Homework 7.1
Assume that there are infinitely many logical atoms. Is there a set $S$ of formulas such that for all $v$ mapping atoms to $\{0, 1\}$, $v \models S$ iff there are exactly three atoms $A, B, C$ with $v(A) = 1$, $v(B) = 1$, $v(C) = 1$?

Homework 7.2
For first-order logic, assume that the logical language has only equality and variables and quantifiers and the logical connectives. The formula $\exists x, y, z \left[ x \neq y \land x \neq z \land y \neq z \right]$ can only be satisfied by a structure with at least three elements. Is there, in this logical language, a formula $\alpha$ which can only be satisfied by structures with infinitely many elements? Is there a set of such formulas?

Homework 7.3
Consider the structure $(\mathbb{N}, +, -, \cdot, <, =, 0, 1, 2, \ldots)$ and the corresponding first-order logical language of arithmetic with constants for every natural number. Make formulas which express the following:

1. Each number is either 0 or 1 or the multiple of a prime number;
2. There are infinitely many prime numbers;
3. Every even number other than 0 and 2 is the sum of two prime numbers;
4. The number 23 is not the sum of three squares;
5. Every number is the sum of four squares;
6. There are infinitely many numbers $x$ such that $x - 1$ and $x + 1$ are both prime numbers.

Homework 7.4
Let $(F, +, -, \cdot, f, =, 0, 1, 2)$ be the finite field with the three elements 0, 1, 2 and a one-place function $f$. Which of the following statements are true for this structure?

1. $\forall x, y \left[ (x + y) \cdot (x + y) = (x \cdot x) + (y \cdot y) - (x \cdot y) \right]$;
2. $\forall x, y \left[ (x + y) \cdot (x + y) \cdot (x + y) = (x \cdot x \cdot x) + (y \cdot y \cdot y) \right]$;
3. $\forall x, y \left[ (x + y) \cdot (x + y) \cdot (x + y) \cdot (x + y) = (x \cdot x \cdot x \cdot x) + (y \cdot y \cdot y \cdot y) \right]$;
4. $\exists a, b, c \forall x \left[ f(x) = a \cdot x \cdot (x - 1) + b \cdot x \cdot (x - 2) + c \cdot (x - 1) \cdot (x - 2) \right]$;
5. $\forall x \left[ x \cdot x \cdot x \neq 2 \right]$. 