Mean Shift Tracking

CS4243 Computer Vision and Pattern Recognition

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Mean Shift

Mean Shift [Che98, FH75, Sil86]

- An algorithm that iteratively shifts a data point to the average of data points in its neighborhood.
- Similar to clustering.
- Useful for clustering, mode seeking, probability density estimation, tracking, etc.

- Consider a set S of n data points $\mathbf{x}_i$ in d-D Euclidean space X.
- Let $K(\mathbf{x})$ denote a kernel function that indicates how much $\mathbf{x}$ contributes to the estimation of the mean.
- Then, the sample mean $\mathbf{m}$ at $\mathbf{x}$ with kernel K is given by

$$\mathbf{m}(\mathbf{x}) = \frac{\displaystyle\sum_{i=1}^{n} K(\mathbf{x} - \mathbf{x}_i)\,\mathbf{x}_i}{\displaystyle\sum_{i=1}^{n} K(\mathbf{x} - \mathbf{x}_i)} \tag{1}$$

- The difference $\mathbf{m}(\mathbf{x}) - \mathbf{x}$ is called mean shift.
- Mean shift algorithm: iteratively move date point to its mean.
- In each iteration, $\mathbf{x} \leftarrow \mathbf{m}(\mathbf{x})$.
- The algorithm stops when $\mathbf{m}(\mathbf{x}) = \mathbf{x}$.

- The sequence $\mathbf{x}, \mathbf{m}(\mathbf{x}), \mathbf{m}(\mathbf{m}(\mathbf{x})), \ldots$ is called the trajectory of $\mathbf{x}$.
- If sample means are computed at multiple points, then at each iteration, update is done simultaneously to all these points.

Kernel

- Typically, kernel K is a function of $\|\mathbf{x}\|^2$:

$$K(\mathbf{x}) = k(\|\mathbf{x}\|^2) \tag{2}$$

- k is called the profile of K.

Properties of Profile:

1. k is nonnegative.
2. k is nonincreasing: $k(x) \geq k(y)$ if $x < y$.
3. k is piecewise continuous and

$$\int_0^\infty k(x)dx < \infty \tag{3}$$

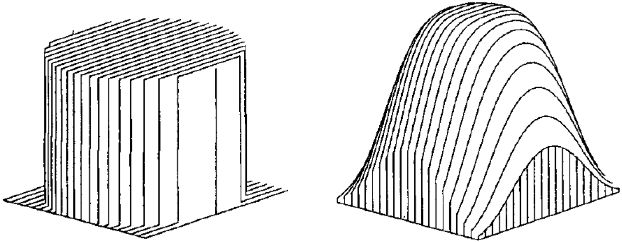
Examples of kernels [Che98]:

- Flat kernel:

$$K(\mathbf{x}) = \begin{cases} 1 & \text{if } \|\mathbf{x}\| \leq 1 \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

- Gaussian kernel:

$$K(\mathbf{x}) = \exp(-\|\mathbf{x}\|^2) \tag{5}$$



(a) Flat kernel (b) Gaussian kernel

Density Estimation

- Kernel density estimation (Parzen window technique) is a popular method for estimating probability density [CRM00, CRM02, DH73].
- For a set of n data points $\mathbf{x}_i$ in d-D space, the kernel density estimate with kernel $K(\mathbf{x})$ (profile $k(x)$) and radius h is

$$\begin{aligned} \tilde{f}_K(\mathbf{x}) &= \frac{1}{nh^d} \sum_{i=1}^{n} K\left(\frac{\mathbf{x}-\mathbf{x}_i}{h}\right) \\ &= \frac{1}{nh^d} \sum_{i=1}^{n} k\left(\left\|\frac{\mathbf{x}-\mathbf{x}_i}{h}\right\|^2\right) \end{aligned} \tag{6}$$

- The quality of kernel density estimator is measured by the mean squared error between the actual density and the estimate.

- Mean squared error is minimized by the Epanechnikov kernel:

$$K_E(\mathbf{x}) = \begin{cases} \frac{1}{2C_d}(d+2)(1-\|\mathbf{x}\|^2) & \text{if } \|\mathbf{x}\| \le 1 \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

where C_d is the volume of the unit d-D sphere, with profile

$$k_E(x) = \begin{cases} \frac{1}{2C_d}(d+2)(1-x) & \text{if } 0 \le x \le 1 \\ 0 & \text{if } x > 1 \end{cases} \tag{8}$$

- A more commonly used kernel is Gaussian

$$K(\mathbf{x}) = \frac{1}{\sqrt{(2\pi)^d}} \exp\left(-\frac{1}{2}\|\mathbf{x}\|^2\right) \tag{9}$$

with profile

$$k(x) = \frac{1}{\sqrt{(2\pi)^d}} \exp\left(-\frac{1}{2}x\right) \tag{10}$$

- Define another kernel $G(\mathbf{x}) = g(\|\mathbf{x}\|^2)$ such that

$$g(x) = -k'(x) = -\frac{dk(x)}{dx}. \tag{11}$$

Important Result

Mean shift with kernel G moves $\mathbf{x}$ along the direction of the gradient of density estimate $\tilde{f}$ with kernel K.

- Define density estimate with kernel K.
- But, perform mean shift with kernel G.
- Then, mean shift performs gradient ascent on density estimate.

Proof:

- Define $\tilde{f}$ with kernel G

$$\tilde{f}_G(\mathbf{x}) \equiv \frac{C}{nh^d} \sum_{i=1}^{n} g\left(\left\| \frac{\mathbf{x} - \mathbf{x}_i}{h} \right\|^2 \right) \tag{12}$$

  where C is a normalization constant.

- Mean shift $\mathbf{M}$ with kernel G is

$$\mathbf{M}_G(\mathbf{x}) \equiv \frac{\sum_{i=1}^{n} g\left(\left\| \frac{\mathbf{x} - \mathbf{x}_i}{h} \right\|^2 \right) \mathbf{x}_i}{\sum_{i=1}^{n} g\left(\left\| \frac{\mathbf{x} - \mathbf{x}_i}{h} \right\|^2 \right)} - \mathbf{x} \tag{13}$$

- Estimate of density gradient is the gradient of density estimate

$$\begin{aligned}
\tilde{\nabla} f_K(\mathbf{x}) &\equiv \nabla \tilde{f}_K(\mathbf{x}) \\
&= \frac{2}{nh^{d+2}} \sum_{i=1}^{n} (\mathbf{x} - \mathbf{x}_i)\, k' \left(\left\| \frac{\mathbf{x} - \mathbf{x}_i}{h} \right\|^2 \right) \\
&= \frac{2}{nh^{d+2}} \sum_{i=1}^{n} (\mathbf{x}_i - \mathbf{x})\, g \left(\left\| \frac{\mathbf{x} - \mathbf{x}_i}{h} \right\|^2 \right) \\
&= \frac{2}{Ch^2} \tilde{f}_G(\mathbf{x}) \mathbf{M}_G(\mathbf{x})
\end{aligned} \tag{14}$$

- Then,

$$\mathbf{M}_G(\mathbf{x}) = \frac{Ch^2}{2} \frac{\tilde{\nabla} f_K(\mathbf{x})}{\tilde{f}_G(\mathbf{x})} \tag{15}$$

- Thus, $\mathbf{M}_G$ is an estimate of the normalized gradient of f_K.
- So, can use mean shift (Eq. 13) to obtain estimate of f_K.

Mean Shift Tracking

Basic Ideas [CRM00]:

- Model object using color probability density.
- Track target candidate in video by matching color probability of target with that of object model.
- Use mean shift to estimate color probability and target location.

Object Model

- Let $\mathbf{x}_i$, $i = 1, \ldots, n$, denote pixel locations of model centered at $\mathbf{0}$.
- Represent color distribution by discrete m-bin color histogram.
- Let $b(\mathbf{x}_i)$ denote the color bin of the color at $\mathbf{x}_i$.
- Assume size of model is normalized; so, kernel radius $h = 1$.
- Then, probability q of color u, $u = 1, \ldots, m$, in object model is

$$q_u = C \sum_{i=1}^{n} k(\|\mathbf{x}_i\|^2)\, \delta(b(\mathbf{x}_i) - u) \tag{16}$$

- C is the normalization constant

$$C = \left[\sum_{i=1}^{n} k(\|\mathbf{x}_i\|^2) \right]^{-1} \tag{17}$$

- Kernel profile k weights contribution by distance to centroid.

- δ is the Kronecker delta function

$$\delta(a) = \begin{cases} 1 & \text{if } a = 0 \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

That is, contribute $k(\|\mathbf{x}_i\|^2)$ to q_u if $b(\mathbf{x}_i) = u$.

Target Candidate

- Let $\mathbf{y}_i$, $i = 1, \ldots, n_h$, denote pixel locations of target centered at $\mathbf{y}$.
- Then, the probability p of color u in the target candidate is

$$p_u(\mathbf{y}) = C_h \sum_{i=1}^{n_h} k\left(\left\|\frac{\mathbf{y} - \mathbf{y}_i}{h}\right\|^2\right) \delta(b(\mathbf{y}_i) - u) \tag{19}$$

- C_h is the normalization constant

$$C_h = \left[\sum_{i=1}^{n_h} k\left(\left\|\frac{\mathbf{y} - \mathbf{y}_i}{h}\right\|^2\right)\right]^{-1} \tag{20}$$

Color Density Matching

- Measure similarity between object model q and color p of target at location $\mathbf{y}$.
- Use Bhattacharyya coefficient ρ

$$\rho(p(\mathbf{y}), q) = \sum_{u=1}^{m} \sqrt{p_u(\mathbf{y})\, q_u} \tag{21}$$

- ρ is the cosine of vectors $(\sqrt{p_1}, \ldots, \sqrt{p_m})^\top$ and $(\sqrt{q_1}, \ldots, \sqrt{q_m})^\top$.
- Large ρ means good color match.
- Let $\mathbf{y}$ denote current target location with color probability $\{p_u(\mathbf{y})\}$, $p_u(\mathbf{y}) > 0$ for $u = 1, \ldots, m$.
- Let $\mathbf{z}$ denote estimated new target location near $\mathbf{y}$, and color probability does not change drastically.

- By Taylor's series expansion

$$\rho(p(\mathbf{z}), q) = \frac{1}{2} \sum_{u=1}^{m} \sqrt{p_u(\mathbf{y})\, q_u} + \frac{1}{2} \sum_{u=1}^{m} p_u(\mathbf{z}) \sqrt{\frac{q_u}{p_u(\mathbf{y})}} \tag{22}$$

- Substitute Eq. 19 into Eq. 22 yields

$$\rho(p(\mathbf{z}), q) = \frac{1}{2} \sum_{u=1}^{m} \sqrt{p_u(\mathbf{y})\, q_u} + \frac{C_h}{2} \sum_{i=1}^{n_h} w_i k \left(\left\| \frac{\mathbf{z} - \mathbf{y}_i}{h} \right\|^2 \right) \tag{23}$$

where weight w_i is

$$w_i = \sum_{u=1}^{m} \delta(b(\mathbf{y}_i) - u) \sqrt{\frac{q_u}{p_u(\mathbf{y})}}. \tag{24}$$

- To maximize $\rho(p(\mathbf{z}), q)$, need to maximize second term of Eq. 23. First term is independent of $\mathbf{z}$.

Mean Shift Tracking Algorithm

Given $\{q_u\}$ of model and location $\mathbf{y}$ of target in previous frame:

(1) Initialize location of target in current frame as $\mathbf{y}$.

(2) Compute $\{p_u(\mathbf{y})\}$, $u = 1, \ldots, m$, and $\rho(p(\mathbf{y}), q)$.

(3) Compute weights w_i, $i = 1, \ldots, n_h$.

(4) Apply mean shift: Compute new location $\mathbf{z}$ as

$$\mathbf{z} = \frac{\displaystyle\sum_{i=1}^{n_h} w_i \, g\left(\left\| \frac{\mathbf{y} - \mathbf{y}_i}{h} \right\|^2\right) \mathbf{y}_i}{\displaystyle\sum_{i=1}^{n_h} w_i \, g\left(\left\| \frac{\mathbf{y} - \mathbf{y}_i}{h} \right\|^2\right)} \tag{25}$$

where $g(x) = -k'(x)$ (Eq. 11).

(5) Compute $\{p_u(\mathbf{z})\}$, $u = 1, \ldots, m$, and $\rho(p(\mathbf{z}), q)$.

(6) While $\rho(p(\mathbf{z}), q) < \rho(p(\mathbf{y}), q)$, do $\mathbf{z} \leftarrow \frac{1}{2}(\mathbf{y} + \mathbf{z})$.

(7) If $\|\mathbf{z} - \mathbf{y}\|$ is small enough, stop.
Else, set $\mathbf{y} \leftarrow \mathbf{z}$ and goto Step 1.

Notes:

- Step 4: In practice, a window of pixels $\mathbf{y}_i$ is considered.
Size of window is related to h.
- Step 6 is used to validate the target's new location.
Can stop Step 6 if $\mathbf{y}$ and $\mathbf{z}$ round off to the same pixel.
- Tests show that Step 6 is needed only 0.1% of the time.
- Step 7: Can stop algorithm if $\mathbf{y}$ and $\mathbf{z}$ round off to the same pixel.
- To adapt to change of scale, can modify window radius h and let algorithm converge to h that maximizes $\rho(p(\mathbf{y}, q))$.

Example 1: Track football player no. 78 [CRM00].

Example 2: Track a passenger in train station [CRM00].

CAMSHIFT

- Mean Shift tracking uses fixed color distribution.
- In some applications, color distribution can change, e.g., due to rotation in depth.

Continuous Adaptive Mean Shift (CAMSHIFT) [Bra98]

- Can handle dynamically changing color distribution.
- Adapt Mean Shift search window size and compute color distribution in search window.

In CAMSHIFT, search window W location is determined as follows:

- Compute the zeroth moment (mean) within W

$$M_{00} = \sum_{(x,y)\in W} I(x,y). \tag{26}$$

- Compute first moment for x and y

$$M_{10} = \sum_{(x,y)\in W} xI(x,y), \quad M_{01} = \sum_{(x,y)\in W} yI(x,y). \tag{27}$$

- Search window location is set at

$$x_c = \frac{M_{10}}{M_{00}}, \quad y_c = \frac{M_{01}}{M_{00}}. \tag{28}$$

CAMSHIFT Algorithm

(1) Choose the initial location of the search window.

(2) Perform Mean Shift tracking with revised method of setting search window location.

(3) Store zeroth moment.

(4) Set search window size to a function of zeroth moment.

(5) Repeat Steps 2 and 4 until convergence.

Example 3: Track shirt with changing color distribution [AXJ03].

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