MA 5219 - Logic and Foundations of Mathematics 1
Homework due in Week 7, Tuesday.

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Hand in each starred homework; 1 mark per homework (if it is correct), up to 10 marks in total for homework.

7.1* Logical Implication. Here let e is a constant and v, w, x, y, z are variables. Let

$$\begin{array}{lll} X & \text{contain} & \forall x, y, z\,[x \circ (y \circ z) = (x \circ y) \circ z],\ \forall x[x \circ e = x],\ \forall x\,[x \circ x = e] \\ \alpha & \text{be} & v \circ w = w \circ v, \\ \beta & \text{be} & v \circ (v \circ v) = v \text{ and} \\ \gamma & \text{be} & v \circ (v \circ v) = e. \end{array}$$

Do $X \models \alpha$, $X \models \beta$ and $X \models \gamma$ hold? Justify for each of the formulas α, β, γ your answers.

7.2* Henkin Sets. Let the logical language $\mathcal{L}$ contain first-order formulas over variables $x_0, x_1, \ldots$ and constants $c_0, c_1, \ldots$ and one predicate symbol P. Check whether the following sets X, Y and Z are Henkin sets. Explain why you think that the corresponding sets are Henkin sets or not.

X contains for all distinct i, j the formula $c_i \neq c_j$ as well as for every k the formulas $x_k = c_k$ and $P(c_k)$.

Y contains the same formulas as X plus the formula $\forall x_0\,[P(x_0)]$.

Z contains the same formulas as X plus the formula $\exists x_0\,[\neg P(x_0)]$.

7.3 Countable models. Assume that X is a Henkin set and $\mathcal{A}$ is a model of X. Show that $\mathcal{A}$ has a substructure $\mathcal{B}$ which is a countable model of X.

7.4 Rules. Enlarge the rule sets on page 92 such that it also deals with or ($\vee$), implication ($\rightarrow$) and existential quantification ($\exists$). Use this enlarged rule system together with the axioms for a groups (where e is the constant denoting the neutral element) in order to prove the following sentence:

$$\forall x\,[x \circ (x \circ x) = e] \rightarrow \forall y, z\,[y \circ (z \circ (y \circ (z \circ y))) = z \circ z].$$