Midterm Examination 2
GEM 1501: Problem Solving for Computing

Wednesday 06.04.2011, duration half an hour

Matriculation Number: _____

Rules
This test carries 12 marks and consists of 6 questions. Each questions carries 2 marks; full marks for a correct solution; a partial solution can give a partial credit.

Question 1 [2 marks].
Let $\varphi_0, \varphi_1, \ldots$ be the list of all syntactical correct Java Script functions with one input variable where the only data type used are natural numbers. Rice's Theorem says that every index sets is either $\emptyset$ or $\{0, 1, 2, \ldots\}$ or undecidable. Explain what an index set is and give an example of an undecidable index set.

Question 2 [2 marks].
Consider the following finite automaton accepting ternary strings, that is, strings consisting of the digits 0, 1 and 2. The set of states of the finite automaton is $\{a, b, c, d, e\}$ and a is the starting state; b, c, d, e are the accepting states.

state	acc/rej	succ at 0	succ at 1	succ at 2
a	reject	a	b	c
b	accept	b	c	d
c	accept	c	d	e
d	accept	d	e	a
e	accept	e	a	b

Which of the following ternary strings is accepted by the automaton?

☐ 211, ☐ 1121, ☐ 10202.

Please give a verbal description when a ternary string $x_0x_1x_2 \ldots x_n$ is accepted by the automaton. What properties must the digits $x_0, x_1, \ldots, x_n$ have?

Question 3 [2 marks].
A researcher proposes the following parallel sorting network for four inputs:

- Input (a_1, a_2, a_3, a_4);
- $b_1 = \min\{a_1, a_2\}$, $b_2 = \min\{a_3, a_4\}$, $b_3 = \max\{a_1, a_2\}$, $b_4 = \max\{a_3, a_4\}$;
- $c_1 = \min\{b_1, b_2\}$, $c_2 = \max\{b_1, b_2\}$, $c_3 = \min\{b_3, b_4\}$, $c_4 = \max\{b_3, b_4\}$;
- Output (c_1, c_2, c_3, c_4).

Is (c_1, c_2, c_3, c_4) always a sorted copy of (a_1, a_2, a_3, a_4)?

☐ Yes; ☐ No.

For answer "yes", explain why the network is correct; for answer "no", provide a counterexample which is not properly sorted.

Question 4 [2 marks].
What are Monte Carlo algorithms and Las Vegas algorithms? What is the relation between these two types of algorithms: Is there a known method to adjust a Monte Carlo algorithm to a Las Vegas algorithm or is there a known method to adjust a Las Vegas algorithm to a Monte Carlo algorithm?

Question 5 [2 marks].
Assume that a sequence is given by

$$f(0) = 0,\ f(1) = 1 \text{ and } f(n+2) = f(n+1) - f(n).$$

Furthermore, assume that adding, subtracting, multiplying, dividing and taking remainders have cost $O(1)$. What is the complexity to compute f in this model? Give the optimal value. Here the parameter n is just the number used as input for $f(n)$.

☐ $O(1)$ ☐ $O(\log(n))$ ☐ $O(n)$ ☐ $O(2^n)$.

Justify your choice: why can the given bound be obtained? A program with a runtime estimate is fine. In the case that you do not take $O(1)$, say why your bound is optimal and cannot be improved.

Hint: Calculate some values $f(0), f(1), f(2), \ldots$ of the sequence before answering this question.

Question 6 [2 marks].
An array x is a subsequence of an array y if one can obtain x by deleting some entries from y without changing the order of the remaining entries. For example, $(2, 8, 7, 9)$ is a subsequence of $(1, 2, 3, 8, 7, 6, 9)$ but $(2, 8, 7, 9)$ is not a subsequence of $(9, 8, 7, 6, 5, 4, 3, 2, 1)$. Furthermore, every x is a subsequence of itself. Write a program which returns 1 if the input array x is a subsequence of the input array y and which returns 0 otherwise; the entries in the arrays x and y are numbers.

```
function issubsequence(x,y)
  { var r;




    return(r); }
```

END OF PAPER