

Midterm Examination 1

GEM 1501: Problem Solving for Computing

Thursday 20.02.2014, duration half an hour

Matriculation Number: TEST SOLUTIONS.

Rules

This test carries 10 marks and consists of 5 questions. Each questions carries 2 marks; full marks for a correct solution; a partial solution can give a partial credit.

Question 1 [2 marks].

The input of a function f consists of for bits x_3, x_2, x_1, x_0 and it computes one bit y such that $y = 1$ if and only if the value of the binary number $x_3x_2x_1x_0$, that is, the number $8 * x_3 + 4 * x_2 + 2 * x_1 + x_0$, is a square number. So $f(1, 0, 0, 1)$ should be 1 as the input represents the square number nine and $f(1, 1, 0, 0)$ should be 0 as the input represents the number twelve which is not a square. Give the formula which computes y from x_3, x_2, x_1, x_0 or draw the corresponding circuit. The connectives (gates) permitted are AND, OR, NOT, XOR, NAND and NOR.

Solution. The function is $f(x_3, x_2, x_1, x_0) = (NOT(x_3) AND NOT(x_1) AND NOT(x_0)) OR (NOT(x_2) AND NOT(x_1) AND x_0)$. The first conjunction outputs 1 on the inputs $(0, 0, 0, 0)$ and $(0, 1, 0, 0)$, the second conjunction outputs 1 on the inputs $(0, 0, 0, 1)$ and $(1, 0, 0, 1)$. These two conjunctions are combined by a disjunction which outputs 1 iff the input is a binary coding of the numbers 0, 1, 4 and 9.

Question 2 [2 marks].

Write down a basic html page which contains a JavaScript program to compute the following: On loading, it prompts for one input x , computes $y = x * x$ and outputs the value y , either by alert or by writing into the document. The html page should have a title and the JavaScript program should be included into the body of the page. Please give the full code of your page.

Solution.

```
<html>
<head><title>Squares</title></head>
<body><script>
document.write("<h1>Computing Squares</h1>");
var x = prompt("Input Number to be squared.");
var y = x*x;
var t = "The square of "+x+" is "+y+".";
alert(t); document.write("<h1>"+t+"</h1>");
</script></body></html>
```

Question 3 [2 marks].

Blaise Pascal, Charles Babbage and Herman Hollerith each constructed mathematical machines. Describe which of these machines were completed, what functionalities they had and how successful they were for solving computation problems of their time.

Solution. Blaise Pascal (1623-1662) was a French mathematician and physicist and philosopher. As his father was a tax collector who had to do many repeating calculations, he constructed a mechanical calculator which could add and subtract six digit numbers. He spent three years in constructing the machines and later produced 20 of them. Besides adding and subtracting, the machines could also do simple multiplications in form of repeated adding. Due to the restricted functionality of mechanical computing, these machines became more a status symbol than a tool used in everyday life.

Charles Babbage (1791-1871) was a British mathematician who devoted a large of his working time for ambitious projects to design and build mechanical computers. He designed two machines, called difference engine and analytical engine. The first one evaluated series of values of polynomial functions. The second one had the full generality of a modern computer. Though he spent a lot of effort on this machines, due to the high complexity of constructing mechanical calculating parts, both machines never got ready. However, in principle, the second machine had the same power as an early electronic computer with severe memory limitations.

Herman Hollerith (1860-1929) was an American statistician who constructed computing machinery in order to process information stored on punch cards. He participated a competition to develop and deploy the machinery to run the census of 1890 in the USA and his machinery brought down the processing time for the census from seven years (for the 1880 one) to few weeks. Although his machinery had not the universal programmability as Charles Babbage designed it for his analytical machine, the machinery was of great substantial help for the census and gave rise to the Tabulating Machine Company which eventually merged with other companies to form IBM.

Question 4 [2 marks].

Write a JavaScript function which computes the sum of the cubes from 0 to x , so the output y should be $0^3 + 1^3 + 2^3 + \dots + x^3$.

Solution.

```
function cubesum(x)
{ var y; var z;
  y = 0;
  for (z=0;z<=x;z++)
    { y = y+z*z*z; }
  return(y); }
```

Question 5 [2 marks].

Assume that a function f satisfies $f(0) = 3$ and $f(1) = 3 + f(0)$ and

$$f(n + 1) = 3 + f(0) + f(1) + \dots + f(n)$$

for all n . Determine the exact order of f and check the correct box (exactly one):

$f(n) \in \Theta(n)$; $f(n) \in \Theta(n^2)$; $f(n) \in \Theta(n^3)$;
 $f(n) \in \Theta(n^4)$; $f(n) \in \Theta(2^n)$; $f(n) \in \Theta(3^n)$.

Furthermore, check all boxes for which the below statement is true:

$f(n) \in O(n)$; $f(n) \in O(n^2)$; $f(n) \in O(n^3)$;
 $f(n) \in O(n^4)$; $f(n) \in O(2^n)$; $f(n) \in O(3^n)$.

Write a few lines how you determined which of the above choices applied.

Solution. The function satisfies the recurrence relations

$$f(n + 1) = 3 + f(0) + f(1) + \dots + f(n)$$

and

$$f(n + 2) = 3 + f(0) + f(1) + \dots + f(n) + f(n + 1)$$

where in the second relation one sees that $f(n+1)$ is equal to $3+f(0)+f(1)+\dots+f(n)$. Thus $f(n + 2) = 2 * f(n + 1)$. One can easily calculate that $f(0) = 3$, $f(1) = 6$, $f(2) = 12$ and then using the induction that $f(n) = 3 * 2^n$. Thus

$$f(n) \in \Theta(2^n), f(n) \in O(2^n), f(n) \in O(3^n)$$

and other choices from the multiple choice part do not apply.

END OF EXAMINATION.