01—Language Processing and Inductive Definitions

CS4215: Programming Language Implementation

Martin Henz

January 13, 2012

Generated on Thursday 12 January, 2012, 22:06

1. Brief Introduction to CS4215
   - Goal: Implementation Principles, Not "Hacking"
   - Method: "Learning by Programming"
   - Style: Incremental and Exploratory
   - Overview of Module Content

2. Administrative Matters

3. Language Processing

4. Inductive Definitions

5. ePL Outlook

Goal: Implementation Principles, Not "Hacking"

- Implementation of major programming language concepts
- As little "clutter" as possible
- Emphasis on the "what" of implementation: correctness w.r.t. given semantics

Learning By Programming

- Goal: get the insider's view on programming languages
- You will implement a sequence of toy languages
- You will write interpreters in Java
- You will write virtual machines in Java
- You will write toy programs in the toy languages
- Extensive software support provided

Incremental^2 and Exploratory^2

- Incremental: Sequence of programming languages, from simple expression-oriented to complex object-oriented
- Incremental: Sequence of implementation techniques, from the simplest interpreter-based implementation to realistic memory-aware virtual machines
- Exploratory: Plenty of scope for exploration, from the most basic to the most advanced topics in each section
- Exploratory: Opportunities for exploring related topics, hands-on, within module framework

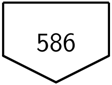
Overview of Module Content

1. Programming language processing tools and inductive definitions (2 hours)
2. ePL: An Expression language (2 hours)
3. simPL: A simple functional language (6 hours)
4. rePL: Records for Functional Programming (2 hours)
5. imPL: A Simple Imperative Language (3 hours)
6. oPL: A Simple Object-oriented Language (3 hours)
7. Memory management, garbage collection (3 hours)
8. Implementation of type systems (3 hours)
9. Combining implementation techniques (2 hours): virtual machines and interpreters, virtual machines and just-in-time compilation

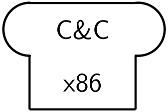
Administrative Matters

- Use `www.comp.nus.edu.sg/~cs4215` and IVLE
- Notes and slides (www; no textbook)
- Assignments (www; intensive work; marked; labs)
- Discussion forums (IVLE)
- Announcements (IVLE)
- Webcast (IVLE)
- No tutorials but labs; register!

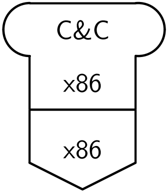
T-Diagrams



x86 Processor



Program “C&C” (x86 code)

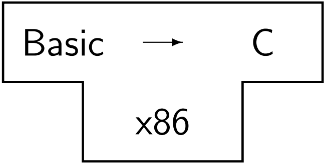


“C&C” running on x86

Translators

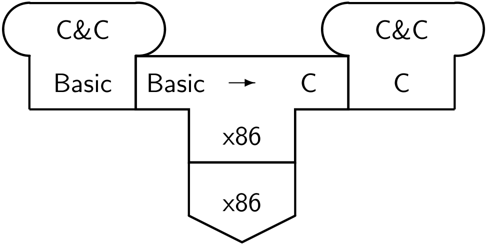
- Translator translates from one language—the *from-language*—to another language—the *to-language*
- Compiler translates from "high-level" language to "low-level" language
- De-compiler translates from "low-level" language to "high-level" language

T-Diagram of Translator



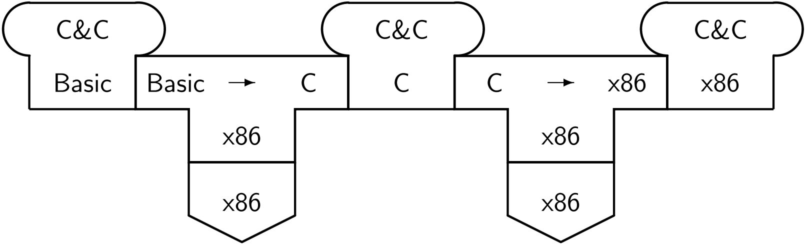
Basic-to-C compiler written in x86 machine code

Compilation



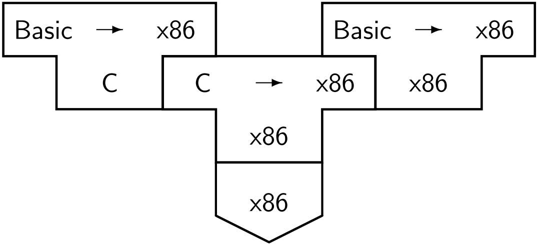
Compiling “C&C” from Basic to C

Two-stage Compilation



Compiling “C&C” from Basic to C to x86 machine code

Compiling a Compiler

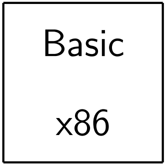


Compiling a Basic-to-x86 compiler from C to x86 machine code

Interpreter

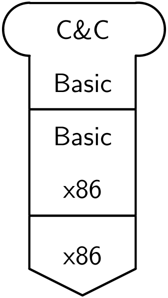
- Interpreter is program that executes another program
- The interpreter's *source language* is the language in which the interpreter is written
- The interpreter's *target language* is the language in which the programs are written which the interpreter can execute

Interpreters



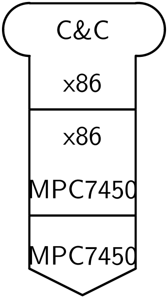
Interpreter for Basic, written in x86 machine code

Interpreting a Program



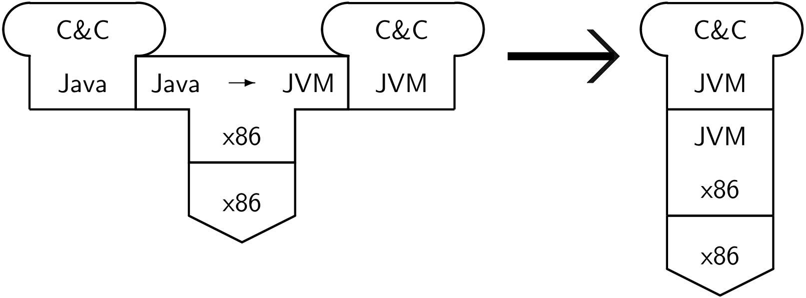
Basic program “C&C”
running on x86 using interpretation

Hardware Emulation



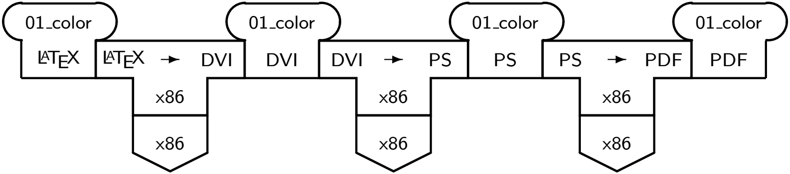
“C&C” x86 executable running on a PowerPC using hardware emulation

Typical Execution of Java Programs



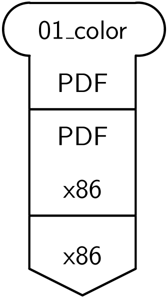
Compiling "C&C" from Java to JVM code, and running the JVM code on a JVM running on an x86

Excursion: Making these Slides



Compiling these slides
from LaTeX to DVI to PostScript to PDF on x86 (MBP)

Excursion: Viewing these Slides



Viewing the slides on a PC

Summary: Language Processing

- Components:
  programs, translators, interpreters, machines
- T-diagrams
- Combination of interpretation
  and compilation is common
- Interpretation and compilation
  are ubiquitous in computing

1. Brief Introduction to CS4215

2. Administrative Matters

3. Language Processing

4. Inductive Definitions
   - What are Inductive Definitions?
   - Extremal Clause
   - Proofs by Induction
   - Defining Sets by Rules in Java

5. ePL Outlook

Inductive Definitions

- We will frequently define a set by a collection of rules that determine the elements of that set.
  Example: the set of machine code programs for a particular virtual machine
- What does it mean to define a set by a collection of rules?

Example: Numerals

Numerals, in unary (base-1) notation

- *Zero* is a numeral;
- if n is a numeral, then so is $Succ(n)$.

Examples

- *Zero*
- $Succ(Succ(Succ(Zero)))$

Example: Binary Trees

Binary trees (w/o data at nodes)

- *Empty* is a binary tree;
- if l and r are binary trees, then so is $Node(l, r)$.

Examples

- *Empty*
- $Node(Node(Empty, Empty), Node(Empty, Empty))$

Examples (more formally)

- Numerals: The set *Num* is defined by the rules

$$\frac{}{Zero} \qquad \frac{n}{Succ(n)}$$

- Binary trees: The set *Tree* is defined by the rules

$$\frac{}{Empty} \qquad \frac{t_l \quad t_r}{Node(t_l, t_r)}$$

Defining a Set by Rules

- Given a collection of rules, what set does it define?
    - What is the set of numerals?
    - What is the set of trees?
- Do the rules pick out a unique set?

Defining a Set by Rules

- There can be many sets that satisfy a given collection of rules.
  - $Num = \{Zero, Succ(Zero), \ldots\}$
  - $StrangeNum = Num \cup \{\infty, Succ(\infty), \ldots\}$, where ∞ is an arbitrary symbol
- Both *Num* and *StrangeNum* satisfy the rules defining numerals (i.e., the rules are true for these sets). Really?

Num Satisfies the Rules

$$\frac{}{Zero} \qquad\qquad \frac{n}{Succ(n)}$$

$Num = \{Zero, Succ(Zero), Succ(Succ(Zero)), \ldots\}$
Does *Num* satisfy the rules?

- $Zero \in Num$. $\surd$
- If $n \in Num$, then $Succ(n) \in Num$. $\surd$

StrangeNum Satisfies the Rules

$$\frac{}{Zero} \qquad \frac{n}{Succ(n)}$$

$StrangeNum =$
$\{Zero, Succ(Zero), Succ(Succ(Zero)), \ldots\} \cup \{\infty, Succ(\infty), \ldots\}$
Does *StrangeNum* satisfy the rules?

- $Zero \in StrangeNum$. $\surd$
- If $n \in StrangeNum$, then $Succ(n) \in StrangeNum$. $\surd$

Defining Sets by Rules

- Both *Num* and *StrangeNum* satisfy all rules.
- It is not enough that a set satisfies all rules.
- Something more is needed: an *extremal* clause.
  - "and nothing else"
  - "the least set that satisfies these rules"

Inductive Definitions

- An inductively defined set is the least set that satisfies a given set of rules.
- Example: Num is the least set that satisfies these rules:
  - $Zero \in Num$
  - if $n \in Num$, then $Succ(n) \in Num$.

Inductive Definitions

Question: What do we mean by “least”?
Answer: The smallest with respect to the subset ordering on sets.

- Contains no “junk”, only what is required by the rules.
- Since $\mathit{StrangeNum} \supsetneq \mathit{Num}$, *StrangeNum* is ruled out by the extremal clause.
- *Num* is “ruled in” because it has no “junk”.

What's the Big Deal?

- Inductively defined sets "come with" an induction principle.
- Suppose I is inductively defined by rules R.
- To show that every $x \in I$ has property P, it is enough to show that P satisfies the rules of R.
- Sometimes called *structural induction* or *rule induction*.

Example: Parity of Numerals

- The numeral *Zero* has parity **0**.
- Any numeral $Succ(n)$ has parity $1 - p$ if p is the parity of n
- Let P be the following property:
  Every numeral has either parity 0 or parity 1.
- Does P satisfy the rules $\dfrac{}{Zero} \qquad \dfrac{n}{Succ(n)}$?

Induction Principle

- To show that every $n \in Num$ has property P, it is enough to show:
  - $Zero$ has property P.
  - if n has property P, then $Succ(n)$ has property P.
- This is just ordinary mathematical induction!

Induction Principle

- To show that every tree has property P, it is enough to show that
  - *Empty* has property P.
  - if l and r have property P, then so does $Node(l, r)$.
- We call this *structural induction on trees*.

Example: Height of a Tree

- To show: Every tree has a height, defined as follows:
  - The height of *Empty* is 0.
  - If l has height h_l and the tree r has height h_r, then the tree $Node(l, r)$ has height $1 + max(h_l, h_r)$.
- Clearly, every tree has at most one height, but does it have a height at all?

Example: height

- It may seem obvious that every tree has a height, but notice that the justification relies on structural induction!
    - An “infinite tree” does not have a height!
    - But the extremal clause rules out the infinite tree!

Example: height

- Formally, we prove that for every tree *t*,
  there exists a number *h* satisfying the specification of height.
- Proceed by induction on the rules defining trees, showing that the property "there exists a height *h* for *t*" satisfies these rules.

Example: height

- Rule 1: *Empty* is a tree.
  Does there exist h such that h is the height of *Empty*?
  Yes! Take h=0.
- Rule 2: $Node(l, r)$ is a tree if l and r are trees.
  Suppose that there exists h_l and h_r, the heights of l and r, respectively.
  Does there exist h such that h is the height of $Node(l, r)$?
  Yes! Take $h = 1 + max(h_l, h_r)$.

Encoding Numerals in Java

```
interface Num {}
class Zero implements Num {}
class Succ implements Num {
   public Num pred;
   Succ(Num p) {pred = p;}
}
Num my_num = new Zero();
Num my_other_num =
   new Succ(new Succ(new Zero()));
```

Encoding Trees in Java

```
interface Tree {}
class Empty implements Tree {}
class Node implements Tree {
   public Tree left, right;
   Node(Tree l,Tree r) {
      left = l; right = r;}
}
Tree my_tree =
   new Node(new Empty(),
            new Node(new Node(new Empty(),
                              new Empty()),
                     new Empty()));
```

Constructors and Rules

- The constructors of the classes correspond to the rules in the inductive definition.
- Numerals
  - `new Zero()` is of type `Num`
  - if `n` is of type `Num`, then `new Succ(n)` is of type `Num`
- Trees
  - `new Empty()` is of type `Tree`
  - if `l` and `r` are of type `Tree`, then `new Node(l,r)` is of type `Tree`

Analogy with Java

- We assume an implicit extremal clause: no other classes implement the interface.
- The associated induction principle may be used to prove termination and correctness of functions.

Example: Height in Java

```
interface Tree {
   public int height();
}
class Empty implements Tree {
   public int height() {return 0;}
}
class Node implements Tree {
   public Tree left, right;
   Node(Tree l,Tree r) \{left = l; right = r;}
   public int height() {
      return 1 + max(height(left),height(right));
   }
}
```

Proving Termination of Java Program

Why does `height(t)` terminate for every tree `t`?

- For every `t` of type `Tree`, does there exist `h` such that `height(t)` returns `h`?
- Proof similar to above!

Summary

- An inductively defined set is the least set that satisfies a collection of rules.
- Rules have the form:
  "If $x_1 \in X$ and $\ldots$ and $x_n \in X$, then $x \in X$."
- Notation: $$\frac{x_1 \qquad \cdots \qquad x_n}{x}$$

Summary

- Inductively defined sets admit proofs by rule induction.
- For each rule

$$\frac{x_1 \qquad \cdots \qquad x_n}{x}$$

  assume that $x_1 \in P$, ..., $x_n \in P$, and show that $x \in P$.
- Conclude that every element of the set is in P.

1. Brief Introduction to CS4215
2. Administrative Matters
3. Language Processing
4. Inductive Definitions
5. ePL Outlook

Syntax of ePL

$$\frac{}{n} \qquad \frac{}{\texttt{true}} \qquad \frac{}{\texttt{false}}$$

$$\frac{E}{p_1[E]} \qquad \frac{E_1 \quad E_2}{p_2[E_1, E_2]}$$

$P_1 = \{\texttt{\textbackslash}\}$,
$P_2 = \{\texttt{|}, \texttt{\&}, \texttt{+}, \texttt{-}, \texttt{*}, \texttt{/}, \texttt{=}, \texttt{>}, \texttt{<}\}$.

Outlook to Week 2

- Syntax (parsing)
- Semantics (static and dynamic)
- Implementations:
  - typing
  - small step interpreter
  - big step interpreter

(compiler-based implementations in Week 3)