LONGYUN DING, On equivalence relations generated by Schauder bases.
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In this talk, a notion of Schauder equivalence relation $R^N/L$ is introduced, where $L$ is a linear subspace of $\mathbb{R}^N$ and the unit vectors $e_n = (0, 0, \cdots, 0, 1, 0, \cdots)$ form a Schauder basis of $L$. The main theorem is to show that the following conditions are equivalent:

1. the unit vector basis is boundedly complete;
2. $L$ is $F_\sigma$ in $\mathbb{R}^N$;
3. $R^N/L$ is Borel reducible to $R^N/\ell_\infty$.

We show that Schauder equivalence relation generalized by any basis of $\ell_2$ is Borel bireducible to $R^N/\ell_2$ itself, but it is not true for bases of $c_0$ or $\ell_1$. Furthermore, among all Schauder equivalence relations generated by sequences in $c_0$, we find the minimum and the maximum elements with respect to Borel reducibility.

We also show that $R^N/\ell_p$ is Borel reducible to $R^N/J$ iff $p \leq 2$, where $J$ is James’ space.