Homework 11.1
(a) Use the Deduction Theorem to show the following:
If $\Gamma \vdash \alpha \rightarrow \beta \rightarrow \gamma \rightarrow \delta$
then $\Gamma \vdash \gamma \rightarrow \alpha \rightarrow \beta \rightarrow \delta$.
(b) Do the same proof using tautologies and modus ponens only.
(c) Which other interchanges of $\alpha, \beta, \gamma, \delta$ are permitted and which not?

Homework 11.2
Let $(A, +, 0)$ be a structure with constant 0 and binary operation +. Make a formal proof for
$\{\forall x [x + x = 0]\} \vdash \forall x [(x + x) + (x + x) = 0]$ 
using axioms from $\Lambda$ and the Generalisation Theorem.

Homework 11.3
Consider all structures $(A, \circ)$ where $A$ has two elements and satisfies the axioms
$\forall x [x \circ x = x]$ and $\forall x \forall y [x \circ y = y \circ x]$.
Show that all these structures are isomorphic.

Homework 11.4
Assume that $(\mathbb{N}, +, <, 0, 1, P)$ is a structure where $\mathbb{N}$ is the set of natural numbers and $+, <, 0, 1$ have the usual meaning on $\mathbb{N}$. Let the powers of 2 be the set $\{1, 2, 4, 8, 16, \ldots\}$ and make a formula $\alpha$ such that $(\mathbb{N}, +, <, 0, 1, P) \models \alpha$ iff $\forall x [Px \leftrightarrow x \text{ is a power of 2}]$.
Note that such a formula only implicitly defines the powers of 2 and not explicitly; therefore this formula $\alpha$ does not say that the powers are definable from addition and order in $\mathbb{N}$.

Homework 11.5
Make a formula $\alpha$ which says that $f : A \to A$ is a one-to-one function but not an onto function. Provide a model $(A, f, =)$ which satisfies $\alpha$. Can $A$ be finite?