MA 5219 - Logic and Foundations of Mathematics 1
Course-Webpage `http://www.comp.nus.edu.sg/~fstephan/mathlogic.html`
Homework due in Week 12, Tuesday 5 November 2013.

Frank Stephan. Departments of Mathematics and Computer Science,
10 Lower Kent Ridge Road, S17#07-04 and 13 Computing Drive, COM2#03-11,
National University of Singapore, Singapore 119076.
Email fstephan@comp.nus.edu.sg
Telephone office 65162759 and 65164246
Office hours Thursday 14.00-15.00h S17#07-04

Hand in each homework which you want to be checked; 1 mark per each correct starred homework; up to 10 marks in total for homework - there will be more than 10 starred homeworks, so you have several chances to try.

12.1* Number of rounds in Ehrenfeucht-Fraïssé games. Give an example of two relational structures $\mathcal{A}$ and $\mathcal{B}$ such that the Duplicator has a winning strategy for the Ehrenfeucht-Fraïssé game of 2 rounds and the Spoiler has a winning strategy for the Ehrenfeucht-Fraïssé game of 3 rounds.

12.2* Finite Structures and the Ehrenfeucht-Fraïssé Game. Are there non-isomorphic finite relational structures $\mathcal{A}$ and $\mathcal{B}$ such that the Duplicator has a winning strategy for every Ehrenfeucht-Fraïssé game? Prove your answer.

12.3* Ehrenfeucht-Fraïssé Games on Infinite Graphs. A graph is given by a set of vertices and an edge-relation E which satisfies $E(x,y) \rightarrow x \neq y$ and $E(x,y) \leftrightarrow E(y,x)$ for all x, y. Give an example of two infinite graphs which are not isomorphic but for which the Duplicator has a winning strategy in every Ehrenfeucht-Fraïssé game.

12.4 Model Complete Theories. A consistent theory is called model complete iff $T + \mathcal{DA}$ is complete in $\mathcal{LA}$ for every model $\mathcal{A}$ of T, that is, if for every $\alpha \in \mathcal{LA}$ either $T + \mathcal{DA} \vdash \alpha$ or $T + \mathcal{DA} \vdash \neg\alpha$. Provide an example of an axiomatisable theory without finite models which is model complete but not complete.