MA 5219 - Logic and Foundations of Mathematics 1
Homework due in Week 4, Tuesday.

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Hand in each starred homework; 1 mark per homework (if it is correct), up to 10 marks in total for homework.

4.1* **Proof systems.** Consider the following rules.

$$\frac{\emptyset}{A \vdash A} \qquad \frac{X \vdash A}{X \cup Y \vdash A}$$
$$\frac{X \vdash A, A \to B}{X \vdash B} \qquad \frac{X, A \vdash B}{X \vdash A \to B}$$
$$\frac{X \vdash A \to \bot}{X \vdash \neg A} \qquad \frac{X \vdash \neg A}{X \vdash A \to \bot}$$
$$\frac{X \vdash \neg(A \to \neg B)}{X \vdash A, B} \qquad \frac{X \vdash A, B}{X \vdash \neg(A \to \neg B)}$$
$$\frac{X \vdash \neg\neg A}{X \vdash A} \qquad \frac{X \vdash A}{X \vdash \neg\neg A}$$
$$\frac{X \vdash \neg A \to B}{X \vdash \neg B \to A} \qquad \frac{X, A \vdash B \text{ and } X, \neg A \vdash B}{X \vdash B}$$

Derive the following rules from the above rules.

$$\frac{X \vdash A \to B, \neg A \to B}{X \vdash B}$$
$$\frac{X \vdash A \to B \to C}{X \vdash B \to A \to C}$$
$$\frac{\emptyset}{X \vdash A \to B \to A}$$
$$\frac{\emptyset}{X \vdash (A \to B \to C) \to (A \to B) \to A \to C}$$

4.2 Models. Consider a set A with an operation $\circ$ and let the lower case letters be variables in the models:

- $\forall x, y, z[(x \circ y) \circ z = x \circ (y \circ z)]$ and $\exists a, b \forall x, y[a \circ x = a \wedge y \circ b = b]$;
- $\forall x, y, z[(x \circ y) \circ z = x \circ (y \circ z)]$ and $\exists e \forall x, y[e \circ x = e \wedge y \circ e = e]$.

Is there a model $(A, \circ)$ which satisfies one set of axioms but not the other one?