Midterm Examination 1
GEM 1501: Problem Solving for Computing

Wednesday 02.03.2011, duration half an hour

Matriculation Number: _______________

Rules
This test carries 12 marks and consists of 6 questions. Each questions carries 1 to 3 marks; full marks for a correct solution; a partial solution can give a partial credit.

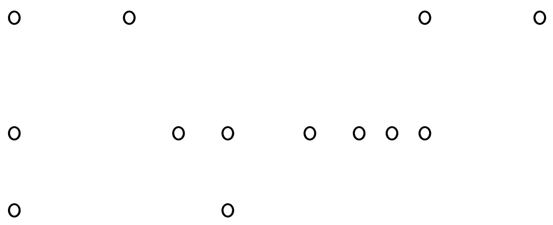
Question 1 [2 marks].
Punch cards have been used in computing to feed computers with data or programs. Describe how punch cards were used in the 19th century: What machines were invented by Joseph Jacquard and by Herman Hollerith, which used the punch cards? For what purpose needed they punch cards?

Question 2 [2 marks].
Call a list of n different pairs of numbers $(a_0, b_0), (a_1, b_1), \ldots, (a_{n-1}, b_{n-1})$ sorted iff there are no i, j with $i < j$, $a_i > a_j$ and $b_i > b_j$. An algorithm to sort the pairs can check whether $a_i < a_j$, $a_i = a_j$ or $a_i > a_j$; similarly the algorithm can check whether $b_i < b_j$, $b_i = b_j$ or $b_i > b_j$. Note that two pairs of numbers can be incomparable as for example $(1, 4)$ and $(2, 2)$. Traditional sorting algorithms do not address such data.

Is it nevertheless possible to sort all n pairs in time $O(n \log(n))$? Give reasons for your answer.

Question 3 [2 marks].
The railroad contractor problem asks for an algorithm to connect nodes in a network such that the resulting network on one hand connects all cities and on the other hand is as short as possible. This algorithm goes in general as follows: Starting with a network consisting of one node, it chooses in each step one node A outside the current network and links it to one node B in the network.

Describe how these two nodes A and B are selected?



Draw the resulting network into the above graphic for the given points, where the algorithm starts in the left lower corner.

Question 4 [3 marks].
The following Java Script function computes some number for an array called "list":

```
function sum(list)
  { var n = list.length;
    var i,j; var sum = 0;
    for (i=0;i<n;i=i+1)
      { for (j=0;j<n;j=j+1)
          { sum = sum+list[i]*list[j]; } }
    return(sum); }
```

What is the order of the running time of the program?

Is there also a program doing the same in time $O(n)$?

If no, explain why such a program cannot exist. If yes, please write the corresponding program below.

Question 5 [2 marks].
Write a program which does the following: It counts how many numbers of the form `x*(x*x+3)` are between `0` and `y`. So if `y` is `14` then the answer should be `3`; the corresponding numbers are `0`, `4` and `14`. Here `y` is always a natural number, that is, `y` is an element of the set $\{0, 1, 2, 3, 4, \ldots\}$.

```
function count(y)
  { var x = 0; var c = 0;
```

```
    return(c); }
```

Question 6 [1 marks].
An NP complete problem is satisfiability. Is the following set of clauses satisfiable?

□ Yes; □ No.

Here the clauses are

1. $x_1 \vee x_2$;
2. $x_2 \vee x_3$;
3. $x_3 \vee x_4$;
4. $x_4 \vee x_5$;
5. $x_5 \vee x_1$;
6. $\neg x_1 \vee \neg x_2$;
7. $\neg x_2 \vee \neg x_3$;
8. $\neg x_3 \vee \neg x_4$;
9. $\neg x_4 \vee \neg x_5$;
10. $\neg x_5 \vee \neg x_1$.