NATIONAL UNIVERSITY OF SINGAPORE

SCHOOL OF COMPUTING

SEMESTER II: 2010–2011

EXAMINATION FOR

GEM 1501 – Problem Solving for Computing

April 2011 – Time Allowed 2 Hours

INSTRUCTIONS TO CANDIDATES

1. This examination paper consists of TEN (10) questions and comprises ELEVEN (11) printed pages.
2. Answer **ALL** questions.
3. This is a **Closed Book** examination.
4. Every question counts FIVE (5) marks. The maximum possible marks are 50.
5. Please write your Matriculation Number below:

   MATRICULATION NO: _____

This portion is for examiner's use only

Qestion	Marks	Remarks	Qestion	Marks	Remarks
Q01:			Q06:		
Q02:			Q07:		
Q03:			Q08:		
Q04:			Q09:		
Q05:			Q10:		
			Total:		

Question 1 [5 marks] **GEM 1501**

What is an array and what is a record? Describe the two data types and explain the differences between them. How is a record realised in Java Script?

Question 2 [5 marks] **GEM 1501**

There are places $a_0, a_1, a_2, \ldots, a_{12}$. Assume that the direct fare for going from a_i to a_j (with $i < j$) is $8 + (j - i) \cdot (j - 1 - i)/2$. Find the cheapest route to go from a_0 to a_{12} consisting of several direct steps using dynamic programming. Determine the price and list one possible route.

Question 3 [5 marks] **GEM 1501**

Consider the following function `ijksearch` which has as input an array x of length n. The array members are numbers.

```
function ijksearch(x)
  { var n = x.length;
    var i; var j; var k;
    for (i=0;i<n;i++)
      { for (j=i+1;j<n;j++)
          { for (k=j+1;k<n;k++)
              { if ((x[i]==x[j])&&(x[i]==x[k])) { return(1); } } } }
    return(0); }
```

What is the worst-case complexity of the running-time of this function? Mark the opitmal answer:

☐ $O(1)$ ☐ $O(\log(n))$ ☐ $O(\log^2(n))$ ☐ $O(n\log(n))$ ☐ $O(n^2)$
☐ $O(n^2\log(n))$ ☐ $O(n^3)$ ☐ $O(n^3\log(n))$ ☐ $O(n^4)$ ☐ $O(n^5)$

What is the worst-case complexity of the problem, that is, the worst-case complexity of the fastest program having the same input-output behaviour? The program is permitted to make copies of the array and compare the array elements; copying each array element and comparing two array elements always induces the cost of one time-unit. Furthermore, all standard comparisons $<$, $<=$, $!=$, $==$, $>$, $>=$ are permitted. Mark the optimal answer describing the complexity of the problem:

☐ $O(1)$ ☐ $O(\log(n))$ ☐ $O(\log^2(n))$ ☐ $O(n\log(n))$ ☐ $O(n^2)$
☐ $O(n^2\log(n))$ ☐ $O(n^3)$ ☐ $O(n^3\log(n))$ ☐ $O(n^4)$ ☐ $O(n^5)$

Give reasons for your answers.

Question 4 [5 marks] **GEM 1501**

What are 2SAT and 3SAT? Furthermore, how does the algorithm "Resolution" behave on these two problems? What difference is observed and what is the reason for it?

Question 5 [5 marks] **GEM 1501**

Write a counter program which computes on input n the number a_n with $a_0 = 1$, $a_1 = 2$ and $a_{n+2} = a_n \cdot a_{n+1}$ for all n. For this task, the counter program is permitted to use operations to compare, to add and to subtract, along with control operations while, for and if. It is permitted to write subfunctions and to call them. All numbers $0, 1, 2, \ldots$ can be used as constants. However, multiplication and division operations are not available. The input n is always a natural number.

Question 6 [5 marks] **GEM 1501**

Make a finite automaton (as a graph or a table) which accepts inputs consisting of binary numbers and operands "+", "-" and "∗". Between any two operands and before the first operand and after the last operand should be a number. Also, except for 0, all binary numbers start with "1", so there are no leading zeroes.

Here some examples. The automaton should accept the following inputs:
`0+11+10*11-101+111*11`
`110101010001-101010010101010`
`1+0+1+0*0-1`
It should reject the following inputs:
`00+10+00101+101+10+110`
`+1+0+11+0+1`
`1**0*1-1101+11`

Question 7 [5 marks] **GEM 1501**

Caesar's cryptography algorithm permutated the letters so that "a" became "d", "b" became "e", "c" became "f", ..., "z" became "c". Write a Java Script function which permutates the (lower case) letters in this way and lets all other symbols unchanged. Input and output are strings x and y. So "the river is yellow." should be mapped to "wkh ulyhu lv bhoorz." by the following function "ceasar".

```
function caesar(x)
  { var y = "";
    var letters = "abcdefghijklmnopqrstuvwxyz";
    var n = x.length;









    return(y); }
```

Question 8 [5 marks] **GEM 1501**

Explain what is Nick's Class.

Given n and n numbers $a_1, a_2, \ldots, a_n \in \{0, 1, 2, 3, 4\}$ the task is to compute the remainder of $a_1 \cdot a_2 \cdot \ldots \cdot a_n$ when divided by 5. For example, if $n = 6$ then the output for $(3, 0, 2, 3, 4, 3)$ is 0 and the output for $(1, 1, 2, 1, 4, 1)$ is 3.

Is this problem in NC (Nick's Class)? ☐ Yes, ☐ No.

Explain why the problem is inside or outside Nick's class.

Question 9 [5 marks] **GEM 1501**

Describe what the Turing test is. Explain the conditions which a computer has to satisfy to pass the test. What are the challenges which one has to handle when making such a computer program.

Question 10 [5 marks] **GEM 1501**

Write a function `count(x)` which counts for input x how many factors p of x exist with the following properties: p is a prime number, $p+2$ is a prime number, $p(p+2)$ is a factor of x. So `count(630)` is 2 as witnessed by $p = 3$ (here 3 is a prime, 5 is a prime and $3 \cdot 5$ divides 630) and $p = 5$ (here 5 is a prime and 7 is a prime and $5 \cdot 7$ divides 630).

```
function count(x)
  { var c = 0;

    return(c); }
```

END OF PAPER