MA 3205 – Set Theory – Homework for Week 9

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Homework. The homework follows the lecture notes. Below the list of the homeworks for the tutorials from 17.10.2006 onwards. It is not mandatory to hand in homework; but it is recommended to solve the questions by yourself before going to the tutorials. Homework can be corrected on request.

Exercise 11.8. The set

$$\{-\tfrac{1}{m_1+1} - \tfrac{1}{m_2+1} - \ldots - \tfrac{1}{m_n+1} \mid n, m_1, m_2, \ldots, m_n \in \mathbb{N}\}$$

is not a well-ordered subset with respect to the natural ordering of $\mathbb{Q}$: show that the set is dense and is not bounded from below.

Exercise 11.14. Define a function $f : \{0,1,\ldots,9\}^* \to \mathbb{N}$ which is order-preserving with respect to the length-lexicographic ordering $<_{ll}$: $v <_{ll} w \Leftrightarrow f(v) < f(w)$. Recall $0 <_{ll} 1 <_{ll} \ldots <_{ll} 9 <_{ll} 00 <_{ll} 01 <_{ll} \ldots <_{ll} 99 <_{ll} 000 <_{ll} \ldots$ and $v <_{ll} w$ if either v is shorter than w or v, w have the same length and $v <_{lex} w$.

Exercise 12.7. Verify the following properties of ordinals.

1. If α is an ordinal, then $S(\alpha)$, which is defined as $\alpha \cup \{\alpha\}$, is also an ordinal.
2. Every element of an ordinal is an ordinal.
3. An ordinal α is transfinite iff $|\alpha| = |S(\alpha)|$.
4. An ordinal α is finite iff $S(\alpha) = \{0\} \cup \{S(\beta) \mid \beta \in \alpha\}$.

Exercise 12.9. Show that the class V_{ord} of all ordinals in V is not a set.