Short Test (Exercise) for CS5232 (appx: 60min)

Surname: __________________ Given Name: __________________________________

Matriculation Number: ___________________________

Note: Write your answers in the blank spaces in this answer book only.

Q1. Decide if the following predicates are true or false (no need to give a reason). Note that $\mathbb{N}$ is the natural number set starting from 0 and $\mathbb{N}_1$ is the natural number set starting from 1.

(a) $\forall n : \mathbb{N}_1 \bullet \exists m, k : \mathbb{N} \bullet nk = m \ \wedge \ n < m$

(b) $\exists n : \mathbb{N} \bullet \forall m : \mathbb{N} \bullet m > n \Rightarrow \exists k : \mathbb{N} \bullet m = 2k$

(c) $\forall S, T : \mathbb{P}\,\mathbb{N} \bullet S \subseteq T \vee T \subseteq S$

(d) $\mathbb{P}\,\mathbb{P}\,\mathbb{P}\,\varnothing = \{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}, \{\{\varnothing\}\}\}$

Q2. Let S be the set of numbers from 1 to 12 inclusive.

(a) Give a formal definition (in the declaration/predicate style) of the relation $R : S \leftrightarrow S$ where x is related to y exactly when y is greater than the square of x but less than the square of $x + 1$.

(b) Write down R as a set of ordered pairs.

Q3. A function *squash* that takes any finite function $f : \mathbb{N} \pfun A$ (for arbitrary set A) and converts it into an element in seq A by squashing the domain to $1 \mathrel{..} \#f$. The formal definition of *squash* is given as:

$$FN == \{f : \mathbb{N} \pfun A \mid f \text{ is finite}\}$$

$$\begin{array}{|l}
squash : FN \rightarrow \operatorname{seq} A \\
\hline
\forall f : FN \bullet \\
\quad f = \varnothing \Rightarrow squash(f) = \langle\,\rangle \\
\quad f \neq \varnothing \Rightarrow \\
\qquad \textbf{let } m = min \operatorname{dom} f \bullet \\
\qquad\quad squash(f) = \langle f(m) \rangle \frown squash(\{m\} \ndres f)
\end{array}$$

(a) Write down the value (result) of $squash(\{(4, a), (2, b), (3, c)\})$.

(b) Specify a function *remove* which, given an element in A and a sequence in seq A, removes from that sequence all occurrences of the element. (hint: using the function *squash*.)

Q4. Let $[People]$ be the set of all possible persons and $[Subjects]$ the set of all possible subjects. A specification for an *University* has a state schema:

$$\begin{array}{|l}
\multicolumn{1}{l}{\textit{University}} \\
students : \mathbb{P}\, People \\
subjects : \mathbb{P}\, Subjects \\
enrolments : People \leftrightarrow Subjects \\
\hline
\operatorname{dom} enrolments \subseteq students \\
\operatorname{ran} enrolments \subseteq subjects
\end{array}$$

For the class *University*, specify following operations. Write your answer on this page, directly in the space provided in the schema definition.

(a) an operation schema *newStudent* such that a person (*pers*?) becomes a student:

newStudent

(b) an operation schema *studentLeaves* such that a person (*pers*?), not enrolled in any subject, ceases to be a student:

studentLeaves

(c) an operation schema *cancel* such that a person (*pers*?) cancels enrolment in a subject (*subj*?):

cancel

(d) an operation schema *personSubjects* which outputs the set of subjects (*subjs*!) in which the person (*pers*?) is enrolled:

personSubjects

Solution

Q1.

(a) True.
given any strictly positive n, let $m = 2n$ and $k = 2$; then $nk = m$ and $n < m$.

(b) False.
take $m = 2n + 1$; then $m > n$ but as $2n + 1$ is odd there is no k with $m = 2k$.

(c) False.
take $S = \{1, 2\}$ and $T = \{3, 4\}$.

(d) True.

Q2.

$$R : S \leftrightarrow S$$

$$\forall\, x, y : \mathbb{N} \bullet$$
$$x \underline{R}\, y \;\Leftrightarrow\; y > x^2 \;\wedge\; y < (x+1)^2$$

$$R = \{(1,2), (1,3), (2,5), (2,6), (2,7), (2,8), (3,10), (3,11), (3,12)\}$$

Q3.

(a) $\langle b, c, a \rangle$

(b)

$remove : A \times \mathrm{seq}\, A \rightarrow \mathrm{seq}\, A$

$\forall\, a : A;\ s : \mathrm{seq}\, A \bullet remove(a, s) = squash(s \rhd \{a\})$

Q4.

(a)

newStudent

$\Delta(students)$
$pers? : People$

$pers? \notin students$
$students' = students \cup \{pers?\}$

(b)

studentLeaves

$\Delta(students)$
$pers? : People$

$pers? \in students$
$pers? \notin \mathrm{dom}\, enrolments$
$students' = students \setminus \{pers?\}$

(c)

cancel

$\Delta(enrolments)$
$pers? : People$
$subj? : subjects$

$pers?\ \underline{enrolments}\ subj?$
$enrolments' = enrolments \setminus \{(pers?, subj?)\}$

(d)

personSubjects

$pers? : People$
$subjs! : \mathbb{P}\, subjects$

$pers? \in students$
$subjs! = \mathrm{ran}(\{pers?\} \lhd enrolments)$