Short Test (Exercise) for CS5232 (appx: 60min)

Matriculation Number:

Surname: _____ Given Name: _____

Note: Write your answers in the blank spaces in this answer book only.

Q1. Decide if the following predicates are true or false (no need to give a reason). Note that \mathbb{N} is the natural number set starting from 0 and \mathbb{N}_1 is the natural number set starting from 1.

- (a) $\forall n : \mathbb{N}_1 \bullet \exists m, k : \mathbb{N} \bullet nk = m \land n < m$
- (b) $\exists n : \mathbb{N} \bullet \forall m : \mathbb{N} \bullet m > n \Rightarrow \exists k : \mathbb{N} \bullet m = 2k$
- (c) $\forall S, T : \mathbb{P} \mathbb{N} \bullet S \subset T \vee T \subset S$
- $(d) \ \mathbb{PPP\varnothing} = \{\varnothing, \{\varnothing\}, \{\varnothing, \{\varnothing\}\}, \{\{\varnothing\}\}\}$

- Q2. Let S be the set of numbers from 1 to 12 inclusive.
 - (a) Give a formal definition (in the declaration/predicate style) of the relation $R: S \leftrightarrow S$ where x is related to y exactly when y is greater than the square of x but less than the square of x+1.

(b) Write down R as a set of ordered pairs.

Q3. A function squash that takes any finite function $f: \mathbb{N} \to A$ (for arbitrary set A) and converts it into an element in seq A by squashing the domain to 1.. # f. The formal definition of squash is given as:

$$FN == \{f : \mathbb{N} \to A \mid f \text{ is finite}\}$$

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squash: FN \to \operatorname{seq} A
\forall f: FN \bullet
f = \varnothing \Rightarrow squash(f) = \langle \rangle
f \neq \varnothing \Rightarrow
\operatorname{let} m = \min \operatorname{dom} f \bullet
squash(f) = \langle f(m) \rangle \cap squash(\{m\} \triangleleft f)
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(a) Write down the value (result) of $squash(\{(4, a), (2, b), (3, c)\})$.

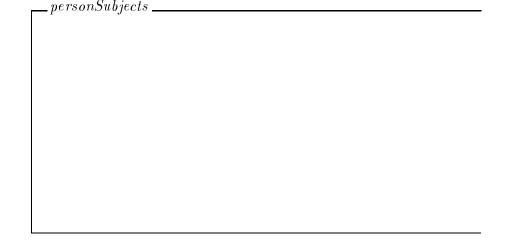
(b) Specify a function remove which, given an element in A and a sequence in seq A, removes from that sequence all occurrences of the element. (hint: using the function squash.)

Q4. Let [People] be the set of all possible persons and [Subjects] the set of all possible subjects. A specification for an University has a state schema:

 $_University _$ $students: \mathbb{P} People$ $subjects: \mathbb{P} Subjects$ $enrolments: People \leftrightarrow Subjects$ $dom\ enrolments \subseteq students$ $ran\ enrolments \subseteq subjects$

	·
	newStudent
	peration schema $studentLeaves$ such that a person ($pers$?), not enrolled in subject, ceases to be a student:
any	
	studentLeaves
(c) an o	peration schema $cancel$ such that a person $(pers?)$ cancels enrolment in a
	$\operatorname{ect} (subj?)$:
	cancel
	Cancer

(d) an operation schema personSubjects which outputs the set of subjects (subjs!) in which the person (pers?) is enrolled:



Solution

Q1.

- (a) True. given any strictly positive n, let m = 2n and k = 2; then nk = m and n < m.
- (b) False. take m = 2n + 1; then m > n but as 2n + 1 is odd there is no k with m = 2k.
- (c) False. take $S = \{1, 2\}$ and $T = \{3, 4\}$.
- (d) True.

Q2.

$$R = \{(1,2), (1,3), (2,5), (2,6), (2,7), (2,8), (3,10), (3,11), (3,12)\}$$

Q3.

- (a) $\langle b, c, a \rangle$
- (b)

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Q4.
 (a)
               \Delta(students)
              \frac{pers?:People}{pers? \notin students}
              students' = students \cup \{pers?\}
 (b)
              \_studentLeaves \_
               \Delta(students)
              pers?: People
              pers? \in students
              pers? \notin dom \ enrolments
              students' = students \setminus \{pers?\}
 (c)
               \Delta(enrolments)
              pers?: People
              subj? : subjects
              pers? enrolments subj?
               enrolments' = enrolments \setminus \{(pers?, subj?)\}
 (d)
              \_personSubjects \_
               pers?: People
              subjs!: \mathbb{P} subjects
              pers? \in students
              subjs! = ran(\{pers?\} \lhd enrolments)
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