}\ \textit{major}}\ [\text{Darii}]$$

```
Axiom Darii : forall major minor middle,
    holds (All middle are major)
 /\ holds (Some minor are middle)
 -> holds (Some minor are major).
```

Closing the Module

```
End TraditionalLogicCheat.
```