CS3234 Logic and Formal Systems

Assignment 03:
Propositional Logic II; Predicate Logic I

Submission on A-4 paper (use as many sheets as you want), to the office COM2, 03-51 (a box is provided). Staple or tie your sheets together and write your name and matriculation number on the top of the front page.
Latest submission: Thursday, 9/9/08, 11:00am.

1. (4 marks) Compute a formula in CNF that is equivalent to the following formula
$$(\neg p) \rightarrow \neg(q \wedge \neg(r \vee \neg(s \wedge u)))$$
by applying the algorithm given in the notes. Indicate the result of each step of the algorithm.

2. (8 marks) Prove Lemma 1 in the notes "Propositional Logic Part II". Remember that "iff" indicates two directions.

3. (7 marks) Let P be a predicate symbol of arity 0, Q be a predicate symbol of arity 2, f be a function symbol of arity 2, and g a function symbol of arity 1. Consider the formula
$$\phi = \neg(\exists w((P \vee (\forall v Q(f(x, v), w))) \wedge (\forall z Q(f(w, x), f(v, z)))))$$

   (a) Draw the parse tree of ϕ.
   (b) Indicate the free and bound variables in this parse tree.
   (c) Compute $[x \Rightarrow g(f(x, x))]\phi$ and $[w \Rightarrow f(z, v)]\phi$.

4. (6 marks) Let ϕ be the sentence
$$\forall x \forall y \exists z (R(x, y) \rightarrow R(y, z))$$
where R is a predicate symbol of arity 2.

   (a) Let $A = \{a, b, c, d\}$ and $R^{\mathcal{M}} = \{(b, c), (b, b), (b, a)\}$. Does $\mathcal{M} \models \phi$ hold? Justify your answer.
   (b) Let $A' = \{a, b, c\}$ and $R^{\mathcal{M}'} = \{(b, c), (a, b), (c, b)\}$. Does $\mathcal{M}' \models \phi$ hold? Justify your answer.

5. (5 marks) Prove that the following entailment does *not* hold:
$$\exists x(\neg P(x) \wedge Q(x)) \models \forall x(P(x) \rightarrow Q(x))$$