CS6234 Advanced Algorithms: Exercises

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1. Matching

Sample Exercise 1. [PS82] Ch-10, Problem 1 (p.243)

Show that in a bipartite graph G, the size of a maximum matching M^* equals the size of the minimum vertex cover C^*. Hence, derive an efficient algorithm for finding a minimum vertex cover in a bipartite graph G. (A vertex cover C of a graph G is a subset of the vertices in G that "covers" all the edges in G, i.e., every edge in G has at least one end-point in C.)

Sample Exercise 2. [PS82] Ch-10, Problem 3 (p.243)

An ***edge cover*** of a graph $G=(V,E)$ is a set C of edges ($C \subseteq E$) such that each vertex in V is incident to at least one edge in C. Give an efficient algorithm for finding a *minimum edge cover* of a graph. (You can assume that G has no isolated vertices.)

(Hint: Let C^* be a minimum edge cover and M^* be a maximum matching. Show that $|C^*| + |M^*| = |V|$.)

Sample Exercise 3. Dancing Couples

There are pq couples at a dance. The pq men are divided according to their ages into p **age-groups** (denoted by $\{M_1, M_2, \ldots M_p\}$). Each age-group M_i has q men. Similarly, the women are divided into p **height-groups** ($\{W_1, W_2, \ldots, W_p\}$) with q women in each height-group, according to their **heights**. Show that p couples can be selected such that every **age-group** and every **height-group** will be represented.

2. Linear Programming, Simplex Algorithm, Primal and Dual LP

Simple Exercise 1 (*Simple*) [PS82] Ch-2, Problem 3, (p.63)

Show that the set of optimal points of an instance of LP is a convex set.

Simple Exercise 2 (*Simple*) Variation of Example 2.6

In Example 2.6 ([PS82]-p49), the book shows how to choose Column 2 to enter the basis. Work the same example, but *choose Column 1* to enter the basis. Show the new tableau after moving to the new bfs and the new value of the objective.

Simple Exercise 3 (*Simple*) Primal-Dual LP

Give the dual LP for Example 2.3, assuming that we use an objective function of $z = 2x_1 + 3x_2 - 4x_3$.

Simple Exercise 4 (*Simple*) Complementary Slackness

Work through Example 3.2 (p.72-73) on Complementary Slackness Theorem.

Simple Exercise 5 (*Simple*) [PS82] Ch-3, Problem 12, (p.85)

Show that the dual of a canonical form LP is also in canonical form.