CS5232 Execise 2

**Q1.** Given $s = \langle a, b, c, d \rangle$ and $t = \langle d, e, f \rangle$, write down

   (1) $s - t$

   (2) $((s \frown t)^{-1} - s^{-1})^{-1}$

   (3) $(\operatorname{dom} t) \lhd s \rhd (\operatorname{ran} t)$

   (4) $s \oplus t$

   (5) $\{1 \mapsto 4, 2 \mapsto 4, 3 \mapsto 4, 6 \mapsto 4\} \mathbin{\mathpalette\superimpose{{\circ}{\circ}}} ((s \frown t) \oplus \{7 \mapsto d\})$

Q2.

The figure below gives the board and starting position for a game of *Shunting*. A move consists of the black piece (the shunter) moving one position either vertically or horizontally provided either

- the position moved to is empty, or

- the position moved to is occupied by a white piece but the position beyond the white piece is empty, in which case the white piece is pushed into the empty position.
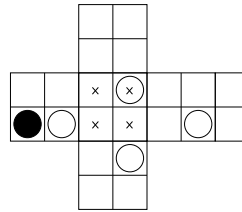
The shunter can not push two white pieces at the same time. At each stage a score is kept of the number of moves made so far. The game ends when the white pieces occupy the four positions marked with a cross.

Give a Z specification of this game. Hints:

$$Board == (1 \mathinner{\ldotp\ldotp} 7 \times 3 \mathinner{\ldotp\ldotp} 4) \cup (3 \mathinner{\ldotp\ldotp} 4 \times 1 \mathinner{\ldotp\ldotp} 6)$$
$$over == \{(3,3), (4,3), (3,4), (4,4)\}$$

$$\begin{array}{|l}
next : Board \leftrightarrow Board \\
beyond : Board \times Board \nrightarrow \mathbb{N} \times \mathbb{N} \\
\hline
\ldots
\end{array}$$

$$\begin{array}{|l}
Shunting \underline{} \\
bposn : Board \\
wposn : \mathbb{P}\, Board \\
score : \mathbb{N} \\
\hline
\ldots
\end{array}
\quad
\begin{array}{|l}
Shunting_{INIT} \\
\ldots \\
\hline
\end{array}
\quad
\begin{array}{|l}
Move \underline{} \\
\ldots \\
\hline
\end{array}
\quad
\begin{array}{|l}
Over \underline{} \\
\ldots \\
\hline
\end{array}$$

CS5232 tut 2 Solution

Q1 Sample Solution:

Given $s = \langle a, b, c, d \rangle$ and $t = \langle d, e, f \rangle$, write out

(1) $s - t$

s

(2) $((s \frown t)^{-1} - s^{-1})^{-1}$

$\{(5, d), (6, e), (7, f)\}$

(3) $(\text{dom } t) \lhd s \rhd (\text{ran } t)$

$\varnothing$

(4) $s \oplus t$

$\langle d, e, f, d \rangle$

(5) $\{1 \mapsto 4, 2 \mapsto 4, 3 \mapsto 4, 6 \mapsto 4\} \fatsemi ((s \frown t) \oplus \{7 \mapsto d\})$

$\{(1, d), (2, d), (3, d), (6, d)\}$

Q2 Sample Solution:

$$Board == (1 \mathbin{..} 7 \times 3 \mathbin{..} 4) \cup (3 \mathbin{..} 4 \times 1 \mathbin{..} 6)$$

$next : Board \leftrightarrow Board$

$\forall (i,j), (k,l) : Board \bullet$
$\quad (i,j) \; \underline{next} \; (k,l) \iff$
$\qquad\qquad i = k \;\wedge\; (j = l+1 \;\vee\; j = l-1)$
$\qquad\qquad \vee$
$\qquad\qquad j = l \;\wedge\; (i = k+1 \;\vee\; i = k-1)$

$over : \mathbb{P} \, Board$

$over = \{(3,3), (4,3), (3,4), (4,4)\}$

$beyond : Board \times Board \nrightarrow \mathbb{N} \times \mathbb{N}$

$\mathrm{dom} \; beyond = \{b, w : Board \mid b \; \underline{next} \; w\}$
$\forall \, b, w : \mathrm{dom} \; beyond \bullet$
$\qquad\qquad beyond(b, w) = 2w - b$

Shunting
$bposn : Board$
$wposn : \mathbb{P} \, Board$
$score : \mathbb{N}$

$bposn \notin wposn$
$\# wposn = 4$

Shunting$_{INIT}$
$bposn = (1,3)$
$wposn = \{(2,3), (4,2),$
$\qquad\qquad (4,4), (6,3)\}$

$score = 0$

Move
$\Delta Shunting$

$wposn \neq over$
$bposn' \; \underline{next} \; bposn$
$bposn' \notin wposn \implies wposn' = wposn$
$bposn' \in wposn \implies$
$\quad wposn' = (wposn - \{bposn'\})$
$\qquad\qquad \cup \{beyond(bposn, bposn')\}$
$score' = score + 1$

Over
$\Xi Shunting$
$score! : \mathbb{N}$

$wposn = over$
$score! = score$