Leftist Heap - Supplements

Facts about leftist heaps:

- Right path is a shortest path from root to leaf.
- Length of this path $\leq$ lg n [Proof: Induction]

What about building a leftist heap ?

- can use repeated insert

$$\sum_{k=1}^{n-1} \lg k = O(n \lg n) \quad \text{[CLR90] Ch 2.}$$

Analysis of Heapify : How did we get

$$O\left(\sum_{i=1}^{\lg k} \frac{k}{2^i} \cdot \max\left\{ 1, \lg \frac{n \cdot 2^i}{k} \right\} \right) = O\left(k \cdot \max\left\{ 1, \lg \frac{n}{k} \right\} \right)$$

$$\sum_{i=1}^{\lg k} \frac{k}{2^i} = k\left(1 + \frac{1}{2} + \frac{1}{2^2} + \ldots + \frac{1}{2^{\lg k}} \right) < 2k$$

Note: $\lg \frac{n \cdot 2^i}{k} = \lg \frac{n}{k} + \lg 2^i = \lg \frac{n}{k} + i$

$$\sum_{i=1}^{\lg k} \frac{k}{2^i} \cdot \lg \frac{n \cdot 2^i}{k} < 2k \cdot \lg \frac{n}{k} + k \cdot \sum_{i=1}^{\lg k} \frac{i}{2^i}$$

$$\sum_{i=1}^{\lg k} \frac{i}{2^i} < \sum_{i=1}^{\infty} \frac{i}{2^i} < 2$$