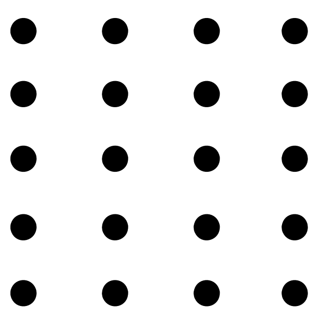
CS5234

Algorithms at Scale

(Fast Algorithms for Slow Problems)

Welcome!

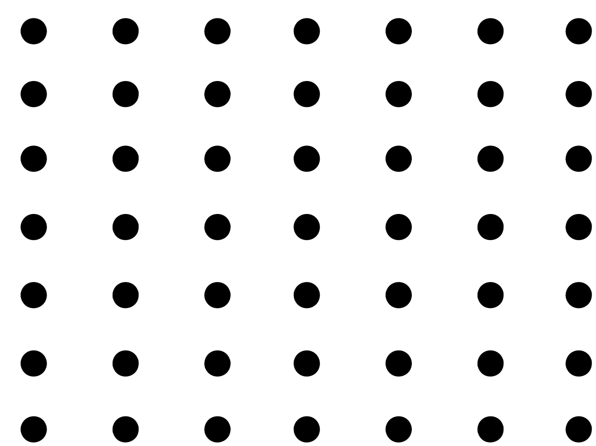


5 x 4:
Use 7 lines.

Puzzle of the Day:

Connect the dots.
Use ?? straight lines.
Don’t lift your pen from the paper.

Can you end at the same place you began?



7 x 7:
Use 12 lines

Why are we here?

Everyone here knows classical algorithms:

- Dijkstra's, Prim's, Red-black trees, Fibonacci Heaps, etc.

What happens when you try to use these algorithms on real-world data sets?

Example:

Imagine a graph containing:

- 4 billion nodes
- 1.2 trillion edges

$\Rightarrow$ E.g., $\cong$ 1 TB of data

(The Facebook graph, for example, is at least that big.)

(Today, a 1TB data set is not that big.)

Example:

Imagine a graph containing:

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⇨ E.g., $\cong$ 1 TB of data

Simple question:

What is the diameter of your graph?

Example:

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Simple question:

What is the diameter of your graph?

Simple answer:

Run breadth-first-search n times. $\Rightarrow$ $O(m \cdot n)$

Some numbers

Assume a graph of size 1TB.

- Disk scan: 200 MB/s ➔ 83 minutes
- Disk seek: 1 MB/s ➔ 11.5 days

Cost of simple breadth-first-search? $\cong$ 1 week

Cost of finding the diameter? $\cong$ years

Why are we here?

To answer simple questions…

- What is the diameter?
- Is the graph connected?
- How many connected components are there?
- What is the shortest path from A to B?
- Find a minimum spanning tree.

… when classical algorithms are too slow.

General strategies?

What can we do?

General strategies?

What can we do?

- Optimize the algorithm.
  - Leverage special structures (e.g., graph is planar).
  - E.g., Use caches more efficiently, keep your data better organized, etc.

- Approximate the answer.
  - E.g., Use random sampling.

- Parallelize the computation.
  - E.g., leverage multicore / multiprocessor / cluster computation

Example: Diameter

Step 1: Approximate

Goal:

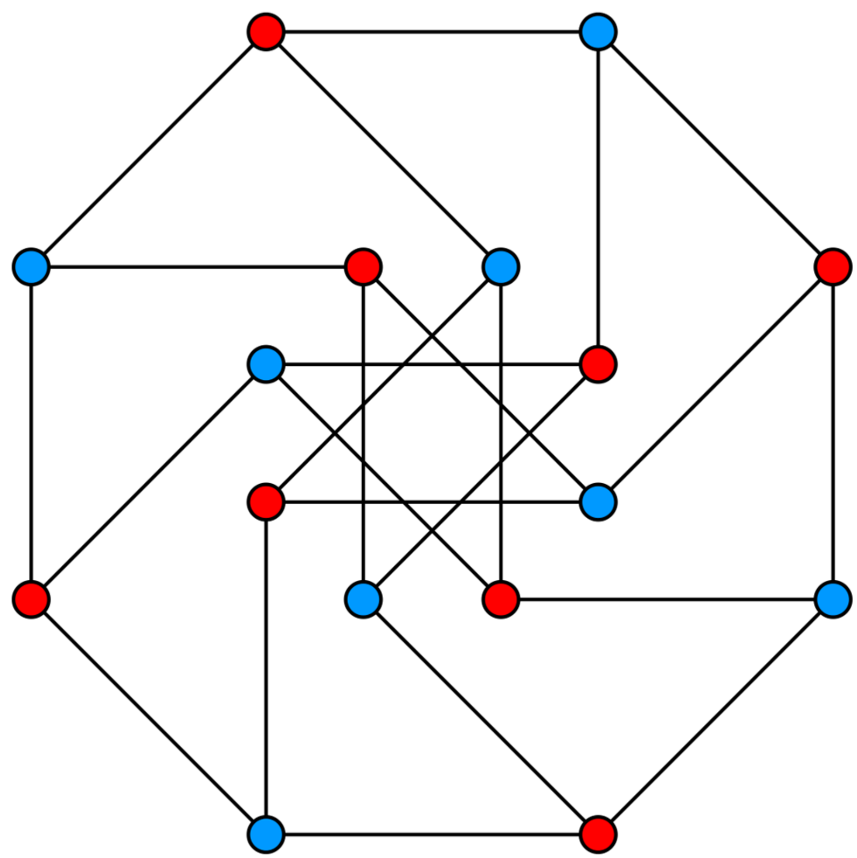
Given graph G=(V,E), find approximate diameter D such that:

$$(\tfrac{1}{2})\cdot \mathrm{diameter}(G) \leq D \leq 2\cdot \mathrm{diameter}(G)$$

$\Rightarrow$ 2-approximation

Example: Diameter

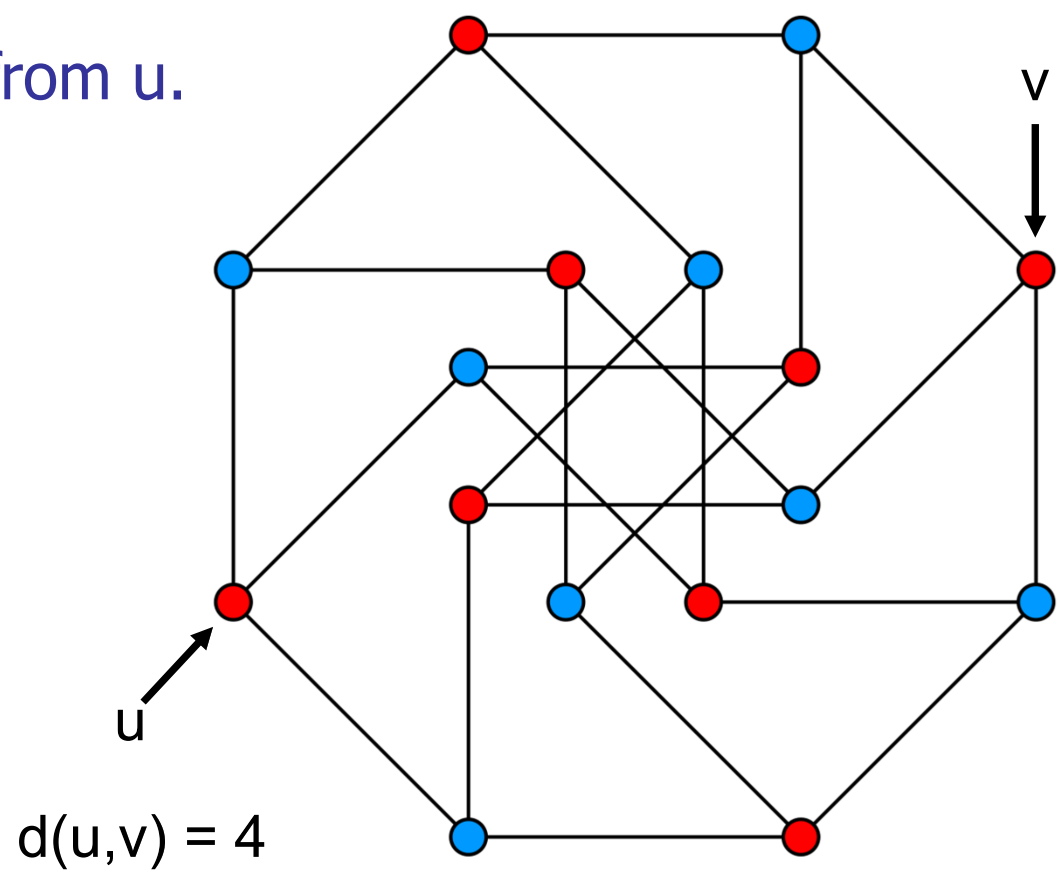
Simple approximation algorithm:



Example: Diameter

Simple approximation algorithm:

1. Choose a node u.
2. Run BFS from u.
3. Let v be farthest node from u.
4. Return d(u,v).



Example: Diameter

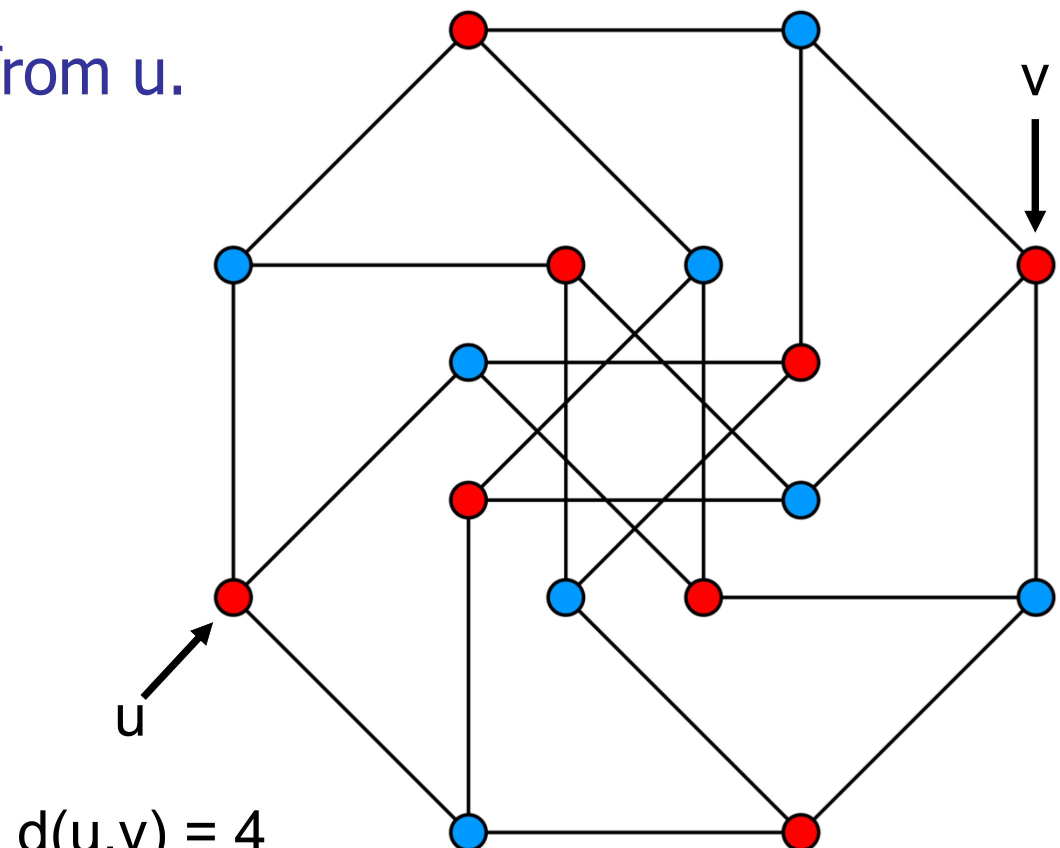
Simple approximation algorithm:

1. Choose a node u.
2. Run BFS from u.
3. Let v be farthest node from u.
4. Return d(u,v).

Proof:

For all pairs (x,y):

$$
\begin{aligned}
d(x,y) &\leq d(x,u) + d(u,y) \\
&\leq d(u,v) + d(u,v) \\
&\leq 2 \cdot d(u,v)
\end{aligned}
$$



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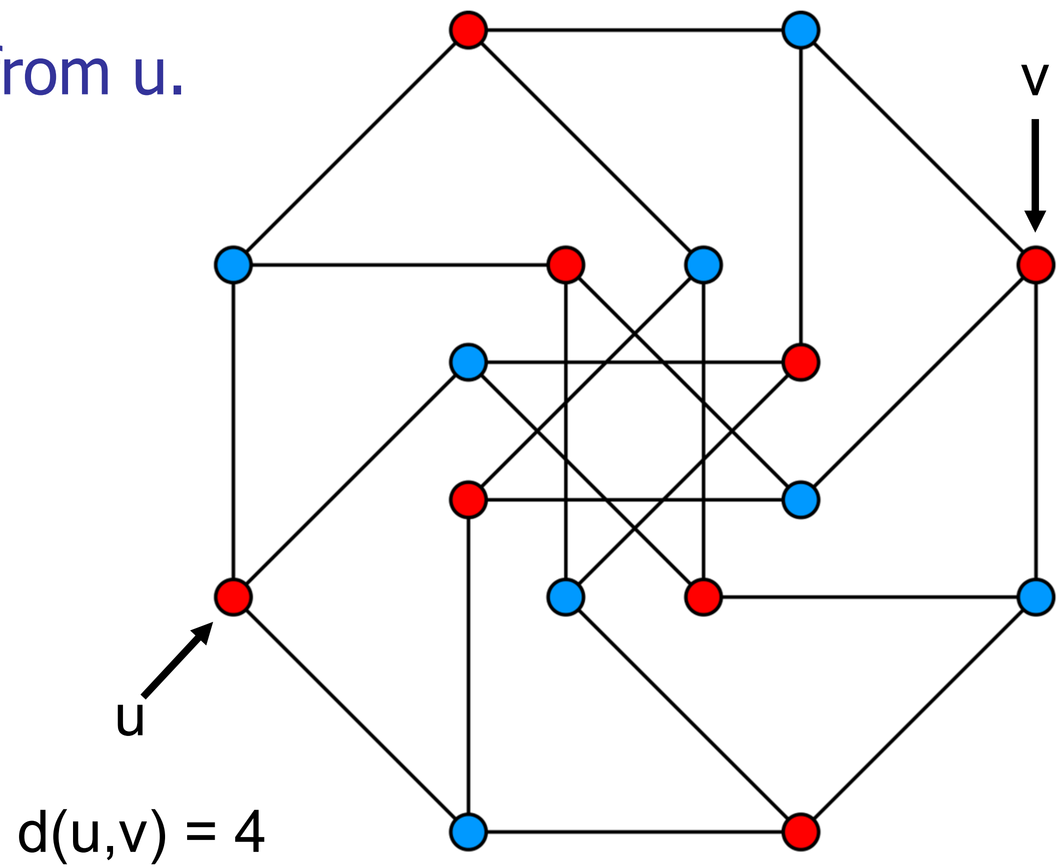
Time:

O(m + n)



Example: Diameter

Simple approximation algorithm:

1. Choose a node u.
2. Run BFS from u.
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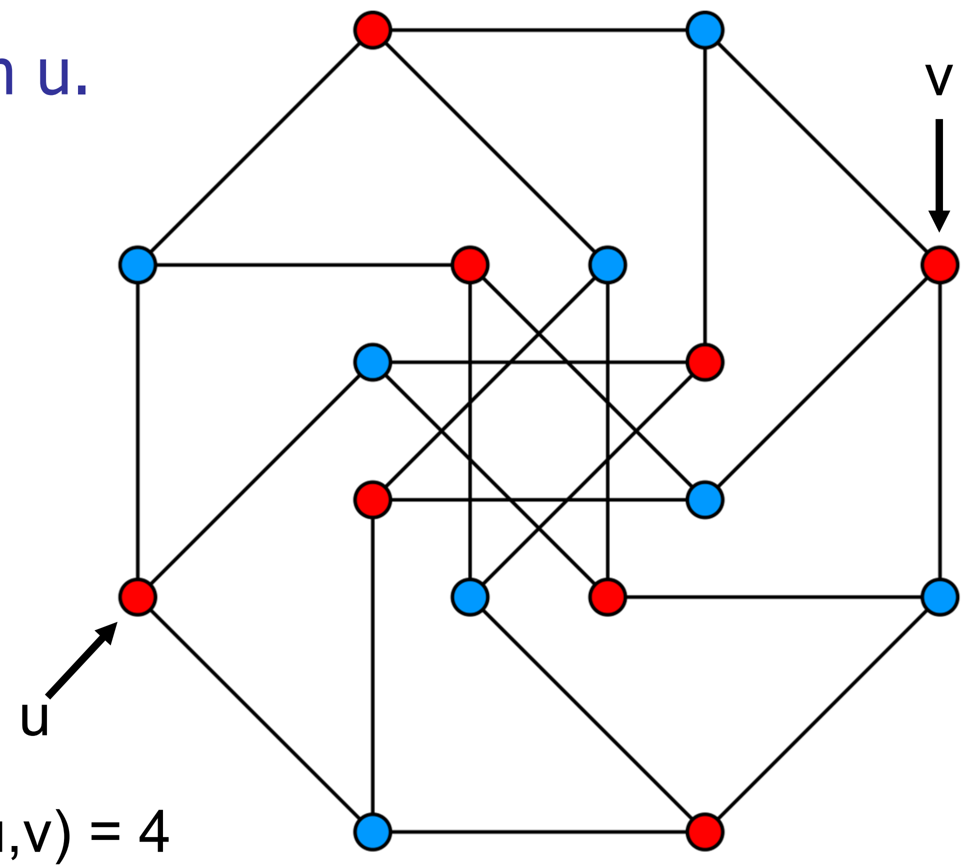
Time:

O(m + n)

Cost of simple BFS?
≅ 1 week
Oops…
Better, but still slow.



d(u,v) = 4

Example: Diameter

Step 2: Scan time is faster than seek time!

- Disk scan: 200 MB/s ➔ 83 minutes
- Disk seek: 1 MB/s ➔ 11.5 days

Goal:

Do BFS by scanning entire graph in any order.

Find log(n)-approximation.

Time: O(n + m) *scan* time.

Speedup of 100x for log-approximate answer.

Example: Diameter

Step 3: Sampling + Approximation

Goal:

If the graph has diameter $\leq$ D return true.

If the graph is FAR from a graph with diameter $\leq 4D + 2$ return true.

Time: $O(1/\varepsilon^3)$

error term / depends on how FAR is FAR

Example: Diameter

Step 4: Optimization + 2-Approximation

Goal:

Use faster BFS to get a 2-approximation.

Assume a cache of size C with block-size B.

Time: $O(n + (m/B) \cdot \log_{C/B}(m/B))$

Speedup of 100x to 1000x when data is on disk.

Example: Diameter

Step 4: Parallelize + 2-Approximation

Goal:

Use parallel-BFS to find a 2-approximation.

Time: $O((m/p)\cdot\log^2 n + D\cdot\log^3 n)$ on p cores.

or

Time: n iterations on Map-Reduce cluster.

Another Ex: Minimum Spanning Tree

Algorithms 101

- Kruskal's Algorithm
- Prim's Algorithm

- Runs in O(m log n) time for *n* nodes and *m* edges.

- Fast enough?

Example: Minimum Spanning Tree

Special Structure

- Is graph planar?

➔ Then we can find an MST in O(m) time.

- Is the graph a social network?

➔ Then graph has special structure too!

Example: Minimum Spanning Tree

Randomization and Approximation

- Can we find a *faster* randomized algorithm?
- Approximate MSG?
- Estimate weight of MST?

Amazingly: $O(dW \log(dW))$

for a graph with degree d and max. edge weight W

No dependence on n!!

Example: Minimum Spanning Tree

Streaming

- What if we only have limited access to data?
- We get to read each edge once in some arbitrary order: $e_1, e_2, e_3, \ldots, e_m$

- We can't store the whole graph!
- Output an (approximate) MST?

Example: Minimum Spanning Tree

Dynamic

- What if edges change over time?
- Edges are continually added and removed from our graph.
- After each change, find a new MST.

Example: Minimum Spanning Tree

Caching

- Caching performance is critical.
- Each time we access part of the graph, a block of memory is loaded.
  - Expensive!
- How can we design an algorithm for finding an MST that uses cache efficiently?

Example: Minimum Spanning Tree

Parallel/GPU/Distributed

- Can we leverage a multicore machine to find an MST faster?
- Can we use GPUs to get faster performance?
- Can we use a distributed cluster (e.g., MapReduce/Hadoop) to find an MST faster?

New Challenges

Scale

- How do we deal with graphs that are big?
- Cannot store entire graph in memory.
- Processing time is large!

New Challenges

Where is the data?

- Data is no longer as easily accessible.
- Is data distributed?
- Is data streaming?
- Is data noisy?

New Challenges

Dynamic world

- Data is no longer static.
- Graphs change over time.
- Edges may be added and removed.
- Users may come and go.

New Challenges

Context matters

- Where did the data come from?
- Is it from a social network?
- Is it from a wireless network?
- Is it from a game?
- How can we leverage the structure to do better?

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… when classical algorithms are too slow by…

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This is a class about algorithms.

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binary search

set cover

big-O notation

recurrences

approximation algorithms

bipartite graphs

Prim's

MST

Dijkstra's

dynamic programming

planar graphs

expected value

P=NP

This is a class about algorithms.

The goal is to deeply understand the algorithms we are studying.

How do they work?

Why do they work?

How do you implement them?

What are the underlying techniques?

What are the trade-offs?

Previous Student feedback:

“What a great class, but so many algorithms. I wish I had known that the class was going to include so many proofs and so much math.”
-- Gil Gunderson

“I loved the class, it was amazing, the algorithms were almost like magic! But unfortunately I had no idea what you were talking about most of the time, since I had never taken an algorithms class before.”
-- Ernie Macmillan

** The above quotes are entirely fabricated and only loosely reflect real student feedback.

For example, if I say:

"To find the shortest path, first use *Prim's algorithm* (with a *Fibonacci Heap*) to find an *MST* in O(m + n log n) time."

You should have two immediate reactions:

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You should have two immediate reactions:

1. Nod your head since you understand exactly what I am saying.

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You should have two immediate reactions:

1. Nod your head since you understand exactly what I am saying.

2. Tell me that this claim seems like NONSENSE (because MSTs are almost never useful for finding a shortest path).

CS5234 : Algorithms at Scale

Target students:

- Advanced (3rd or 4th year) undergraduates
- Master's students
- PhD students
- Interested in algorithms
- Interested in tools for solving hard problems

Prerequisites:

- CS3230 (Analysis of Algorithms)
- Mathematical fundamentals

"If you need your software to run twice as fast, hire better programmers.

But if you need your software to run more than twice as fast, use a better **algorithm.**"

-- *Software Lead at Microsoft*

“... pleasure has probably been the main goal all along.

”

-- *D. E. Knuth*

CS5234 Overview

- Mid-term exam

  October 16   In class, Week 9

- Final exam

  November 27

  (Please double-check the official schedule in case it changes or in case this is wrong!)

CS5234 Overview

- Lecture

  Thursday 6:30-8:30pm

- Extra time

  Thursday 8:30-9:30pm

  Extra time will be used for discussion, reviewing problem sets, answering questions, solving riddles, doing crossword puzzles, eating cookies, etc.

CS5234 Overview

- Assessments

  Problem sets + Mini-Project

  Mid-term exam

  Final exam

- Problem sets
  - 5-6 sets (roughly every week)
  - Focused on algorithm design and analysis.
  - Perhaps a few will require coding.

CS5234 Overview

- Mini-Project

  Small project

  Idea: put together some of the different ideas we have used in the class.

  Time scale: last 4 weeks of the semester.

CS5234 Overview

Survey: Google form.

On the web page.

What is your background?

Not more than 10 minutes.

PS1: Released tomorrow.

CS5234 Overview

- Problem set grading

  Simple scheme:

  3 : excellent, perfect answer

  2 : satisfactory, mostly right

  1 : many mistakes / poorly written

  0 : mostly wrong / not handed in

  -1 : utter nonsense

CS5234 Overview

- What to submit:

  Concise and precise answers:

  Solutions should be rigorous, containing all necessary detail, but no more.

  Algorithm descriptions consist of:

  1. Summary of results/claims.
  2. Description of algorithm in English.
  3. Pseudocode, if helpful.
  4. Worked example of algorithm.
  5. Diagram / picture.
  6. Proof of correctness and performance analysis.

CS5234 Overview

- How to draw pictures?

  By hand:

  Either submit hardcopy, or scan, or take a picture with your phone!

  Or use a tablet / iPad…

  Digitally:

  1. xfig (ugh)
  2. OmniGraffle (mac)
  3. Powerpoint (hmmm)
  4. ???

CS5234 Overview

- Policy on plagiarism:

  Do your work yourself:

  Your submission should be *unique*, unlike anything else submitted, on the web, etc.

  Discuss with other students:

  1. Discuss general approach and techniques.
  2. Do not take notes.
  3. Spend 30 minutes on facebook (or equiv.).
  4. Write up solution on your own.
  5. List all collaborators.

  Do not search for solutions on the web:

  Use web to learn techniques and to review material from class.

CS5234 Overview

- Policy on plagiarism:

  Penalized severely:

  First offense: minimum of one letter grade lost on final grade for class (or referral to SoC disciplinary committee).

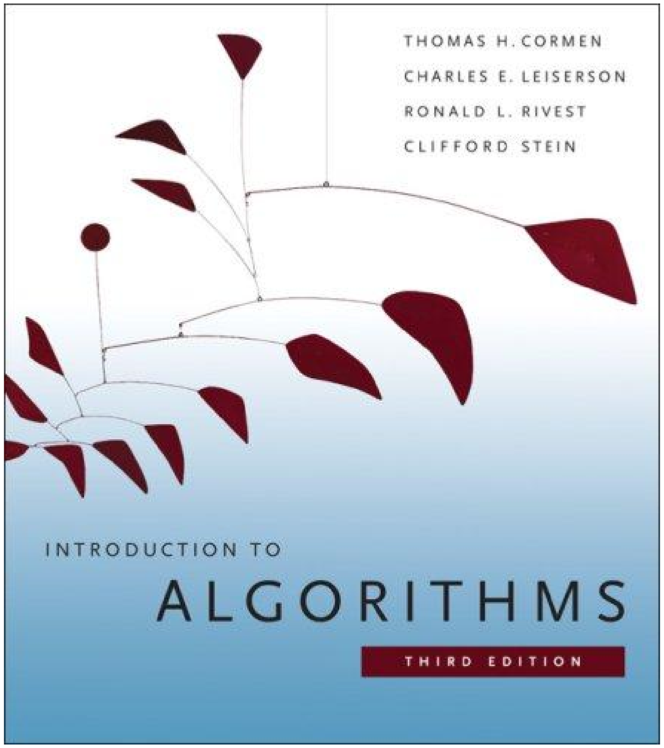
  Second offense: F for the class and/or referral to SoC.

  Do not copy/compare solutions!

Algorithms Review

Introduction to Algorithms

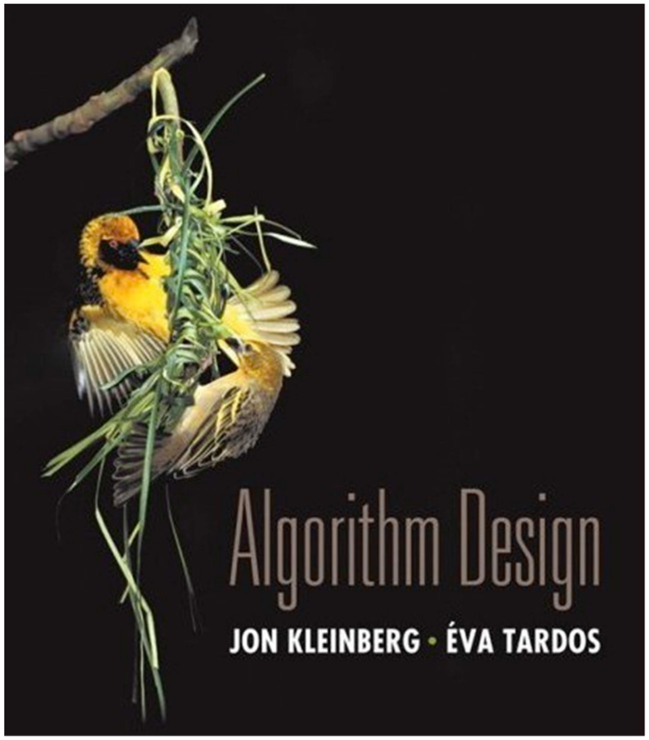
- Cormen, Leiserson, Rivest, Stein



Algorithms Review

Algorithm Design

– Kleinberg and Tardos



Topics (tentative)

- Sampling and Sketching Very Big Graphs

- Efficient Algorithms for Modern Machines

A modern twist on classic problems…

BFS, DFS, MST, Shortest Path, etc.

Topics (tentative)

- Sampling and Sketching Very Big Graphs

  Part 1: Graph properties in less than linear time

  - Connectivity
  - Connected components
  - Minimum spanning tree
  - Average degree
  - Approximate diameter
  - Matching

Topics (tentative)

- Sampling and Sketching Very Big Graphs

  Part 2: Sketches and streams

  - Sampling from a stream
  - L0-samplers
  - Graph sketches
  - Connectivity
  - Minimum spanning trees
  - Triangle counting

Topics (tentative)

- Efficient Algorithms for Modern Machines

  Part 3: Caching

  - Cache-efficient algorithms
  - BFS
  - Priority queues
  - Shortest path
  - Minimum spanning trees

Topics (tentative)

- Efficient Algorithms for Modern Machines

  Part 4: Parallel Algorithms

  - Fork-join parallelism
  - Map-Reduce
  - BFS / DFS
  - Shortest path

CS5234 Overview

- **Webpage:**

  http://www.comp.nus.edu.sg/~gilbert/CS5234

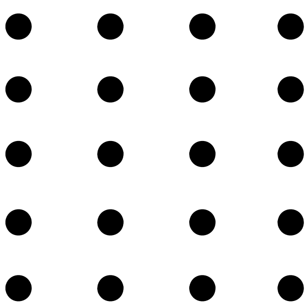
- **Instructor: Seth Gilbert**

  Office: COM2-323

  Office hours: by appointment

CS5234
Algorithms at Scale
(Fast Algorithms for Slow Problems)

Welcome!



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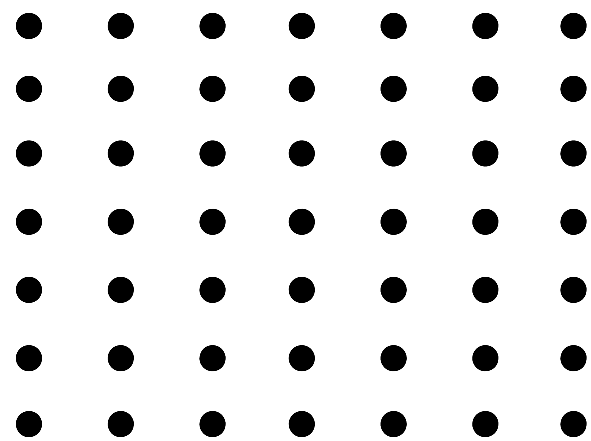

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