Midterm Examination 2
GEM 1501: Problem Solving for Computing

Wednesday 07.04.2010, duration half an hour

Matriculation Number: _______________

Rules
Each correct question, 1 mark. Maximum score: 12 marks.
Programming Language for Questions 7–12 is Java Script.

Question 1. Computations modulo a prime number are important in computer science. Show that you understand how to do this. Compute $f(9)$ modulo 11 where $f(x) = 220*(x-1)*x*(x+1)+5*(x-2)*(x+2)*x^{1819}+242*x^{23}+50*(x-8)*x+5*x+18$. That is, find that number y such that there is an integer z with $f(9) = 11 * z + y$. The number y is

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5
☐ 6 ☐ 7 ☐ 8 ☐ 9 ☐ 10.

Question 2. Which of the following problems on multivariate quadratic polynomials $f(x_1, x_2, \ldots, x_n)$ and $g(x_1, x_2, \ldots, x_n)$ with integer coeffitients between $-n^n$ and $+n^n$ are known to be in RP? More precisely, for which of the three formulas below there is a randomised polynomial time algorithm that outputs with high probability "yes" if the formula with the given parameter polynomials f and g is true and always "no" if the formula with parameters f and g is false?

☐ $\exists x_1, x_2, \ldots, x_n[f(x_1, x_2, \ldots, x_n) = g(x_1, x_2, \ldots, x_n)]$;
☐ $\exists x_1, x_2, \ldots, x_n[f(x_1, x_2, \ldots, x_n) \neq g(x_1, x_2, \ldots, x_n)]$;
☐ $\exists x_1, x_2, \ldots, x_n[f(x_1, x_2, \ldots, x_n) * g(x_1, x_2, \ldots, x_n) = 0]$.

Question 3. What is the best possible complexity of parallel sorting of n inputs?

☐ $O(n \log(n))$ processors and $O(\log \log(n))$ parallel time;
☐ $O(n)$ processors and $O(\log(n))$ parallel time;
☐ $O(\sqrt{n})$ processors and $O(\sqrt{n})$ parallel time;
☐ $O(\log(n))$ processors and $O(n \log^3(n))$ parallel time.

Question 4. Which of the following problems are known to be solvable in NC? Check two out of five:

- ☐ Finding the shortest roundtour of n cities on a map;
- ☐ Computing greatest common divisor of two natural numbers;
- ☐ Multiplication two $n * n$ matrices;
- ☐ Playing checkers on an $n * n$ board optimally;
- ☐ Summation of n numbers.

Question 5. Consider the following complexity classes: LOGSPACE, NC, NP, P, PSPACE, RP. Find two of these classes which are known to be different:

The class ________________ is different from the class ________________.

Question 6. The set $\{T : T$ is the text of a program which halts on input $1965\}$ is recursively enumerable but not decidable. Not every set is recursively enumerable. Please provide a definition of a set of texts which is not recursively enumerable.

The set $\{T : T$ is a text of a program which ____

____$\}$ is not recursively enumerable.

Question 7. Counter programs modify the value of a variable by at most one and all variables have natural numbers as a value. What is the function computed by the following counter program?

```
function f(n)
  { var m=2; var k=0; var s=0; var t;
    if ((n == 0)||(n == 1)) { return(n); }
    while (m != n) { m = m+1; k = k+1; }
    t = f(k); k = k+1; s = f(k);
    while(t != 0) { t = t-1; s = s+1; }
    return(s); }
```

The function value f(n) for input n is the following (check 1):

- ☐ the exponential function of n $(1, 2, 4, 8, 16, \ldots)$;
- ☐ the factorial of n, that is, $1 * 2 * \ldots * n$;
- ☐ the n-th Fibonacci number $(0, 1, 1, 2, 3, 5, \ldots)$;
- ☐ the square of n;
- ☐ the sum $0 + 1 + \ldots + n$ of the numbers from 0 to n;
- ☐ the downrounded squareroot of n.

Question 8. Consider the following program where the input u is always a natural number:

```
function g(u)
  { var s = u%10;
    if (u > 9)
      { return(s+g(Math.floor(u*0.1+0.0005))); }
    else
      { return(s); } }
```

Check the five correct answers:

- [ ] $g(12345)$ terminates and returns 10;
- [ ] $g(12345)$ terminates and returns 12;
- [ ] $g(12345)$ terminates and returns 15;
- [ ] $g(12345)$ terminates and returns 19;
- [ ] $g(u)$ is never strictly larger than u;
- [ ] $g(u)$ is equal to u for all u;
- [ ] $g(u)$ takes infinitely often the value 0;
- [ ] $g(u)$ takes infinitely often the value 1;
- [ ] $g(u)$ takes infinitely often the value 2;
- [ ] $g(10 * u + r) = g(u) + r$ for all u and $r = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$;
- [ ] $g(10 * u + r) = g(u) * r$ for all u and $r = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$;
- [ ] $g(10 * u + r) = g(u)^r$ for all u and $r = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9$;

Question 9. What is the runtime complexity of this program on inputs x,y where x+y has n decimal digits? Every input is a natural number.

```
function h(x,y)
  { if ((x<1)||(y<1)) { return(0); }
    var v = 0; var w = 0; var u = 0;
    while (u+y <= x)
      { while (u+v <= x) { w = v; v = v+v+y; }
        u = u+w; v = 0; w = 0; }
    return(x-u); }
```

The runtime complexity is

- [ ] Logarithmic or polylogarithmic in n;
- [ ] Polynomial in n but not polylogarithmic in n;
- [ ] Exponential in n but not polynomial in n;
- [ ] Worse than exponential in n.

Question 10. Write a program which finds the largest prime number p below the input n; the program should return n if n is prime number. In the case that $n < 2$ the program should return 0.

```
function primesearch(n)
  { var p = 0;



    return(p); }
```

Question 11. Write a program which finds the largest m such that in the array a some number occurs m times. So if a is $(1, 2, 3, 2, 2, 3, 4, 3, 3, 4, 53, 58)$ then $count(a)$ should return 4 as 3 occurs 4 times in a.

```
function count(a)
  { var n = a.length; var m;




    return(m); }
```

Question 12. Write a function *search* which computes for input y the largest integer x such that $x * x * x - 10000 * x \leq y$.

```
function search(y)
  { var x;




    return(x); }
```

Worksheet

Do not remove this sheet from the test.

You can use this sheet to do calculations, but you should write the answers into the space provided. Answers found here are not evaluated.