CS4234: Optimization Algorithms Nov. 5, 2015

Tutorial Week 12: Game Day

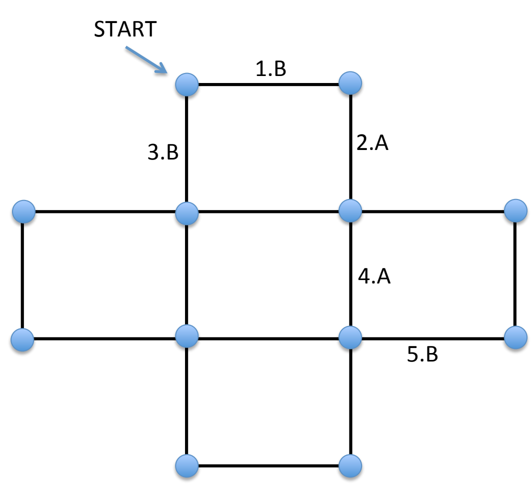
Slither

The game of Slither is played on a graph. The goal is to build a snake that is slithering all over the graph. It is played by two players: Player A and Player B. The game proceeds as follows:

- Player A chooses a node in the graph. This is the designated *start node*.
- Player B chooses an edge adjacent to the start node.
- The players alternate choosing edges that extend the path.

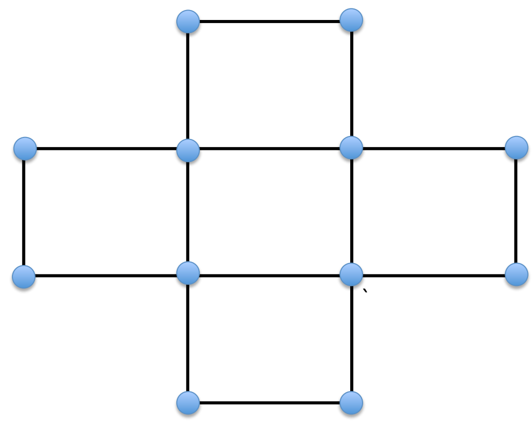
The edges selected must form a simple path: there may be no cycles. The loser is the first player who cannot make a move.

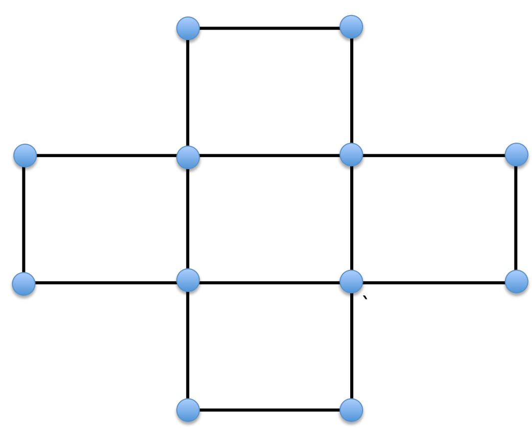
In the example below, Player A chooses the top-left node as the start node. Player B then chooses the edge to the right, and player A moves down. Player B then decides to extend the other side of the snake downward. And so on.

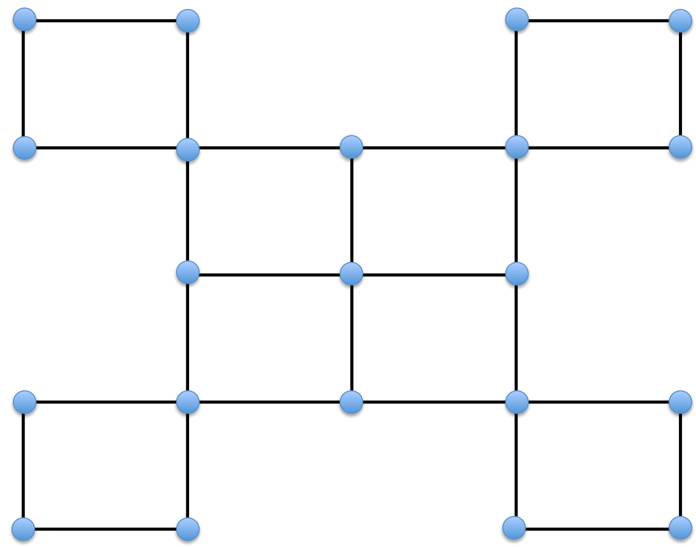


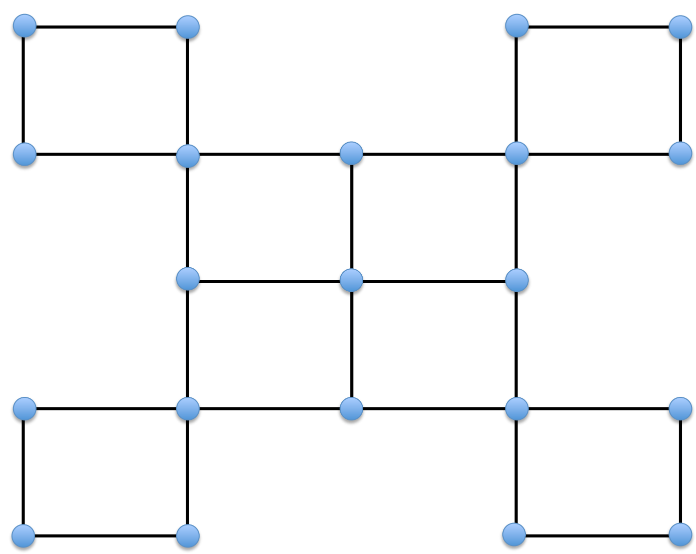
On the next several pages, you will find several possible playing boards. (The triangle version is just for fun.) Try to answer the following questions:

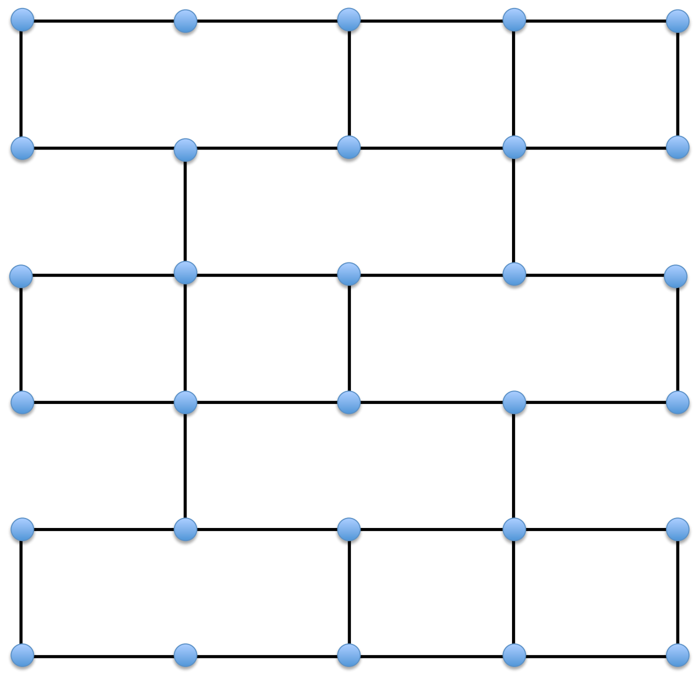
1. For each playing board, which player wins? For each graph, one of the two players always has a winning strategy.
2. Give an algorithm to decide which player will win for a given graph.
3. What is the winning strategy for each player?

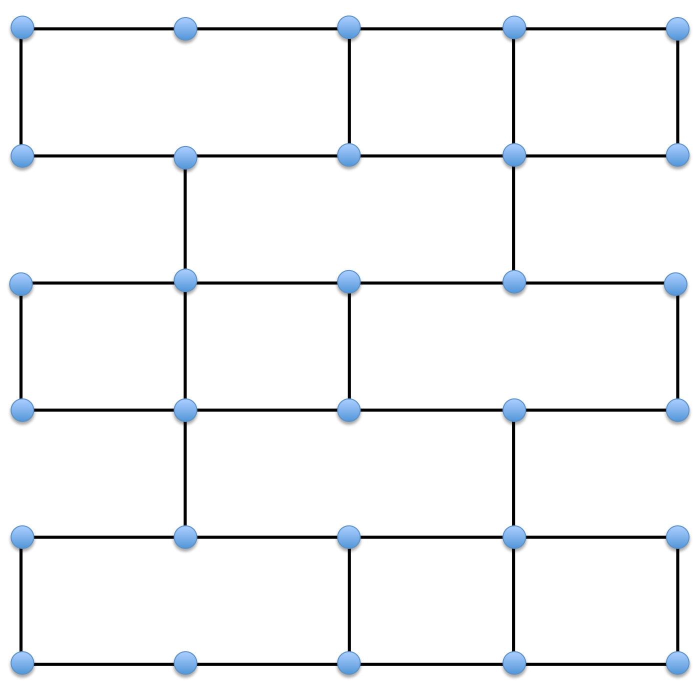


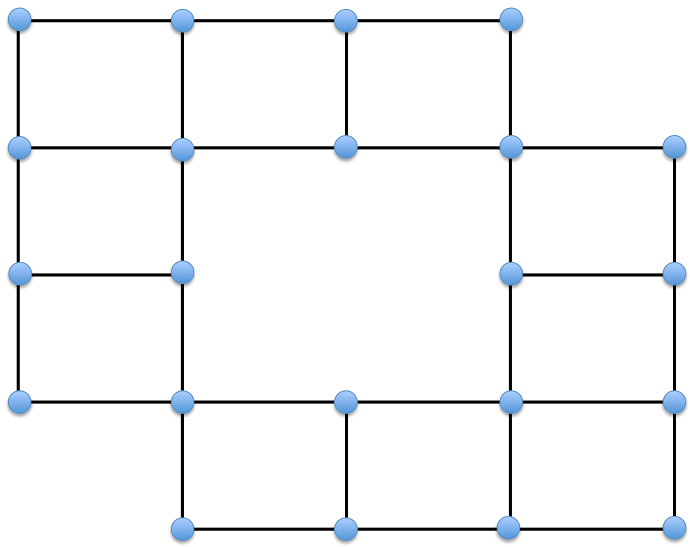


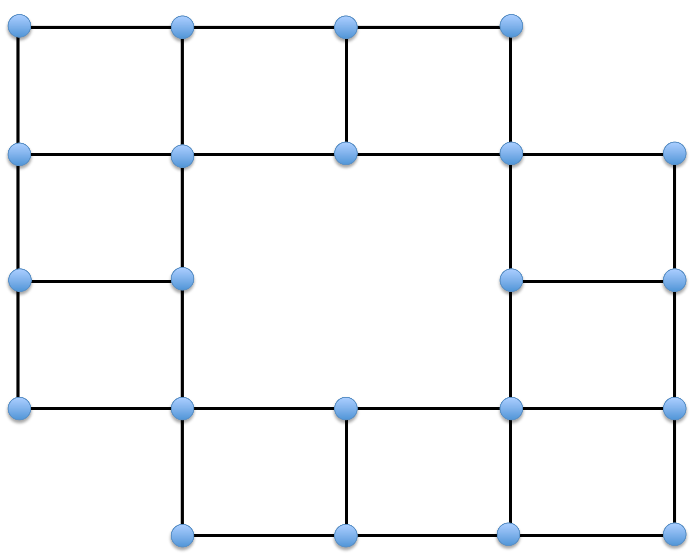


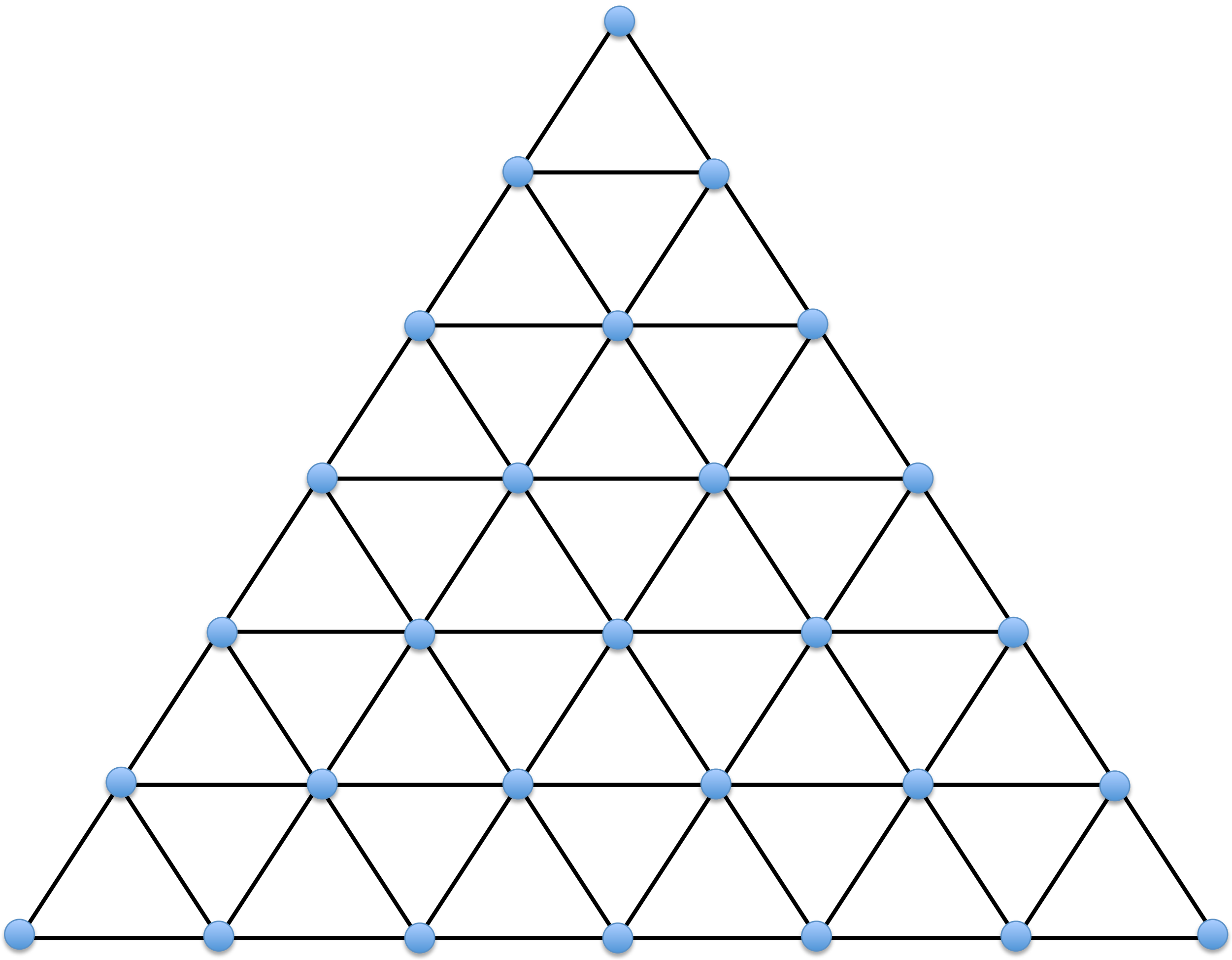








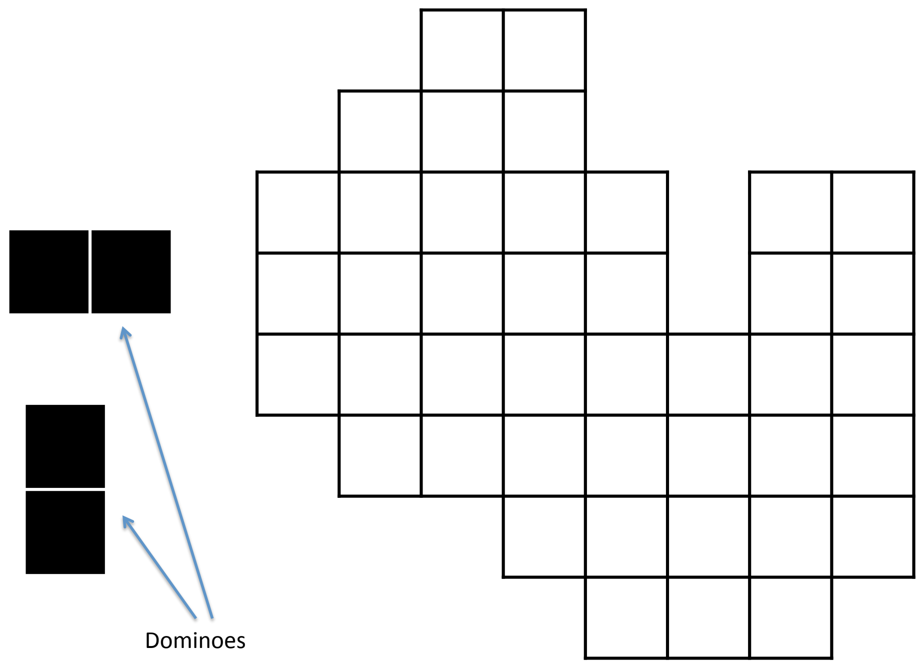




(Beware that the triangle graph is a little different than the previous graphs.)

Dominoes

Imagine you are playing a game of dominoes. Each domino covers exactly two adjacent squares. Dominos may not overlap. Give an algorithm for determining whether it is possible to cover all the squares with dominoes. Is it possible in the example below? If so, show a tiling. If not, prove it.



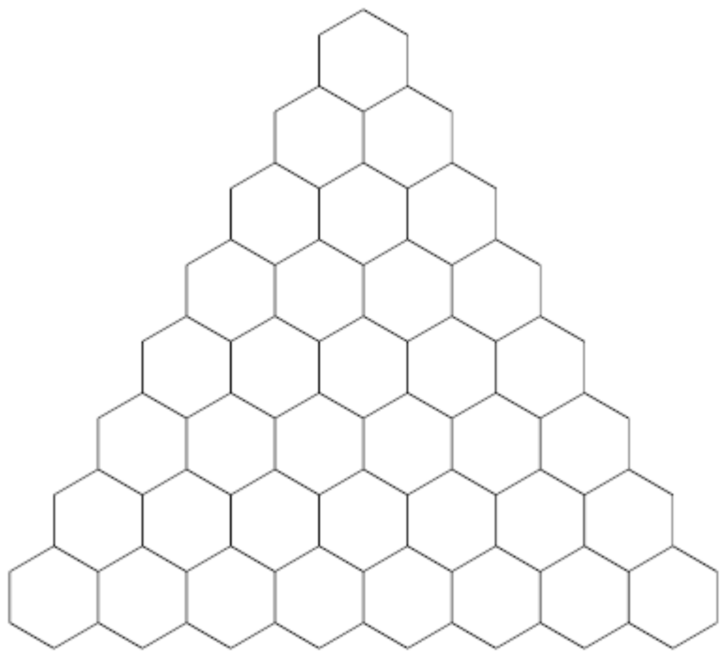
Hint: A bipartite graph $G = (A, B, E)$ has a perfect matching if and only if for every subset $a \subseteq A$, $|N(a) \geq |a|$, where $N(a)$ refers to all the neighbors of nodes in a (i.e., all the nodes in B with edges to nodes in a).

Can you prove this? The hard part is to show that if there is no perfect matching, then there exists a set a where $|a| > |N(a)|$. Think about the construction for bipartite vertex cover, the connection between vertex cover and matchings, and show that $S_B > S_A$.

MiserY

Warning: This problem is just for fun; it is unrelated to the earlier problems.

The game of *MiserY* is the "misere" version of the game Y. It is played on a triangular shaped board consisting of hexagons. It is played by two players, white and black. Each player takes turns placing a token (or writing "W" or "B") on an empty hexagon. (Once a token has been played, it never moves.) The loser is the first player to create a single connected component that connects all three sides of the board. A piece in the corner automatically connects two sides. (For example, if a player owns all the hexagons on one side, then they lose. If a player builds a Y that touches all three sides, they lose. Etc.)



It is possible to prove that:

- There is always a winner (and a loser)—the game never results in a tie.
- If there are an even number of positions on the board, then the first player has a winning strategy; if there are an odd number of positions on the board, then the second player has a winning strategy.

These proofs do not rely on material we have covered this semester, and are not easy to derive.

To the best of my knowledge, no one knows how to actually *find* a winning strategy. We can prove that it must exist, but we do not know what it is!