

NATIONAL UNIVERSITY OF SINGAPORE
SCHOOL OF COMPUTING
EXAMINATION FOR
Semester 1, 2008/2009
CS 5201 - FOUNDATION IN THEORETICAL CS

November 2008

Time Allowed: 3 Hours

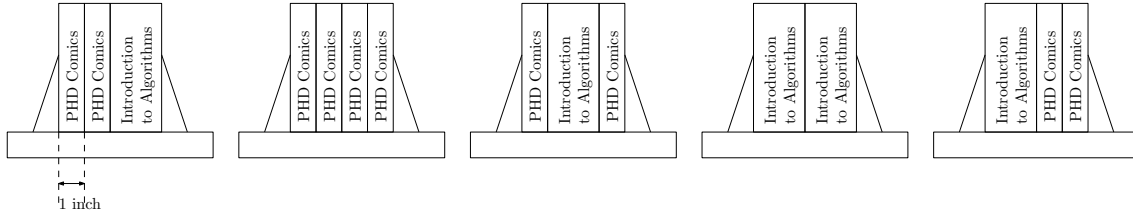
INSTRUCTIONS TO CANDIDATES

1. This examination paper contains **four**(4) long questions and comprises **six (6)** pages, including this page.
2. Answer **three out of four** questions.
3. Each question should be answered in a **separate** answer book.
4. This is an *OPEN BOOK* examination.
5. Write your Matriculation number in **all** the answer books.

A: Algorithm ((2+2.5)+2.5+3 = 10 marks)

Notice the *requirements* for each question, marks will be deducted if they are violated. Also, all graphs are assumed to be simple and undirected.

1. There is a bookshelf lining up a lot of books. However, all the books have only two types of books and thickness. Namely, some books are 1 inch thick and the others are 2 inches thick. When the books are placed on a shelf, there are a lot of ways to put the books. For example, if the shelf is 4 inches long, there are 5 ways of arrangements as the followings:



- (a) (2 marks) Write an algorithm in pseudo code to compute the number of ways of arranging the books for a shelf that is n inches long in $O(n)$ time with $O(1)$ memory space (including calling stack memory). (Hint: Consider the number of ways if we know the type of the last book.)
- (b) (2.5 marks) Write an algorithm to compute the same number of ways in $O(\log n)$ time (without the constraint of memory spaces). (Hint: consider the matrix $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$ and its power.)

Requirement: Your codes for (a) and (b) should not exceed 10 and 20 lines respectively. You can assume the matrix data structure and its addition, subtraction, multiplication, inverse and determinant are provided, but NOT its power.

2. (2.5 marks) Give an $O(n)$ algorithm to color the n vertices of a planar graph $G = (V, E)$ with 6 colors (assuming adjacency list structure for the graph) and each vertex has a different color from its neighbors.

Requirement: Your code or description must be cleared and justified to be $O(n)$.

3. Description of the longest path problem (LPP): Given a graph $G = (V, E)$ and a cost function $c : E \rightarrow \mathbb{Z}^+$ which maps every edge to a positive integer, does G contain a simple path (that is, a path encountering no vertex more than once) with a total cost greater than or equal to a given number k .

(3 marks) Prove that LPP is NP-complete. You can assume the following problems are NP-complete:

3-Satisfiability (3-SAT): Given a set U of variables, collection C of clauses over U such that each clause $c \in C$ has $|c| = 3$. Is there a satisfying truth assignment for C ?

Subgraph Isomorphism problem (SIP): Given two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, does G_1 contain a subgraph isomorphic to G_2 , that is, two subsets $V' \subseteq V_1$ and $E' \subseteq E_1$ such that $|V'| = |V_2|$, $|E'| = |E_2|$, and \exists a one-to-one function $f : V_2 \rightarrow V'$ satisfying $\{u, v\} \in E_2$ iff $\{f(u), f(v)\} \in E'$?

Hamiltonian circuit (HC): Given a graph $G = (V, E)$, does G contain a Hamiltonian circuit?

Vertex Cover (VC): Given a graph $G = (V, E)$ and a positive integer $k \leq |V|$, is there a vertex cover of size k or less for G , i.e., a subset $V' \subseteq V$ with $|V'| \leq k$ such that for each edge $\{u, v\} \in E$ at least one of u and v belongs to V' ?

Travelling Salesman Problem (TSP): Given a complete graph K_n with n vertices and the cost for each edge. Does there exist a simple cycle that traverses every vertex with a total cost less than or equal to a number k ?

Requirement: Your answer to this question is limited to **one** page.

B: Theory of Computation (10 marks)**Q1:**

Let $\Sigma = \{0, 1, 2, \dots, 9\}$. For a string $w \in \Sigma^*$, let $num(w)$ denote the decimal number represented by the string w . Thus, $num(252)$ is the number 252, and $num(534)$ is the number 534.

Give a Deterministic Finite State Automata to accept the language $\{w \in \Sigma^* : w \text{ starts with a non-zero character and } num(w) \text{ is divisible by } 3\}$.

Q2:

Show that $\{w \in \{a, b\}^* : \text{every prefix of } w \text{ has at least as many } a\text{'s as } b\text{'s}\}$ is a context free language.

Q3:

For a language L define $Suff(L) = \{y : (\exists x)[xy \in L]\}$.

(a) Prove or Disprove: If L is recursively enumerable, then $Suff(L)$ is also recursively enumerable.

(b) Prove or Disprove: If L is recursive, then $Suff(L)$ is recursive.

C: Principles of Programming Languages (10 marks)

A. Find a Haskell expression whose type is:

`(a -> a -> a) -> a -> a`

where `a` is a type variable. [1 mark]

B. Describe in detail the process of inferring the type of the following Haskell expression.

`\f -> \g -> \x -> (f (g f) (g f)) x`

[2 marks]

C. Consider the following C function.

```
void f(int a[], int n) {
    int i, j ;
    i = 1 ;
11:   if ( i >= n ) goto 12 ;
        j = i -1 ;
        k = a[i] ;
13:   if ( j < 0 ) goto 14 ;
        if (a[j]<=k) goto 14 ;
        a[j+1] = a[j] ;
        j-- ;
        goto 13 ;
14:   a[j+1] = k ;
        i ++ ;
        goto 11:
12:   return ;
}
```

Translate this function into Java.[4 marks]

D. Based on your experience in solving the previous question, propose a general scheme for translating C programs that use `goto` statements into Java. You may assume that the C programs have no loops.[3 marks]

D. Logic and Artificial Intelligence (3 + 4 + 3 = 10 marks)

(A) Determine for each of the following formulas the number of functions $f : \{0,1,2,3\} \rightarrow \{0,1,2,3\}$ such that the corresponding formula is true; the variables range over $\{0,1,2,3\}$.

(I) $\forall x [x = f(f(x))]$.

(II) $\forall x, y \exists z [f(x) \neq f(z) \wedge f(y) \neq f(z)]$.

(III) $f(0) = f(1) \wedge f(2) = f(3) \wedge f(0) + f(1) + f(2) + f(3) = 6$.

(B) Make first-order formulas in the language of nonnegative integers with plus, minus and times which describe the following Java Script functions f and g; the formulas can use the symbols “f” and “g”, respectively. The values of the inputs x and y are from $\{0,1,2,3,4,5,\dots\}$.

(I) Program for f.

```
function f(x)
{ var y = 0; var z = 0;
  while (z < x)
    { y = y+4*z+2; z = z+1; }
  return(y); }
```

(II) Program for g.

```
function g(x,y)
{ if (x < 1) { return(0); }
  if (y < 1) { return(0); }
  var v = x; var w = y;
  while (v != w)
    { if (v > w) { v = v-w; }
      else { w = w-v; } }
  z = x*y; w = 0;
  while (z > 0)
    { w = w+1; z = z-v; }
  return(w); }
```

(C) Let $\Phi = (x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee x_2 \vee x_4) \wedge (x_1 \vee x_3 \vee x_4) \wedge (x_2 \vee x_3 \vee x_4) \wedge (\neg x_1 \vee \neg x_2) \wedge (\neg x_3 \vee \neg x_4)$ be a formula in conjunctive normal form.

(I) Determine one solution of Φ .

(II) How many solutions does Φ have?

(III) Write Φ in disjunctive normal form (where the roles of \wedge and \vee are interchanged).