

Midterm Examination 1

GEM 1501: Problem Solving for Computing

Wednesday 03.03.2010, duration half an hour

Matriculation Number: _____

Rules

Each correct question, 1 mark. Maximum score: 12 marks.

Programming Language for Questions 7–12 is JavaScript.

Question 1. Use Euclid's Algorithm to determine the greatest common divisor of 184 and 299. Write down the intermediate steps.

$$299 - 184 = 115$$

$$184 - 115 = 69$$

$$115 - 69 = 46$$

$$69 - 46 = 23$$

$$46 - 23 = 23$$

The result is 23.

Question 2. Write the intermediate results of the following Pivot Sort algorithm.

Input 13 8 12 99 11 24 27 88 77.

Pivot Element: 13 .

Sublist One: 8,12,11 .

Sublist Two: 99,24,27,88,77 .

Sorted version of Sublist One: 8,11,12 .

Sorted version of Sublist Two: 24,27,77,88,99 .

Output: 8,11,12,13,24,27,77,88,99 .

Question 3. Arrange the following list of words to obtain the (long) names of four programming languages: All-Purpose Beginner's Business-Oriented Code Common Formula Instruction Java Language Script Symbolic Translator. The solutions are:

1. Beginner's All-Purpose Symbolic Instruction Code (BASIC) .
2. Common Business-Oriented Language (COBOL) .
3. Formula Translator (FORTRAN) .
4. Java Script .

Question 4. Let $a_1, a_2, a_3, a_4, a_5, a_6$ be a list of places where each two places a_i, a_j with $i \neq j$ are connected by a direct line. The costs are $40 - (i - j)^2$ SGD / month to rent. Now a company wants to rent so many lines that it owns a network connecting all places. The constraint is that the monthly pay is as low as possible. It is not necessary to have direct connections between each two places, but indirect connections perhaps going through other places should always be there.

How many lines does the company have to order? 5 .

How much money does the company pay per month? 125 .

Question 5. Check the right answer (1 out of 4).

The word “algorithm” was first used by Euclid when he formulated his famous algorithm to find the greatest common divisor of two integers.

The word “algorithm” stems from the name of Mohammed al-Khowârizmî who formulated algorithms for many mathematical problems like multiplying and dividing numbers or finding solutions to quadratic equations.

The word “algorithm” is due to a computer called “ALGORIT H3 M12” used by Alan Turing to crack German codes in World War II.

The word “algorithm” stems from the programming language “ALGOL 58” which was invented in 1958.

Question 6. Which of the following statements about interpreters and compilers are true? Check 2 out of 5.

An interpreter can be used to translate a Pascal program into a C program.

Compilers check systematically for syntax errors in the program to be compiled; if no syntax errors are there, then they translate the given program into a code which can be executed by a computer.

Interpreters and compilers are the same; the only difference is that some companies prefer to use the word “compiler” and others prefer to use the word “interpreter” for marketing reasons.

Browsers contain interpreters for JavaScript which are used to interpret the JavaScript code inside the html-files of webpages.

Ada Lovelace programmed for Charles Babbage’s “Analytical Engine” a compiler translating C programs into machine code for that computer.

Question 7. Gauss found that the following function computes a sum of another function from 0 to n :

```
function gauss(n)
  { return(0.5*n*(n+1)); }
```

Which sum is computed by this formula? Check 1 out of 7:

$\text{gauss}(100)$ is $\frac{0}{1} + \frac{1}{2} + \frac{2}{3} + \dots + \frac{100}{101}$;

$\text{gauss}(100)$ is $3 + 3 + 3 + \dots + 3 = 303$;

$\text{gauss}(100)$ is $0 + 1 + 2 + \dots + 100$;

$\text{gauss}(100)$ is $\sin(0) + \sin(1) + \sin(2) + \dots + \sin(100)$;

$\text{gauss}(100)$ is $e^{-0^2} + e^{-1^2} + e^{-2^2} + \dots + e^{-100^2}$;

$\text{gauss}(100)$ is $2^0 + 2^1 + 2^2 + \dots + 2^{100}$;

$\text{gauss}(100)$ is $\log(1) + \log(2) + \log(3) + \dots + \log(101)$.

Question 8. Consider the following function:

```
function numbersort(a)
{ var n = a.length; var i = 0; var j = n-1; var b;
  while (i<j)
    { if (a[i]>=0 && a[j]<0)
      { b = -a[i]; a[i] = a[j]; a[j] = -b; }
      if (a[i]<0) { i = i+1; }
      if (a[j]>=0) { j = j-1; } }
  return; }
```

Analyze the program and its runtime. The basic operations like negating, adding, assigning values and accessing array members are all 1 time step. Check the right answers (2 out of 6):

- The program sorts the integers in the array;
- The program puts the negative integers before the positive ones;
- The program makes all integers to be positive;
- The runtime of the program is $O(n)$;
- The runtime of the program is $O(n \log n)$;
- The runtime of the program is $O(n^2)$.

Question 9. Put the keywords into the following program which checks whether the input x is a square number. All inputs are considered to be integers.

```
function issquare(x)
{ ____var____ y; ____var____ z;
  y = 0;

  ____while____ (y*y<x) { y = y+1; }

  ____if____ (y*y == x)
  { z = 1; }

  ____else____
  { z = 0; }

  ____return__(z); }
```

Question 10. What can be said about the correctness of the following function to compute the output x^4 from input x ? The input x can be any integer number.

```
function squareofsquare(x)
  { var y = 0; var z;
    for (z=0;z!=x*x*x;z=z+1)
      { y = y+x; }
    return(y); }
```

Check the correct answers (2 out of 5).

- The function is partially correct.
- The function terminates on all inputs.
- The function outputs on some inputs wrong outputs.
- The function runs on input 5 into an infinite loop.
- The function outputs on input 8 the value 4096.

Question 11. Write a JavaScript function which counts how many members of an array x between positions n and m are 3. So if x is (3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4, 3, 4) and n is 2 and m is 4 then the correct answer is 2, as $x[2]$ and $x[4]$ are both 3. The output is in the variable y .

```
function threecount(x,n,m)
  { var y = 0;
    var z = n;
    while ((z <= m) && (z < x.length))
      { if (x[z]==3) { y = y+1; }
        z = z+1; }
    return(y); }
```

Question 12. The following program has running time exponential in the length n of the input. The input x has at least 3 array elements. The second input parameter m is an integer.

```
function f(x,m)
  { var n = x.length;
    if (m < 0) { return(0); }
    if (m == 0) { return(x[0]); }
    if (m == 1) { return(x[0]+x[1]); }
    if (m < n) { return(f(x,m-1)+f(x,m-2)+x[m]); }
    return(f(x,m-1)+f(x,m-2)); }
```

Write a better program for the same function which has running time polynomial in the length n of the input x .

```
function f(x,m)
  { var y; var k; var n = x.length;
    if (m<0) { return(0); }
    if (m==0) { return(x[0]); }
    if (m==1) { return(x[0]+x[1]); }
    y = 0; v = x[0]; w = x[0]+x[1];
    for (k=2;k<=m;k++)
      { if (k<n)
          { y = v+w+x[k]; }
        else
          { y = v+w; }
        v = w; w = y; }
    return(y); }
```