Homework 3.1
Construct a bijection from the finite sequences of natural numbers to the set of natural
numbers. Here a finite sequence is a sequence \((x_1, x_2, \ldots, x_n)\) where this sequence is
empty in the case that \(n = 0\). There are in principal three ways to do the coding:
First by modifying the way prime factors are used; second by coding the numbers
using binary digits; third by iterating Cantor’s pairing function. Use at least two of
these methods to construct the corresponding functions, that is, give two bijections
using these methods and explain why they are bijections.

Homework 3.2
Show that a power set has always more elements than the given set, that is, fill out
the missing details at the following proof-sketch. Recall that \(|A| \leq |B|\) iff there is a
one-one function from \(A\) to \(B\) and show that \(|P_A| \nless |A|\).

Proof-Sketch: The \(\emptyset\) has 0 and \(P\emptyset\) has one element, namely \(\emptyset\), hence one cannot have
a one-one mapping from \(P\emptyset\) to \(\emptyset\). Now assume that \(A\) is not empty and \(f : A \to P A\)
is a function. Show that there is a set \(B \subseteq A\) which is not in the range of \(f\). Then
consider any function \(g : PA \to A\) and prove that this function cannot be one-one, as
otherwise a surjective \(f\) from \(A\) to \(PA\) would exist. Hence \(|PA| \nless |A|\).

Homework 3.3
Consider the following formulas:

\[
\phi_1 = (((A_1 \lor A_2) \lor A_3) \land ((A_4 \lor A_5) \lor A_6)); \\
\phi_2 = (((A_1 \lor A_2) \land (A_3 \lor A_4)) \land (A_5 \lor A_6)); \\
\phi_3 = (((A_1 \oplus A_2) \oplus A_3) \oplus A_4) \oplus A_5) \oplus A_6).
\]

There are \(2^6 = 64\) ways to assign the truth-values to the sentence symbols (or atoms)
\(A_1, \ldots, A_6\). Determine for each of the formulas \(\phi_1, \phi_2, \phi_3\), how many of these assignments
make the formula true and how many of these assignments make the formula
false.

Homework 3.4
For the formulas from Homework 3.3, are the following statements true or false?

\[
\{\phi_1, \phi_2, \phi_3\} \models (((A_1 \land A_2) \land A_3) \land A_4) \land A_5) \land A_6); \\
\{\phi_1, \phi_2, \phi_3\} \models (((A_1 \lor A_2) \lor A_3) \lor A_4) \lor A_5) \lor A_6).
\]