Homework for 28.10.2004

Frank Stephan

Homework. The homework follows the lecture notes. What cannot be done as scheduled, will be done the week afterwards.

Lecture is Mon 16.00h - 17.30h and Thu 16.00h - 16.45h. Tutorial is Thu 16.45h - 17.30h. The room is S13#05-03.

http://www.comp.nus.edu.sg/˜fstephan/homework.ps
http://www.comp.nus.edu.sg/˜fstephan/homework.pdf

Exercise 13.4. For any ordinal α, consider the successor function S restricted to α, that is, consider the set

$$S\restriction\alpha = \{\{\beta, \{\beta, S(\beta)\}\} \mid \beta \in \alpha\}.$$

Determine $\mathcal{H}(S\restriction\alpha)$ for $\alpha = 42, 1905, 2004, \omega, \omega+1, \omega+131501, \omega^2+\omega\cdot 2+1, \omega^{17}+\omega^4$.

Exercise 13.6. V_ω has been defined twice. Let A be the version of V_ω as defined in Definition 7.5, that is let A consist of all hereditarily finite sets. Let $B = \bigcup\{V_n \mid n < \omega\} = \{x \in V \mid \mathcal{H}(x) < \omega\}$ be the version defined here. Show that both definitions coincide, that is, show $A \subseteq B \wedge B \subseteq A$.

Show that B contains $\emptyset$, is closed under unions of two sets and is closed under the operation forming $\{v\}$ from v. Thus, by Theorem 7.8, $A \subseteq B$.

Show by induction that all members of V_n with $n < \omega$ are hereditarily finite. Thus $B \subseteq A$.