

GEM 1501 Problem Solving With Computers

Lecture 2:

Algorithmics

Martin Henz

Summary of Previous Lecture

- Computers can do simple steps very fast
- Computers can solve those problems that can be broken down into sequences of simple steps
- Some problems cannot be broken down into sequences of simple steps

Overview of this Lecture

- History of algorithmics
- Motivation
- Control structures

Focus Today: How To Take Steps?

- What are the kinds of steps that we take?
- How to control the steps?
- How to describe the steps to be taken?

Focus Next Lecture: How To Execute These Steps?

- Programming languages
- Algorithmic methods
- Correctness and efficiency

Overview of this Lecture

- **History of algorithmics**
- Motivation
- Control structures

History of Algorithms

- First algorithm
- First description of algorithmics
- First machine to execute algorithms
- First computing machine
- First programming

First Algorithm

- Euclid of Alexandria (born about 325 BC, died about 265 BC)
- Most famous for his "Elements" (used as standard textbook in geometry in schools until early 20th century!)
- Follower of Plato
- Invented the first algorithm, for finding the Greatest Common Divisor (GCD) of two positive integers
- See <http://aleph0.clarku.edu/~djoyce/java/elements/bookVII/propVII2.html>

First Description of Algorithmics

- Mohammed al-Khowārizmī (born about 780 AD in Baghdad, died about 850)
- Founder of “school algebra” (as he saw it, theory of linear and quadratic equations with a single unknown, and the elementary arithmetic of relative binomials and trinomials)
- Described many algorithms for adding, dividing, equation solving etc.

First Machine to Execute Algorithms

- Invented by Joseph Jacquard in 1801
- Domain of application: Weaving
- Algorithms described “declaratively” by holes punched in stiff cards
- Holes determine weaving pattern
- Executed by a mechanical loom

First Computing Machine

- Invented by Charles Babbage (born in 1791, Died 1871)
- Precursor was “difference engine”
- General computing device called “analytical engine”
- Programmed by punch cards
- Never completed (parts were recently built by a team of historians and mechanics)

First Computing Machine (II)

- Invented by Herman Hollerith (born 1860, died 1929)
- Based on relays (electromechanical devices)
- Used by American Census Bureau
- Gave rise to company “Tabulating Machine Company” in 1896

First Computing Machine (III)

- In the late 40s, early 50s, several machines were built that allowed general computing
- Among them ENIAC (assembled late 1945), and the Z series of computers in Germany
- Programmed using punch cards
- General “von Neumann” architecture evolved from these machines in the late 1940s

First Programming

- Ada Lovelace “programmed” Babbage’s analytical engine
- Logicians Turing, Gödel, Markov, Church, Post, Kleene all worked on formalisms for general computing and thus “programmed”
- Zuse developed “Plankalkül in 1944
- Grace Hooper’s MATH-MATIC (1950s)
- FORTRAN (1957)

Overview of this Lecture

- History of algorithmics
- **Motivation for algorithms**
- Control structures

Algorithms are Recipes

Melt chocolate and 2 tablespoons water in double boiler. When melted, stir in powdered sugar; add butter bit by bit. Set aside. Beat egg yolks until thick and lemon-colored, about 5 minutes...

Loops

Loops allow short descriptions of long processes. Example: compute the total sum of all salaries in a list of employees

1. make a note of the number 0
2. proceed through the list, adding each employee's salary to the noted number
3. having reached the end of the list, produce the noted number as output

Algorithmic Problems

- Characterize input/output behavior (algorithmic problem)
- Formulate an algorithm A
- A should be given input and produce output

Overview of this Lecture

- History of algorithmics
- Motivation for algorithms
- **Control structures**

Control Structures

- Direct sequencing
- Conditional branching
- Bounded iteration
- Conditional iteration
- Subroutines
- Recursion

Direct Sequencing

- “do A followed by B”
- “gently fold in chocolate; reheat slightly”
- In programming, often the ; symbol is enough
- A; B
- Sometimes, just juxtaposition does the job
- A B

Conditional Branching

- “if Q then do A otherwise do B”
- “if Q then do A” (otherwise do nothing)
- “reheat slightly to melt chocolate, if necessary”

Combining Sequencing and Branching

- A; if Q then B; C else D; E
- The precedence of the operators matters
- if Q then A; B
- (if Q then A); B

Bounded Iteration

- “do A exactly N times”
- N is a fixed (or computed) number
- allows to describe long computations with short programs

Conditional Iteration

- “repeat A until Q”
- “while Q do A”
- “beat egg whites until foamy”
- combines iteration with conditional branching

Example: Salary Computation

1. make a note of 0; point to the first salary on the list;
2. do the following $N - 1$ times
 - (a) add the salary pointed at to the noted number;
 - (b) point to the next salary;
3. add the salary pointed at to the noted number
4. produce the noted number as output

Another Example: Bubble Sort

- Given: A list of numbers
- Desired: Sorted list with the same numbers
- Algorithm: go “often enough” through the list and exchange neighboring numbers

Bubble Sort

1. do the following $N - 1$ times
 - (a) point to the first element
 - (b) do the following $N - 1$ times
 - i. compare the element pointed to with the next element
 - ii. if the compared elements are in the wrong order, exchange them
 - iii. point to the next element

Diagrams from Algorithms

- Describe algorithms in 2-D
- Use arrows to describe the flow of control
- Not all diagrams can be written in any programming language
- Arrows correspond to “goto” instructions on some languages

Subroutines: Example

Let us count the number of sentences in which the word “money” occurs.

Subroutine: search-for X

1. do the following until either the combination X is being pointed at, or the end of the text is reached:
 - (a) advance the pointer one symbol in the text;
2. if the end of the text is reached, output the counter’s value and stop;
3. otherwise return to the main algorithm

Main Algorithm

1. set counter to 0; pointer to the start of the text
2. do the following until the end of the text is reached
 - (a) search-for “money”
 - (b) search-for “.”
 - (c) increment counter

Recursion: Towers of Hanoi

1. Move a tower of rings from A to B using C
2. No larger ring can sit on top a smaller ring
3. Idea: use subroutine for moving smaller tower

Subroutine: move N from X to Y using Z

1. if N is 1 then output "move X to Y"
2. otherwise
 - (a) call move $N - 1$ from X to Z using Y
 - (b) output "move X to Y"
 - (c) call move $N - 1$ from Z to Y using X

Main Algorithm

call move 3 from A to B using C

Next Week

- Programming languages
- Introduction to Oz