

MA 5219 - Logic and Foundations of Mathematics 1

Homework due in Week 9, Tuesday.

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Office hours Thursday 14.00-15.00h

Hand in each starred homework; 1 mark per homework (if it is correct), up to 10 marks in total for homework.

9.1* Register Programs.

Write register programs which compute the below functions; the programs are permitted to add, subtract and compare variables and constants. As the data-type is natural numbers, 5-8 gives 0. Please be aware of that.

(a) $f(x, y)$ is the remainder when x is divided by y ; as exception handling, $f(x, 0)$ and $f(x, 1)$ are always 0.

(b) $g(x) = x^2 + 18 * x + 22$.

(c) $h(x) = 6^x$.

9.2* Halting Problem.

On which inputs does the following program halt? What is the output?

1. Function $f(x, y, z)$
2. If $y < x$ then goto 5;
3. If $z < x$ then goto 5;
4. $y = y - x$; $z = z - x$; Goto 2;
5. If $z < y$ then goto 7;
6. $y = z$;
7. If $y < 2$ then goto 9;
8. $y = y - 2$; Goto 7;
9. Return(y).

9.3 Primitive Recursive Functions.

Assume that you know that all linear functions like $a(x, y, z) = 5 + 3 * x + 2 * y - z$ are primitive recursive and that case distinctions are primitive recursive, like

$$b(x, y) = \begin{cases} c(x, y) & \text{if } x < y; \\ d(x, y) & \text{if } x = y; \\ e(x, y) & \text{if } x > y; \end{cases}$$

where c, d, e are already known to be primitive recursive functions. Now show that the following functions are primitive recursive.

(a) $f(x)$ is the remainder when x is divided by 5.

(b) $g(x) = x^2 + 18 * x + 22$.

(c) $h(x) = 6^x$.