MA 3205 – Set Theory – Homework due Week 9

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Homework. The homework follows the lecture notes. You have to hand in one starred homework in Weeks 3–6, in Weeks 7–9 and in Weeks 10–13. Further homework can be checked on request. Homework to be marked should be handed in after the lecture on Tuesday of the week when the homework is due.

Exercise 11.8. The set

$$\{-\tfrac{1}{m_1+1} - \tfrac{1}{m_2+1} - \ldots - \tfrac{1}{m_n+1} \mid n, m_1, m_2, \ldots, m_n \in \mathbb{N}\}$$

is not a well-ordered subset with respect to the natural ordering of $\mathbb{Q}$: show that the set is dense and is not bounded from below.

Exercise 11.14. Define a function $f : \{0, 1, \ldots, 9\}^* \to \mathbb{N}$ which is order-preserving with respect to the length-lexicographic ordering $<_{ll}$: $v <_{ll} w \Leftrightarrow f(v) < f(w)$. Recall $0 <_{ll} 1 <_{ll} \ldots <_{ll} 9 <_{ll} 00 <_{ll} 01 <_{ll} \ldots <_{ll} 99 <_{ll} 000 <_{ll} \ldots$ and $v <_{ll} w$ if either v is shorter than w or v, w have the same length and $v <_{lex} w$.

Exercise 12.7*. Verify the following properties of ordinals.

1. If α is an ordinal, then $S(\alpha)$, which is defined as $\alpha \cup \{\alpha\}$, is also an ordinal.
2. Every element of an ordinal is an ordinal.
3. An ordinal α is transfinite iff $|\alpha| = |S(\alpha)|$.
4. An ordinal α is finite iff $S(\alpha) = \{0\} \cup \{S(\beta) \mid \beta \in \alpha\}$.

Exercise 12.9. Show that the class V_{ord} of all ordinals in V is not a set.