CS3234 Logic and Formal Systems

Self-Assessment 01: Predicate Calculus

The following questions serve the students of CS3234 to assess their understanding of the course material. This self-assessment is not marked. Please use the corresponding discussion forum to discuss this self-assessment.

1. In the lecture, you have seen the soundness result for predicate logic. Soundness provides you with a powerful tool to show that a proof of the validity of a sequent $\phi \vdash \psi$ is not possible. To do so, you only need to build a model in which the sequent’s premise holds, but in which the sequent’s conclusion does not hold.

   Use soundness to show the impossibility to prove the following sequents:
   
   (a) $\exists x(P(x), \exists y Q(y)) \vdash \exists z (P(z) \land Q(z))$
   
   (b) $\forall x \exists y S(x, y) \vdash \exists y \forall x S(x, y)$

2. Recall from class that the validity of an arbitrarily chosen sentence $\phi$ is undecidable. From this, show that the satisfiability of an arbitrarily chosen sentence $\phi$ is also undecidable.

3. Write a program in the language of your choice that takes two numbers $n$ and $m$. It then generates a solvable Post correspondence problem of length $n$, whose shortest solution (index sequence) has a size of at least $m$.

   Alternatively, you may use the program PcpSieve given in http://www.theory.informatik.uni-kassel.de/~stamer/pcp/pcpcontest_en.html.