Berth Allocation Problem –

A Case Study in Algorithmic Approaches

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Content:

- ❖ **Problem**
- ❖ **Complexity & Relationships**
- ❖ **Approaches**
- ❖ **Findings**

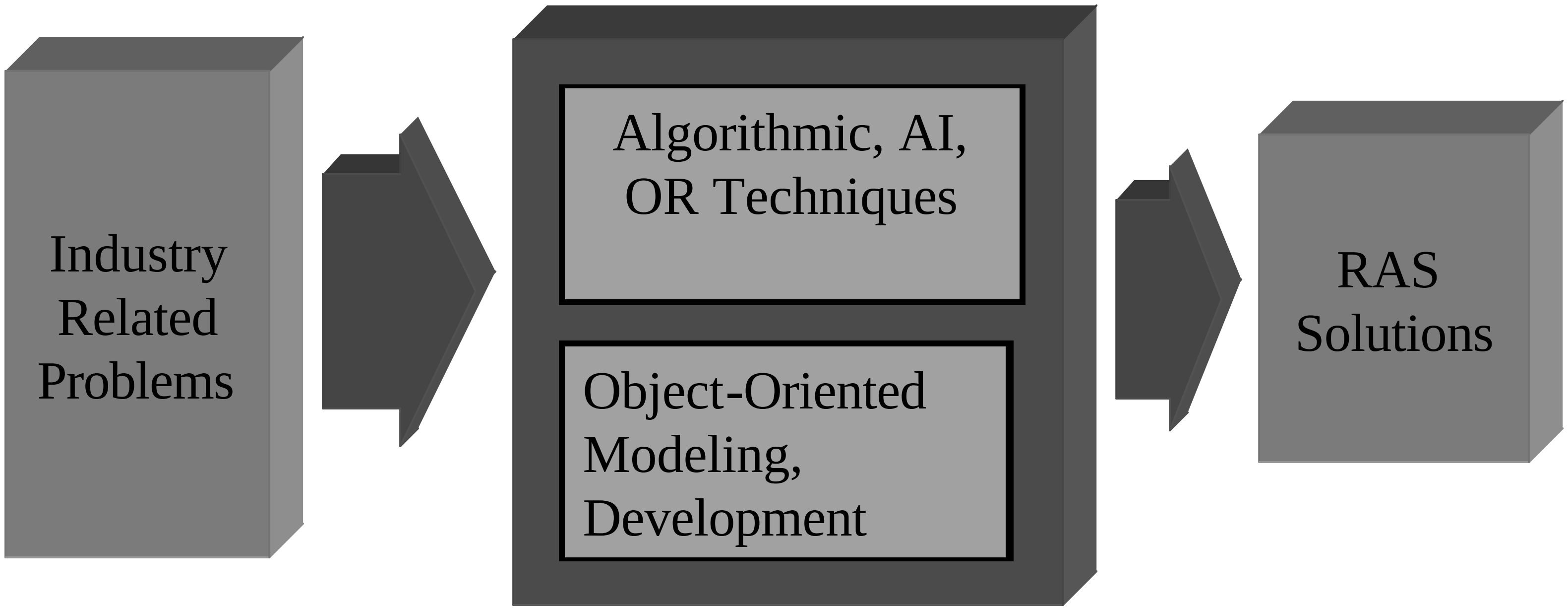
The RAS Group

❖ Real World Problems

- ❑ *Resource Allocation / Scheduling / Planning*
- ❑ *Resource Optimization Problems*

❖ Solution Technologies:

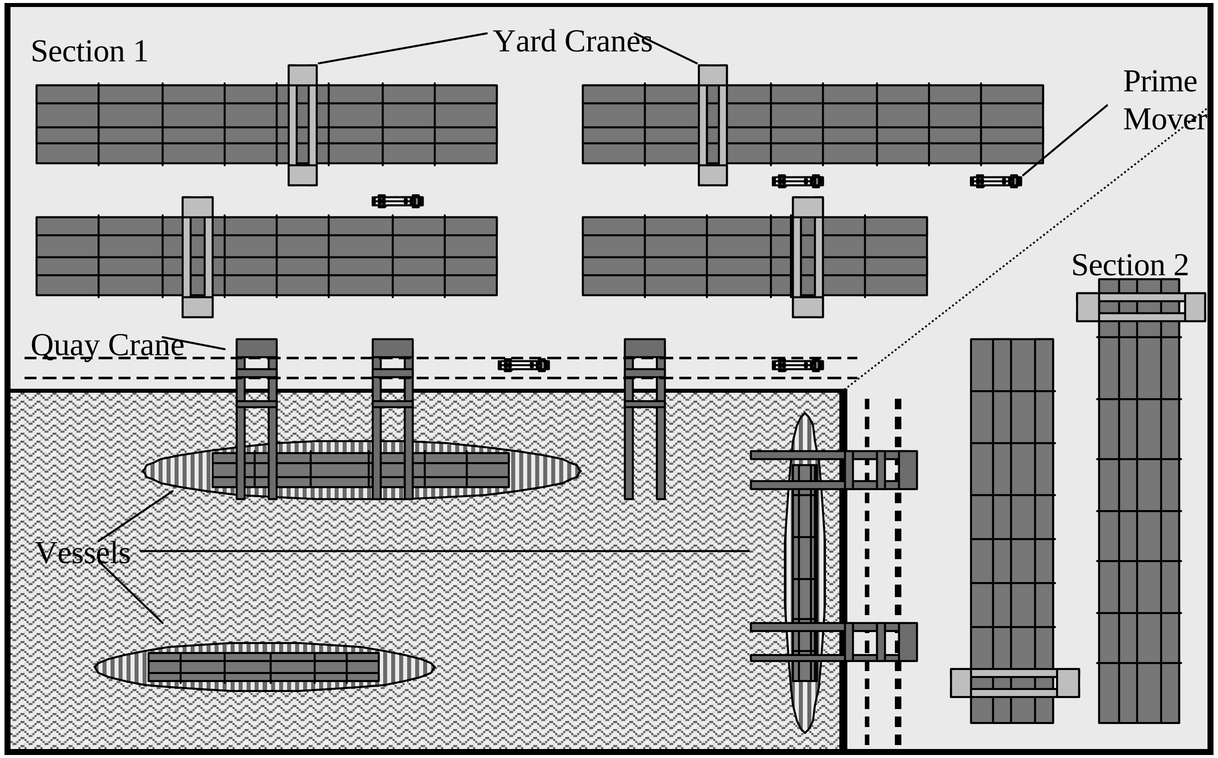
- ❑ *Algorithms, AI, OR/MP methods;*
- ❑ *Object-Oriented System Development*



The Berth Allocation Problem

Problem:

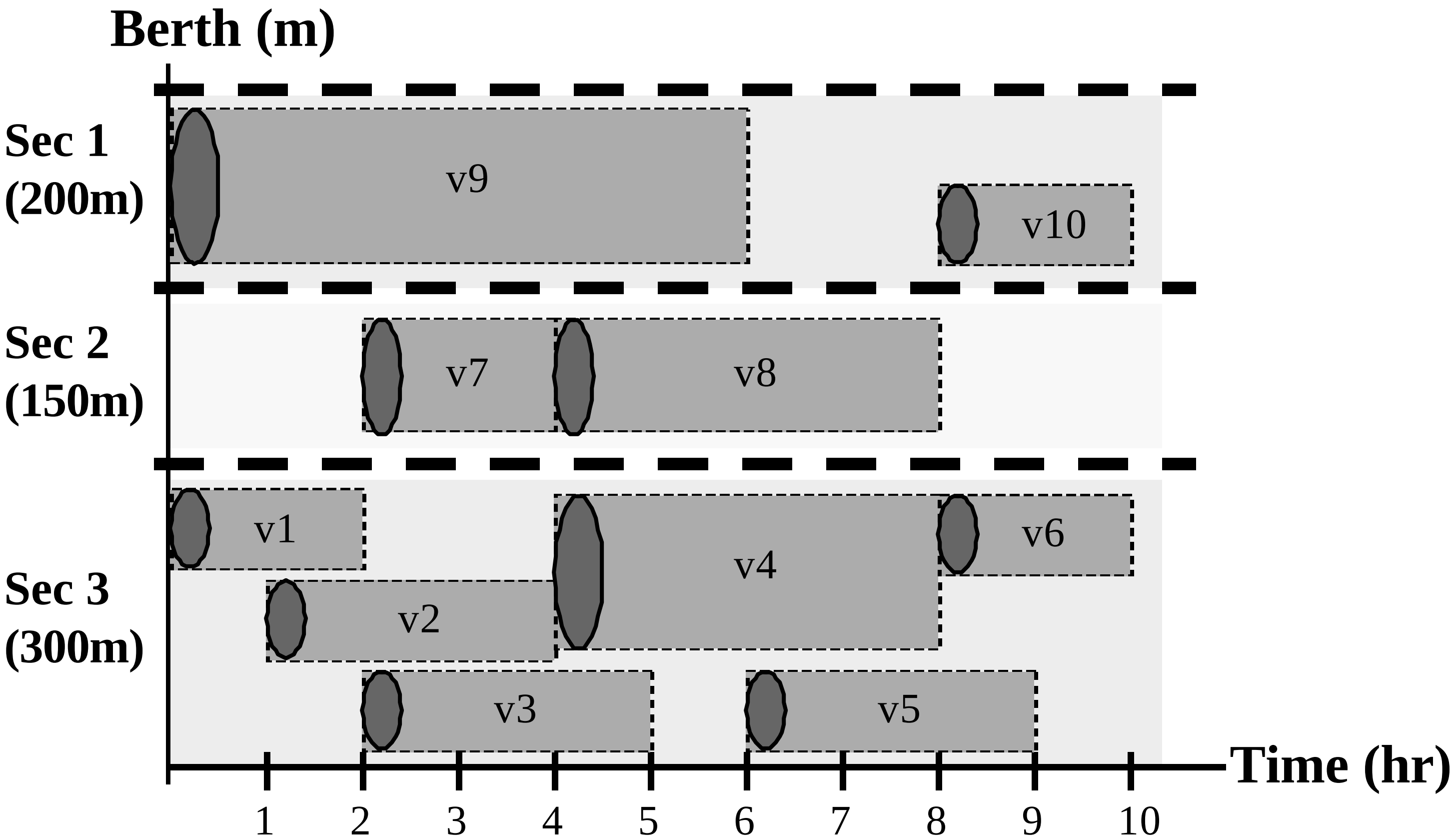
Vessels arriving at a container transhipment port are berthed in a section of wharf, and the containers they ferry are then transferred to other vessels or to the port.



Objective:

The BAP involves finding good berth allocations so as to minimise the movement of container traffic across wharf sections.

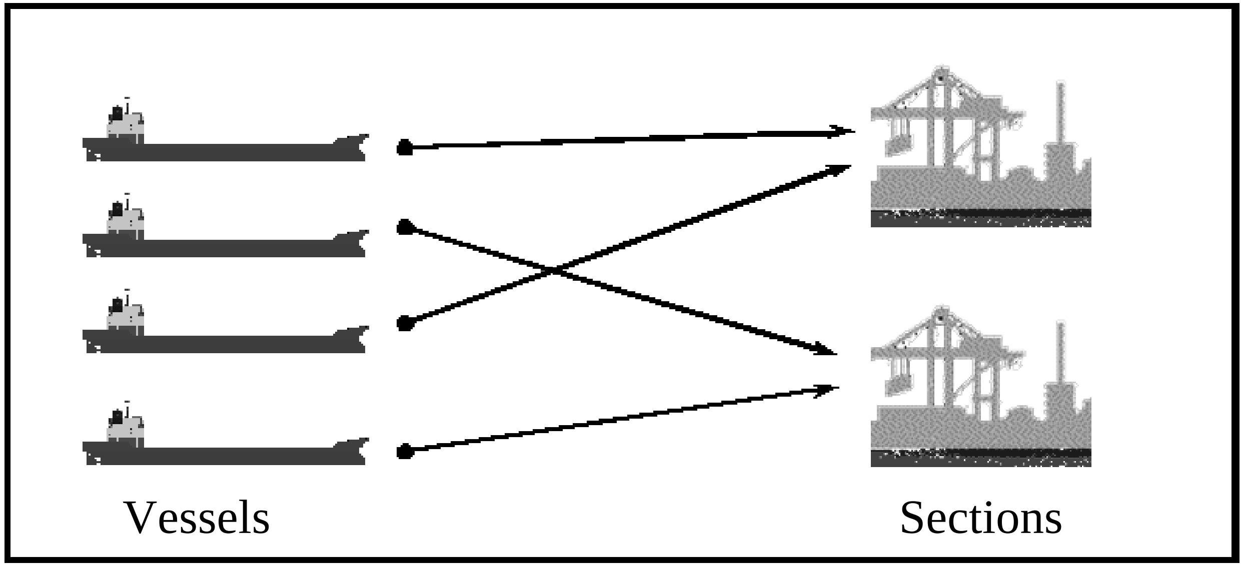
Schematic View of BAP Solution



Summary:

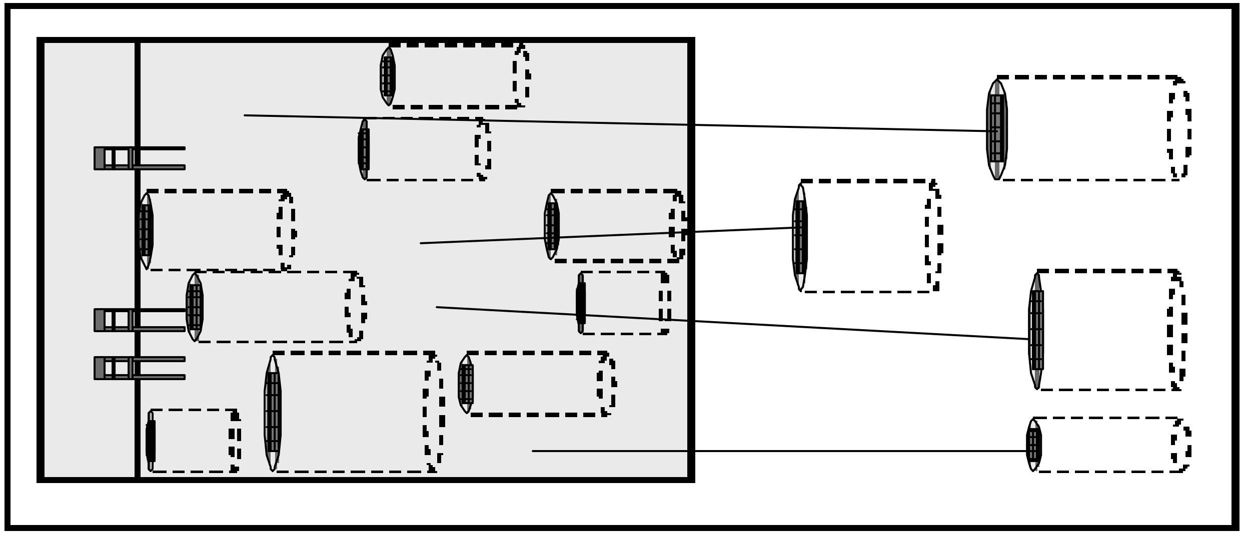
- Each vessel is assigned to a section (partition) and a wharfmark within the section (pack)

Two-Stage Approach



Partitioning

Assign vessels to sections



Packing

Assign wharfmarks to vessels

BAP: Planning vs Operations

- **BAPS (Planning System)**
  - *Longer Planning Time Window;*
  - *More interested in overall capacity handling*
  - *Global Key Performance Indicators (KPI's)*
  - *Used for Planning Resources and Manpower*

- **iBAPS (interactive System)**
  - *Shorter Time Horizon*
  - *Fast Handling of change requests*
  - *Other Operational issues*

BAP Research Direction

- **Rigorous Research**
- **Realistic Domain Modeling**
  - *BAP Port Survey Document*
- **Comprehensive Quality Benchmarking**
  - *BAP Data Generation Sub-System*
  - *BAP Experiment Sub-System*
- **Research Tool**
  - *Ability to Study "What-If" Scenarios*
  - *Planning vs. Operational Systems*
- **Oriented towards Technology Transfer**
  - *Prototype Development (Proof-of-concept)*
  - *Deployment potential*

The BAP Road Map

[Loh96]

	BAPS 1.0 (Date: Aug 1997)	BAPS 2.0 (Date: May 2000)
Partitioning	Modified [Loh96]	BAP-BB [Xu99]; BAP-GP [Ong00]; BAP-GA [Foo00]
Packing	Heuristic Packer [LeKu96]	DSA [Quek98]; BAP-MP [Li99]; BAP-Pack [Chen99]
Expt.	Port Survey [Foo96]; Data Gen. [OnFo97]; Expt. & Analysis [OnFo97]	Expt. & Analysis [OnFo98]
System	BAPS GUI [FYP]	BAPS GUI 2.0 [Ong00]
Interactive/Delay		Interactive iBAPS [Sim99]; Interactive iBAPS 2.0 [Nisha00]

BAPS 1.0 Date: Aug 1997

BAPS 2.0 Date: May 2000

Date:

BAP Modelling

❖ Port & Section Information

$S = \{S_1, S_2, \ldots, S_m\}$ -- sections in the port P

- ***Section S_k has length L_k***
- ***S_0 is a pseudo-section***

$D = [\, d_{kl} \,]$ – inter-section distance matrix (m+1 x m+1)

❖ Vessel Information

$V = \{V_1, V_2, \ldots, V_n\}$ -- vessels arriving at a port P

- ***Vessel V_k -- length l_k, arrival time a_k, departure time d_k,***
- ***V_0 is a pseudo-vessel (model import-export containers)***

❖ Container Information

$F = [\, f_{ij} \,]$ -- container flow matrix (n+1 x n+1)

BAP Modelling (continued….)

- **Planning Information**
  - *A given planning time horizon*

- **TO DO:**
  - *Assign each vessel* V_i *to a section* S_k *and a wharfmark* w_i *within that section*

- **Objective:**
  - *Minimize the Transhipment Cost*
  - *Secondary: Minimize number of unassigned vessels*

BAP Modelling (continued…)

❖ **Decision Variables:**

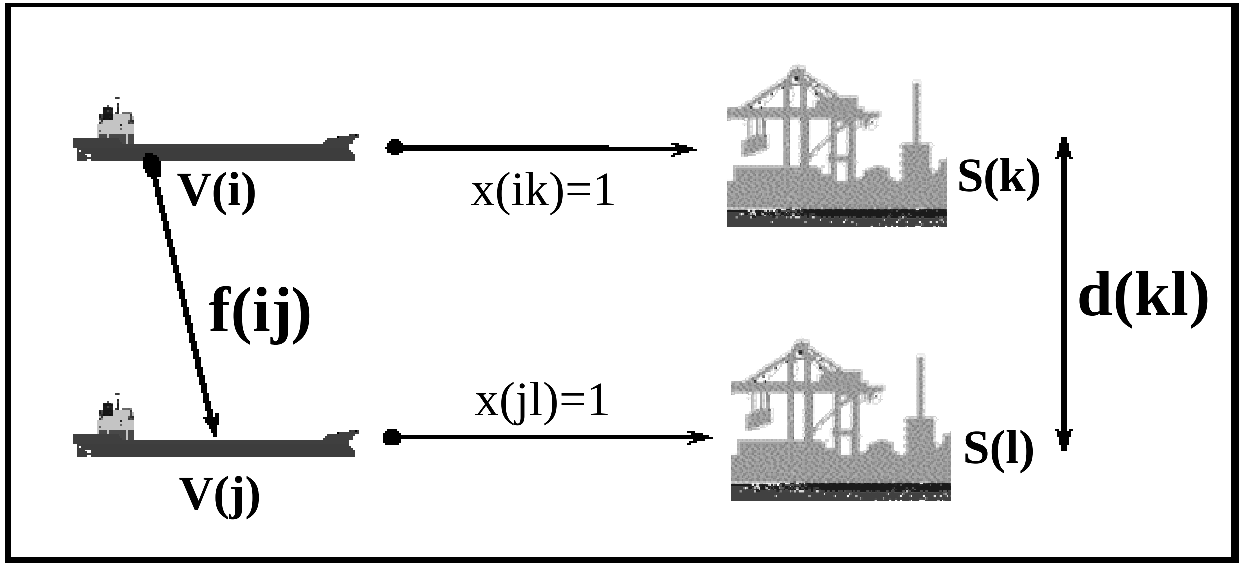
$$X = [\, x_{\text{ik}} \,], \quad \text{where } x_{\text{ik}} = \begin{cases} 1 & \text{if } v_i \text{ is assigned to } s_k \\ 0 & \text{otherwise} \end{cases}$$

w_i = wharf mark assigned to vessel v_i

❖ **Transshipment Cost: (QAP-cost?)**

$$Minimize \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=0}^{m} \sum_{l=0}^{m} f_{ij} d_{kl} x_{ik} x_{jl}$$

Transshipment Cost



- Assume that
  - ✓Transshipment f(ij) containers from V(i) to V(j)
  - ✓Vessel V(i) is assigned to section S(k) [x(ik)=1]
  - ✓Vessel V(j) is assigned to section S(l) [x(jl)=1]
  - ✓Distance from S(k) to S(l) is d(kl)
- Then Transshipment cost is given by

f(ij) * d(kl) * x(ik) * x(jl)

The BAP Problem Model

❖ Objective

- *Minimize Transshipment Cost*

$$\text{Minimize} \sum_{i=0}^{n}\sum_{j=0}^{n}\sum_{k=0}^{m}\sum_{l=0}^{m} f_{ij} d_{kl} x_{ik} x_{jl}$$

❖ Constraints

- *One section per vessel*
- *Vessels berthed cannot exceed section capacity*
- *One berthing location per vessel*
- *No overlap of vessels*
- *No straddling across section boundaries*

Constraints (Math Modelling)

❖ **Vessels assigned to one section each**

$$\sum_{k=1}^{m} x_{ik} = 1 \qquad \text{for all } i = 1, 2, \ldots, n$$

❖ **Vessels do not straddle between sections**

$$(w_i + l_i)*x_{ik} \le L_k*x_{ik} \quad \text{for all } i = 1,2,\ldots,n$$

❖ **Section capacity is not exceeded**

$$\sum_{i=1}^{n} x_{ik}\, l_i\, y_{it} \le L_k \qquad \text{for all } t = 0,\ldots,T \text{ and } k=1,\ldots,m$$

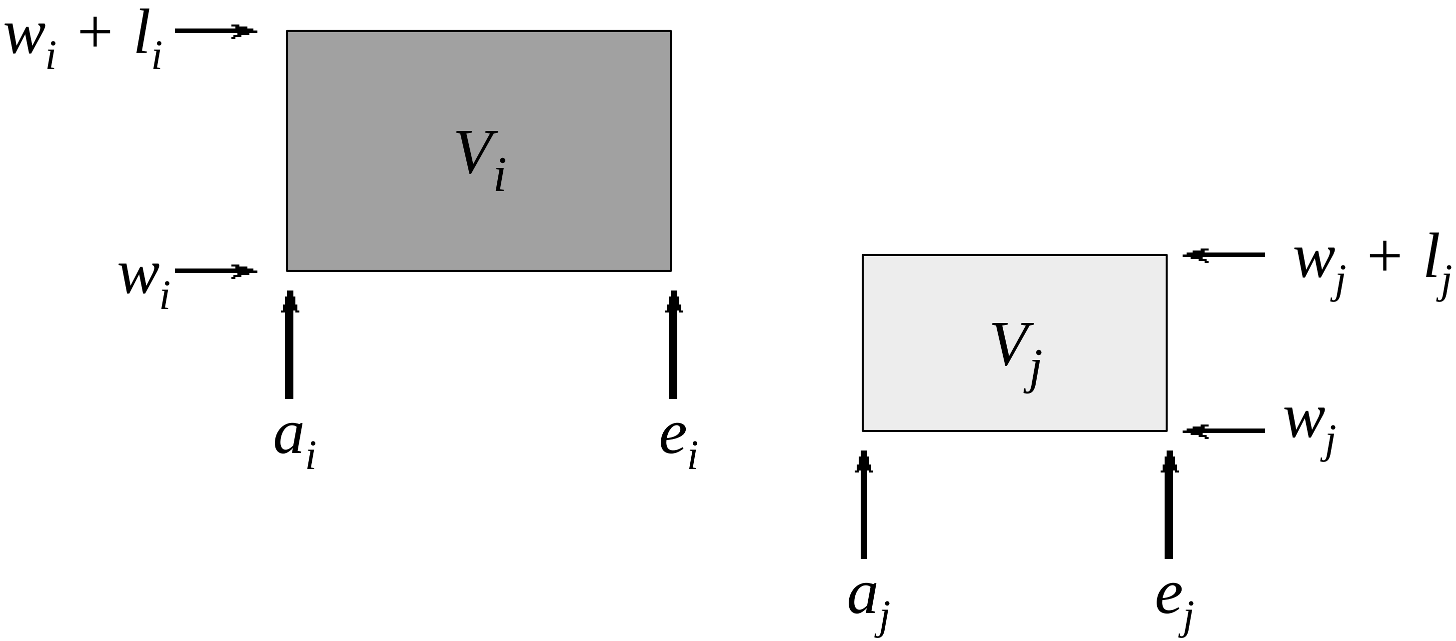
Overlap Constraints

- **For all distinct vessels i and j,**

$(x_{ik}\, x_{jk} = 1)$ → one of the following must be true

$e_i < a_j$ $\qquad$ $w_i + l_i < w_j$

$a_i > e_j$ $\qquad$ $w_i > w_j + l_j$



(Summary of Constraints)

- **Constraints**
  - *Vessel must lie entirely within a section*
  - *Concurrent vessels does not exceed section capacity*
  - *Implicit: No collision (space-and-time overlap)*
- **Assumptions**
  - *Sections are straight lines*
  - *Any vessel can berth anywhere in any section*
  - *Does not consider quay crane / prime movers*
  - *Arrival and departure times are **fixed in advance***
- **Simplifications**
  - *Approx inter-section distances (centre-to-centre)*

Model Augmentation

- **To Model "Import/Export" Containers**
  - *Use a Pseudo Vessel (assigned to pseudo Section 0)*

- **To Handle Unassigned Vessels**
  - *Have a Pseudo Section (very far away)*
  - *Incorporate a Penalty in the Cost Function*

BAP Literature Review

- **Related Problems**
  - *QAP (Quadratic Assignment Problem)*
  - *GAP (airport Gate Allocation Problem)*
- **QAP (Quadratic Assignment Problem)**
  - *aka Facility Layout Problem*
  - *Widely studied*
  - *Given n facilities, to be located in n fixed places*
- **GAP (*airport* Gate Allocation Problem)**
  - *arriving airplanes to be allocated gates in airport*
- **NP-hard Problems**
  - QAP → GAP → BAP
  - QAP is NP-Hard, so are GAP, BAP

BAP Problem Hierarchy

- **QAP (Quadratic Assignment Problem)**
  - *Mapping: n facilities → n fixed places*
  - *(facilities=vessel); (places=sections)*
  - *No time dimension; 1 vessel per section;*
- **GAP (*airport* Gate Allocation Problem)**
  - *Mapping: airplanes → gates in airport*
  - *(airplaces=vessel); (gates=sections)*
  - *Time Dim; 1 vessel per section at any given time*
- **BAP (*seaport* Berth Allocation Problem)**
  - *Mapping: vessels → wharfmarks in sections*
  - *Time Dimension;*
  - *Many vessels per section at any given time*

BAP – Research into Solutions

- **BAP Domain Survey**
  - *Understand domain entity, problem, issues*
  - *Know the data*

- **BAP Literature Research**
  - *QAP → GAP → BAP*
  - *GP (graph partitioning) → BAP-Partitioning*
  - *Both QAP and GP are well-known problems*

  - *BAP-Packing aka DSA (dynamic storage allocation)*
  - *BAP-Packing – rectangle packing and approx alg.*

BAP Partitioning Algorithms

- **BAP Partitioning**
  - *Generalization of QAP and Graph Partitioning*
  - *NP-hard*
- **Several Different Approaches**
  - *Graph Partitioning* *(fast, good results)*
  - *Genetic Algorithm* *(slower, very good results)*
  - *Branch-and-Bound* *(very slow, optimal results)*
- **User can do Time-vs-Quality Tradeoffs**

BAP Partitioning Alg (cont…)

- **Graph Partitioning Alg [OTW]**
  - *Adapted from GP algorithm by KL and FM*
  - *Constructive Methods for Initial Solutions*
  - *Iterative Improvement using Modified FM*
  - *Post-Processor using Multiple-Knapsack*
- **Genetic Algorithm [FHM]**
  - *Expt with standard GA: group-based, ordered-based*
  - *Domain-specific GA: Grouping GA*
- **Branch-and-Bound & Sim. Annealing [XDG]**
  - *Works only for small problem sizes*
  - *Useful tool for checking solutions*

BAP Packing Algorithms

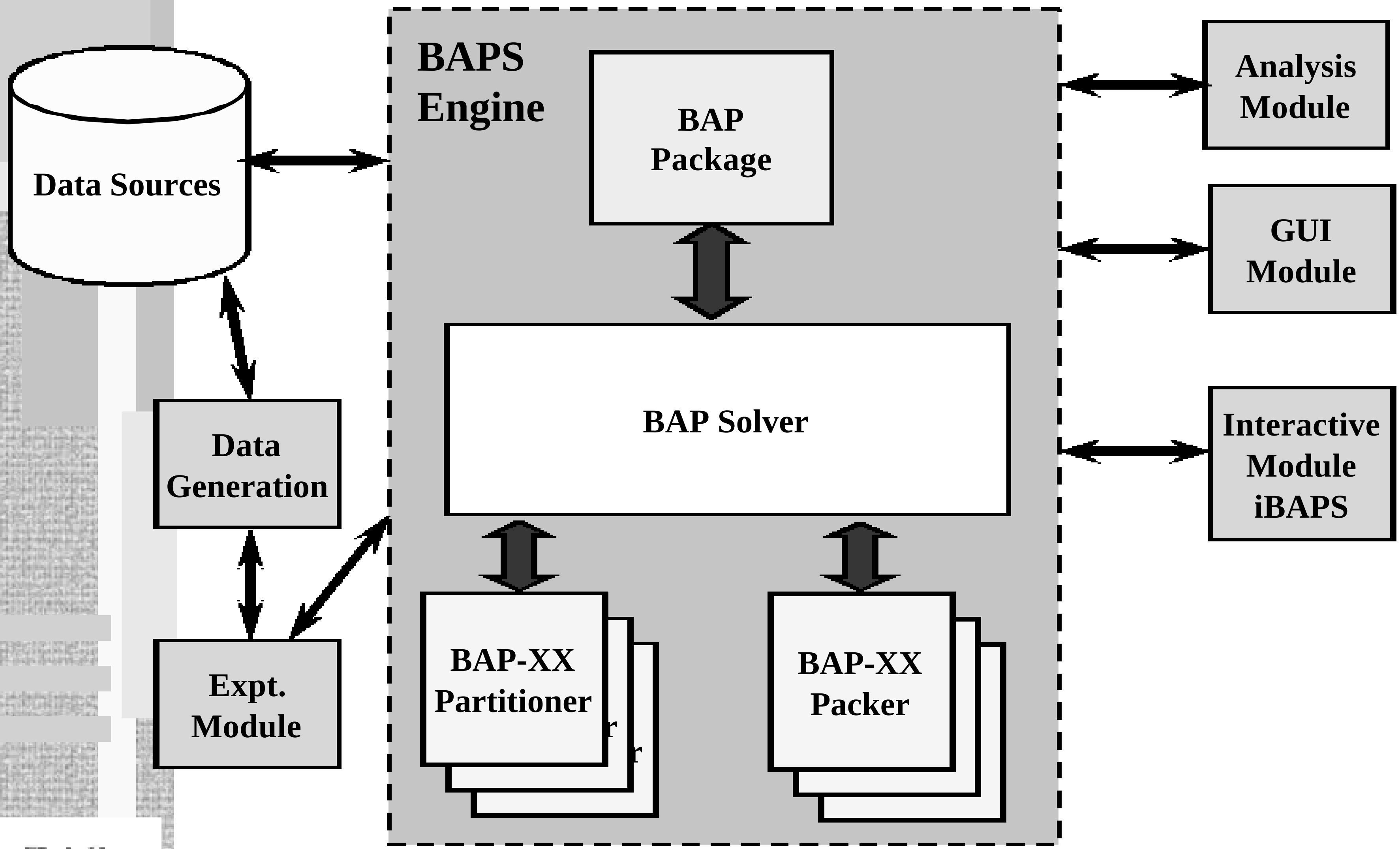
- **BAP Packing is NP-hard**
  - *Studied as DSA problem (memory management)*
  - *Similar to rectangle packing with constraints*
  - ***Best known is 3-approx algorithm* [Gergov]**
  - *Other Recent Work: Tabu Search, Ant Colony, etc…*
- **Research Bin Packing & Approx Algs [CLW]**
  - *Comprehensive Study of bin-packing methods*
  - *Implemented and Integrated*
- **Major Findings**
  - *Online Alg: (Best: Best-Fit, First-Fit)*
  - *Offline Alg: (Best: First-Fit)*
  - *Approx. Alg (Good results after "Compaction")*

BAP Packing (continued…)

❖ Theoretical Study of BAP-Packing [LSC]

- ***Define BAP-Pack(W, D, h)***
  - h = Max Length of any vessel
  - D = “Max-Density” of the Set of Vessels
  - W = Section Length Needed to Pack all the Vessels
- ***Study problem for small h***
  - Negative Results: eg: BAP-Pack(4,4,2) not feasible
  - Positive Results: eg: BAP-Pack(1.5D, D, 2) is feasible
  - Positive Results: eg: BAP-Pack(2D, D, 3) is feasible
- ***Extrapolate Results from Insights Gained***

The BAP Architecture



BAP Data Generation

- **Real world data are classified,**
  - *No published benchmarks*
- **Our Group Spent ~1/2 yr to collect data**
  - *Port periodicals, PSA reports, internet web sites*
- **Developed BAP Data Generation Sub-System**
  - *Data set = (Section+Vessel+Transshipment) scenarios*
  - *Realistic statistics: vessel length, #containers, etc.*
- **Fast-Generation and Repeatability**
  - *Can Generate Port Scenarios*

BAP Port Scenario Generated

- **Simple Case (SC1)**
  - *Two Sections: 600m each*

- **PSA Brani-Terminal Case (SC5)**
  - *Four Sections: 600m, 480m, 800m, 200m*

- **Experimentation Cases (SC2-4)**
  - *5 Sections: 1-1-1-1-1 600m each*
  - *3 Sections: 1-3-1 600m, 1800m, 600m*
  - *3 Sections: 2-1-2 1200m, 600m, 1200m*

Other Port Parameters

- **Vary the business of the port**
  - *Define ABD (Average-Berthing-Demand)*
    - Does not include vessel-to-vessel tolerance

ABD	SC1 2 sections	SC2 5 sections	SC3 3 sections	SC4 3 sections	SC5 4 sections
0.25	*d101-d105* 60				(Brani)
0.40	*d106-d110* 96	*d121-d125* 240	*d141-d145* 240	*d161-d165* 240	*d181-d185* 166
0.50	*d111-d115* 120	*d126-d130* 302	*d146-d150* 302	*d166-d170* 302	*d186-d190* 302
0.60	*d115-d120* 144	*d131-d135* 362	*d151-d155* 362	*d171-d175* 362	*d191-d195* 362
0.70		*d136-d140* 422	*d156-d160* 422	*d176-d180* 422	*d196-d200* 422

Acknowledgements

- Steven Quek
- Ku Liang Ping
- Loh Swee Nam
- David Ong Tat Wee
- Foo Hee Meng
- Li Shuai Cheng
- Chen Li Wen
- Xu Degui

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