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Proofs of Propositions and Lemmas, Comparative Statics, and Extensions

Proof of Proposition 1. Consider a high-type consumer's effort in concealment. We analyze two cases:

Case (i): $U_h(k_j, e_j)$ is maximized at $k_j = 0$. Then, the high-type consumer's effort in concealment would not be affected by the sellers' solicitations.

Case (ii): $U_h(k_j, e_j)$ is maximized at some $k_j^* > 0$. Then, by (10), $\partial U_h / \partial k_j = 0$, which by (4) and (5), simplifies to

$$[S_1 + \ldots + S_N]\frac{d\varphi}{dk_j}\left\{[1-\varphi(k_j)\rho(e_j)]^{S_1+\ldots+S_N-1}V_h - w\right\} = \frac{1}{\rho(e_j)} \cdot \frac{d}{dk_j}C_K(k_j) > 0, \quad \text{(A1)}$$

By (1), (2), and (3), for all *j*,

$$\frac{d}{dk_j}\varphi(k_j) = \frac{d}{dk_j}\left(\frac{\alpha(k_j)}{\Lambda}\right) = \frac{1}{\Lambda}\left[1-\frac{\alpha(k_j)}{\Lambda}\right]\frac{d\alpha}{dk_j} = \frac{1}{\Lambda}[1-\varphi(k_j)]\frac{d\alpha}{dk_j} < 0. \quad \text{(A2)}$$

Hence, (A1) and (A2) together imply that at $k_j = k_j^*$, for any e_j,

$$[1-\varphi(k_j)\rho(e_j)]^{S_1+\ldots+S_N-1}V_h - w < 0. \quad \text{(A3)}$$

Now, differentiate (10) with respect to seller *m*'s solicitations, S_m,

$$\frac{\partial^2 U_h}{\partial S_m \partial k_j} = \rho(e_j)\frac{d\varphi}{dk_j}\left\{\left\{[1-\varphi(k_j)\rho(e_j)]^{S_1+\ldots+S_N-1}V_h - w\right\}\right. \\ \left. + [S_1 + \ldots + S_N]\ln(1-\varphi(k_j)\rho(e_j))[1-\varphi(k_j)\rho(e_j)]^{S_1+\ldots+S_N-1}V_h\right\}. \quad \text{(A4)}$$

By (A3), the first set of terms in braces on the right-hand side of (A4) is negative. The second set of terms in braces on the right-hand side of (A4) is negative, as the logarithmic term is negative. Hence, by (A2), the cross-partial, $\partial^2 U_h / \partial S_m \partial k_j > 0$, which means that the high-type consumer's concealment effort is a strategic complement with sellers' solicitations.

Similarly, we analyze two cases for the high-type consumer's effort in deflection:

Case (i): $U_h(k_j, e_j)$ is maximized at $e_j = 0$. Then, the high-type consumer's effort in deflection would not be affected by the sellers' solicitations.

Case (ii): $U_h(k_j, e_j)$ is maximized at some $e_j^* > 0$. Then, by (11), $\partial U_h / \partial e_j = 0$, which by (4) and (5), simplifies to

$$[S_1 + \ldots + S_N]\varphi(k_j)\left\{[1-\varphi(k_j)\rho(e_j)]^{S_1+\ldots+S_N-1}V_h - w\right\} = \left[\frac{d\rho}{de_j}\right]^{-1}\frac{d}{de_j}C_E(e_j) < 0, \quad \text{(A5)}$$

which implies that for any k_j, (A3) holds at $e_j = e_j^*$.

Now, differentiate (11) with respect to seller *m*'s solicitations, S_m,

$$\frac{\partial^2 U_h}{\partial S_m \partial e_j} = \varphi(k_j)\frac{d\rho}{de_j}\left\{\left\{[1-\varphi(k_j)\rho(e_j)]^{S_1+\ldots+S_N-1}V_h - w\right\}\right.$$
$$\left. + [S_1 + \ldots + S_N]\ln(1-\varphi(k_j)\rho(e_j))[1-\varphi(k_j)\rho(e_j)]^{S_1+\ldots+S_N-1}V_h\right\}. \quad \text{(A6)}$$

By similar reasoning as following (A4), $\partial^2 U_h / \partial S_m \partial e_j > 0$. This proves that the high-type consumer's deflection effort is a strategic complement with sellers' solicitations.

Finally, for low-type consumers, by differentiating (13) and (14) with respect to S_m,

$$\frac{\partial^2 U_l}{\partial S_m \partial k_i} = -\rho(e_i)\frac{d\varphi}{dk_i}w > 0, \quad \text{(A7)}$$

$$\frac{\partial^2 U_l}{\partial S_m \partial e_i} = -\varphi(k_i)\frac{d\rho}{de_i}w > 0, \quad \text{(A8)}$$

by (1), (5) and (A2). These prove the result for low-type consumers. □

LEMMA 1. *For any integer, M, and* $\gamma \in [0, 1]$,

$$\sum_{z=0}^{M-1}\frac{1}{z+1}\binom{M-1}{z}\gamma^z[1-\gamma]^{M-1-z} = \frac{1}{M\gamma}\left\{1-[1-\gamma]^M\right\}.$$

Proof. Note that

$$\frac{1}{z+1}\binom{M-1}{z}=\frac{1}{z+1}\frac{(M-1)!}{z!(M-1-z)!}=\frac{(M-1)!}{(z+1)!(M-1-z)!}=\frac{1}{M}\frac{M!}{(z+1)!(M-1-z)!}=\frac{1}{M}\binom{M}{z+1},$$

and hence,

$$\begin{aligned}&\sum_{z=0}^{M-1}\frac{1}{z+1}\binom{M-1}{z}[1-\gamma]^{M-1-z}\gamma^{z}\\ &\quad=\frac{1}{M}\sum_{z=0}^{M-1}\binom{M}{z+1}[1-\gamma]^{M-1-z}\gamma^{z}=\frac{1}{M\gamma}\sum_{z=0}^{M-1}\binom{M}{z+1}[1-\gamma]^{M-1-z}\gamma^{z+1}.\end{aligned}\tag{A9}$$

Now,

$$\begin{aligned}&\sum_{z=0}^{M-1}\binom{M}{z+1}[1-\gamma]^{M-1-z}\gamma^{z+1}\\ &\quad=\sum_{z=1}^{M}\binom{M}{z}[1-\gamma]^{M-z}\gamma^{z}=\sum_{z=0}^{M}\binom{M}{z}[1-\gamma]^{M-z}\gamma^{z}-[1-\gamma]^{M}\\ &\quad=1-[1-\gamma]^{M}\ ,\end{aligned}\tag{A10}$$

where the first step changes the index of summation, the second step uses

$$\binom{M}{0}=1,$$

and the third step applies the binomial theorem. Substituting from (A10) in (A9) yields the result. □

Proof of Proposition 2. Let h denote one high-type consumer and $j \neq h$ index the other $[H-1]$ high-type consumers. Then (18) can be re-written as

$$\begin{aligned}\frac{\partial\Pi}{\partial S_m}=&\left\{-\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m}\ln(1-\varphi(k_h)\rho(e_h))\frac{S_m}{S_{\sim m}+S_m}\right.\\ &\left.+\left\{1-\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m}\right\}\frac{S_{\sim m}}{[S_{\sim m}+S_m]^2}\right\}pq_h\\ &+[H-1]\left\{-[1-\varphi(k_j)\rho(e_j)]^{S_{\sim m}+S_m}\ln(1-\varphi(k_j)\rho(e_j))\frac{S_m}{S_{\sim m}+S_m}\right.\\ &\left.+\left\{1-[1-\varphi(k_j)\rho(e_j)]^{S_{\sim m}+S_m}\right\}\frac{S_{\sim m}}{[S_{\sim m}+S_m]^2}\right\}pq_h-\frac{\partial}{\partial S_m}C(S_m,\Lambda)=0.\end{aligned}\tag{A11}$$

Differentiating (A11) with respect to one high-type consumer *h*'s effort in deflection, e_h, and simplifying,

$$\frac{\partial^2 \Pi}{\partial e_h \partial S_m} = [1-\varphi(k_h)\rho(e_h)]^{S_{\sim m}+S_m-1}\left[S_m \ln(1-\varphi(k_h)\rho(e_h))+1\right]\varphi(k_h)\frac{d\rho}{de_h}pq_h. \quad \text{(A12)}$$

By (6), the cost of solicitation is convex in S_m, and hence, in equilibrium, seller *m* would send out a finite number of solicitations. By (1), when the number of consumers in the population is reasonably large, $\varphi(k_h)$ would be very small, which implies that

$$S_m \ln(1-\varphi(k_h)\rho(e_h))+1>0. \quad \text{(A13)}$$

By (5), $d\rho / de_h < 0$, and hence together with (A13), the right-hand side of (A12) is negative, that is, $\partial^2\Pi / \partial e_h \partial S_m < 0$. Further, by (18), it is obvious that $\partial^2\Pi / \partial e_l \partial S_m = 0$. This proves the proposition with respect to deflection.

Similarly, differentiating (A11) with respect to one high-type consumer *h*'s effort in concealment, k_h, and simplifying,

$$\begin{aligned}\frac{\partial^2 \Pi}{\partial k_h \partial S_m} &= [1-\varphi(k_h)\rho(e_h)]^{S_{\sim m}+S_m-1}\left[S_m \ln(1-\varphi(k_h)\rho(e_h))+1\right]pq_h\rho(e_h)\frac{d}{dk_h}\varphi(k_h)\\ &\quad +[H-1][1-\varphi(k_j)\rho(e_j)]^{S_{\sim m}+S_m-1}\left[S_m \ln(1-\varphi(k_j)\rho(e_j))+1\right]pq_h\rho(e_j)\frac{d}{dk_h}\varphi(k_j)\\ &\quad -\frac{\partial^2}{\partial\Lambda\partial S_m}C(S_m,\Lambda)\frac{d\alpha}{dk_h}, \qquad \text{(A14)}\end{aligned}$$

where, by (3), $d\Lambda / dk_h = d\alpha / dk_h$. By (1), for $j \neq h$,

$$\frac{d}{dk_h}\varphi(k_j) = -\frac{\alpha(k_j)}{\Lambda^2}\frac{d}{dk_h}\alpha(k_h) = -\frac{1}{\Lambda}\varphi(k_j)\frac{d}{dk_h}\alpha(k_h) > 0. \quad \text{(A15)}$$

Substituting from (A2) for $j = h$ and (A15) for $j \neq h$ in (A14), in symmetric equilibrium,

$$\frac{\partial^2 \Pi}{\partial k_h \partial S_m} = \lambda [1 - \varphi(k_h)\rho(e_h)]^{S_{\sim m}+S_m-1} \left[S_m \ln(1-\varphi(k_h)\rho(e_h)) + 1 \right] \rho(e_h) p q_h - \frac{\partial^2}{\partial \Lambda \partial S_m} C(S_m, \Lambda) \frac{d\alpha}{dk_h}, \tag{A16}$$

where

$$\lambda = \frac{1}{\Lambda}[1-\varphi(k_h)]\frac{d\alpha}{dk_h} - [H-1]\frac{1}{\Lambda}\varphi(k_h)\frac{d\alpha}{dk_h} = \frac{1}{\Lambda}[1 - H\varphi(k_h)]\frac{d\alpha}{dk_h} < 0 \tag{A17}$$

by (1) and (2). By (A13) and (A17), the first term on the right-hand side of (A16) is negative. By (2) and (6), the second term on the right-hand side of (A16) is also negative, and hence $\partial^2 \Pi / \partial k_h \partial S_m < 0$, which proves that seller solicitation is a strategic substitute with high-type consumers' effort in concealment.

Finally, differentiating (A11) with respect to a low-type consumer *l*'s effort in concealment, k_l, substituting from (A15), and simplifying,

$$\frac{\partial^2 \Pi}{\partial k_l \partial S_m} = -H [1 - \varphi(k_h)\rho(e_h)]^{S_{\sim m}+S_m-1} [S_m \ln(1-\varphi(k_h)\rho(e_h)) + 1] p q_h \rho(e_h) \frac{\varphi(k_h)}{\Lambda} \cdot \frac{d\alpha}{dk_l} - \frac{\partial^2}{\partial \Lambda \partial S_m} C(S_m, \Lambda) \frac{d\alpha}{dk_l}. \tag{A18}$$

By (2) and (A13), the first term on the right-hand side of (A18) is positive. By (2) and (6), the second term on the right-hand side of (A18) is negative. Accordingly, if the second term on the right-hand side of (A18) outweighs the first term, then $\partial^2 \Pi / \partial k_l \partial S_m \le 0$, and seller solicitation is a strategic substitute with low-type consumers' effort in concealment.

However, if the marginal cost of solicitation does not increase too fast with consumer effort in concealment, that is, $\partial^2 C(S_m, \Lambda) / \partial \Lambda \partial S_m$ is sufficiently small, then the first term on the right hand side of (A18) outweighs the second term, and hence, $\partial^2 \Pi / \partial k_l \partial S_m > 0$, which proves that seller solicitation is a strategic complement with low-type consumers' effort in concealment. □

LEMMA 2. *There exists a non-trivial equilibrium.*

Proof. In a symmetric equilibrium, $k_j = k_h$ for high-type consumers, $k_i = k_l$ for low-type consumers, and $S_m = S$. For ease of presentation and without loss of generality, we sketch the following proof with the individual reaction functions k_h, k_l, and *S*.

By (13), the low-type consumer concealment function $k_l(S \mid e_l)$ is continuous and, by Proposition 1, increasing in *S*. Further, if all $S = 0$, then $k_l = 0$, and if any $S \to \infty$, then $k_l \to \infty$. Hence, referring to Figure A, the $k_l(S \mid e_l)$ curve starts from the origin and has positive slopes at all *S*.

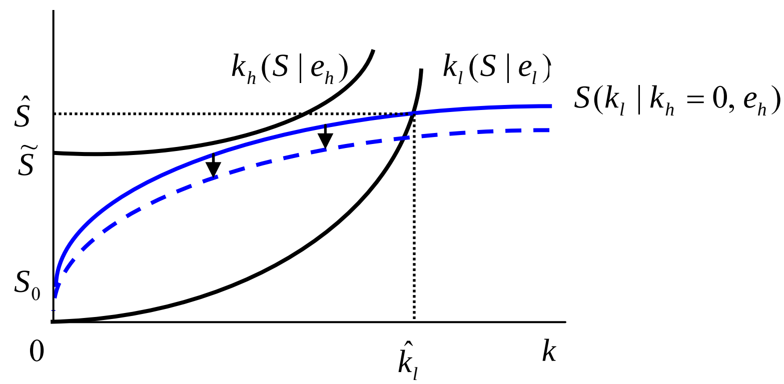

Figure A. Adjustment to equilibrium

Next, by (18), the seller solicitation function is continuous and, by Proposition 2, increasing in k_l. By (1), (2), (3), (5), and (6), if all $k_j = 0$, $k_i = 0$, $e_j = 0$, and $S_m = 0$, then $\alpha(k_j) = 1$, $\varphi(k_j) = 1/[H + L]$, $\rho(e_j) = 1$, and $\partial C(S_m, \Lambda)/\partial S_m = 0$, and hence, by (18),

$$\frac{\partial \Pi}{\partial S_m} = \left\{ 1 - \left[1 - \frac{1}{H + L} \right]^{S_{\sim m}} \frac{1}{S_{\sim m}} \right\} H p q_h > 0. \tag{A19}$$

Accordingly, the function $S(k_l \mid k_h, e_h)$, with $k_h = 0$ and $e_h = 0$, intersects the *S*-axis at some $S_0 > 0$. By (2), if $k_l \to \infty$, then $d\alpha / dk_l \to 0$, and so, by (A18), $\partial^2 \Pi / \partial k_l \partial S_m \to 0$, which means that $\Delta S / \Delta k_l \to 0$. That is, sellers' solicitations converge to an asymptote as k_l increases.

Therefore, when $k_h = 0$ and $e_h = 0$, the seller solicitation and low-type consumer concealment functions would intersect at some $(\hat{S}, \hat{k}_l)$, where $\hat{S} > 0$ and $\hat{k}_l > 0$. Recall from (10) and (13) that the high-type consumers' concealment function lies to the left of the low-type consumers', that is, $k_h(S \mid e_h) < k_l(S \mid e_l)$ for all *S*. By (9), when *S* is sufficiently large, $U_h(k_j, e_j) < 0$, and high-type consumers would choose positive efforts in concealment and deflection. Accordingly, there exists some $\tilde{S} \geq 0$ such that for all $S > \tilde{S}$, $k_h(S \mid e_h) > 0$ and $e_h(S \mid k_h) > 0$.

If $k_h(\hat{S} \mid e_h) = 0$ and $e_h(\hat{S} \mid k_h) = 0$, then $(\hat{S}, k_h, e_h, \hat{k}_l, \hat{e}_l)$, with $k_h = e_h = 0$ and where $\hat{e}_l$ solves (14), constitutes the consumer-seller equilibrium.

However, if $\hat{S} > \tilde{S}$, then $k_h(\hat{S} \mid e_h) > 0$ and $e_h(\hat{S} \mid k_h) > 0$, which is inconsistent with the original supposition. Let $k_h = \Delta k$ and $e_h = \Delta e$, and re-compute the sellers' solicitation, $S(k_l \mid k_h, e_h)$. By Proposition 2, the increase in k_h and e_h would shift the seller solicitation function downwards, as illustrated by the broken curve in Figure A. Hence, its intersection with the low-type consumer concealment function will now shift to some $(\hat{S} - \Delta S, \hat{k}_l - \Delta k_l)$.

Regarding the high-type consumers' efforts in concealment and deflection, if

$$k_h(\hat{S} - \Delta S \mid e_h) = \Delta k \text{ and } e_h(\hat{S} - \Delta S \mid k_h) = \Delta e, \qquad \text{(A20)}$$

then $(\hat{S} - \Delta S, \Delta k, \Delta e, \hat{k}_l - \Delta k_l, \hat{e}_l - \Delta e_l)$, where $\hat{e}_l - \Delta e_l$ solves (14), constitutes the consumer-seller equilibrium. If, however, $k_h(\hat{S} - \Delta S \mid e_h) > \Delta k$ or $e_h(\hat{S} - \Delta S \mid k_h) > \Delta e$, then k_h and e_h should be raised by small increments and the above procedure repeated until the intersection of the seller solicitation and consumer concealment functions satisfy the equivalent of (A20). The Figure in the text illustrates the equilibrium. □

Proof of Proposition 3. Let h denote one high-type consumer and $j \neq h$ index the other $[H - 1]$ high-type consumers, and l denote one low-type consumer and $i \neq l$ index the other $[L - 1]$ low-type consumers. From (19), the direct privacy harm caused by solicitations is

$$\begin{aligned}\Gamma = &[S_1 + \ldots + S_N]\varphi(k_h)\rho(e_h)w + [S_1 + \ldots + S_N][H-1]\varphi(k_j)\rho(e_j)w \\ &+ [S_1 + \ldots + S_N]\varphi(k_l)\rho(e_l)w + [S_1 + \ldots + S_N][L-1]\varphi(k_i)\rho(e_i)w.\end{aligned} \tag{A21}$$

Differentiating (A21) with respect to a high-type consumer h's effort in concealment, k_h,

$$\begin{aligned}\frac{\partial \Gamma}{\partial k_h} = [S_1 + \ldots + S_N]w\Bigg\{&\frac{d}{dk_h}\varphi(k_h)\rho(e_h) + [H-1]\frac{d}{dk_h}\varphi(k_j)\rho(e_j) \\ &+ \frac{d}{dk_h}\varphi(k_l)\rho(e_l) + [L-1]\frac{d}{dk_h}\varphi(k_i)\rho(e_i)\Bigg\}.\end{aligned} \tag{A22}$$

Using (A2) and (A15), and that, in a symmetric equilibrium, $e_j = e_h$ for all high-type consumers and $e_i = e_l$ for all low-type consumers, (A22) simplifies to

$$\begin{aligned}\frac{\partial \Gamma}{\partial k_h} &= [S_1 + \ldots + S_N]w[\rho(e_h) - H\rho(e_h)\varphi(k_h) - L\rho(e_l)\varphi(k_l)]\frac{1}{\Lambda}\frac{d\alpha}{dk_h} \\ &< [S_1 + \ldots + S_N]w[\rho(e_h) - H\rho(e_h)\varphi(k_h) - L\rho(e_h)\varphi(k_l)]\frac{1}{\Lambda}\frac{d\alpha}{dk_h} = 0,\end{aligned} \tag{A23}$$

since, by (11) and (14), high-type consumers choose less effort in deflection than low-type consumers, $e_h < e_l$, and so, $\rho(e_h) > \rho(e_l)$, while, by (1) and (3), $H\varphi(k_h) + L\varphi(k_l) = 1$, and, finally, by (2), $d\alpha / dk_h < 0$.

By similar reasoning, differentiating (A21) with respect to a low-type consumer l's effort in concealment, k_l,

$$\begin{aligned}\frac{\partial \Gamma}{\partial k_l} &= [S_1 + \ldots + S_N]w[\rho(e_l) - L\rho(e_l)\varphi(k_l) - H\rho(e_h)\varphi(k_h)]\frac{1}{\Lambda}\frac{d\alpha}{dk_l} \\ &> [S_1 + \ldots + S_N]w[\rho(e_l) - L\rho(e_l)\varphi(k_l) - H\rho(e_l)\varphi(k_h)]\frac{1}{\Lambda}\frac{d\alpha}{dk_l} = 0.\end{aligned} \tag{A24}$$

Finally, differentiating (A21) with respect to a high-type consumer *h*'s effort in deflection, e_h, and a low-type consumer *l*'s effort in deflection, e_l,

$$\frac{\partial \Gamma}{\partial e_h} = [S_1 + \ldots + S_N] w\varphi(k_h)\frac{d\rho}{de_h} < 0, \tag{A25}$$

$$\frac{\partial \Gamma}{\partial e_l} = [S_1 + \ldots + S_N] w\varphi(k_l)\frac{d\rho}{de_l} < 0. \quad \square \tag{A26}$$

Proof of Proposition 4. First, we prove the properties of the optimal charge. Re-writing (19) to distinguish seller *m* from the other sellers, social welfare is

$$\begin{aligned} W = {} & H\left\{1-[1-\varphi(k_h)\rho(e_h)]^{S_{\sim m}+S_m}\right\}[V_h + pq_h]\frac{S_m}{S_{\sim m}+S_m} \\ & + H\left\{1-[1-\varphi(k_h)\rho(e_h)]^{S_{\sim m}+S_m}\right\}[V_h + pq_h]\frac{S_{\sim m}}{S_{\sim m}+S_m} - \sum_{m=1}^{N} C(S_m, \Lambda) \\ & -[S_{\sim m}+S_m]H\varphi(k_h)\rho(e_h)w - HC_K(k_h) - HC_E(e_h) \\ & -[S_{\sim m}+S_m]L\varphi(k_l)\rho(e_l)w - LC_K(k_l) - LC_E(e_l). \end{aligned} \tag{A27}$$

Differentiating (A27), the effect of seller *m*'s solicitation on welfare,

$$\begin{aligned} \frac{\partial W}{\partial S_m} = {} & H\Bigg\{-[1-\varphi(k_h)\rho(e_h)]^{S_{\sim m}+S_m}\ln(1-\varphi(k_h)\rho(e_h))\frac{S_m}{S_{\sim m}+S_m} \\ & + \left\{1-[1-\varphi(k_h)\rho(e_h)]^{S_{\sim m}+S_m}\right\}\frac{S_{\sim m}}{[S_{\sim m}+S_m]^2}\Bigg\}[V_h + pq_h] \\ & + H\Bigg\{-[1-\varphi(k_h)\rho(e_h)]^{S_{\sim m}+S_m}\ln(1-\varphi(k_h)\rho(e_h))\frac{S_{\sim m}}{S_{\sim m}+S_m} \\ & - \left\{1-[1-\varphi(k_h)\rho(e_h)]^{S_{\sim m}+S_m}\right\}\frac{S_{\sim m}}{[S_{\sim m}+S_m]^2}\Bigg\}[V_h + pq_h] \\ & - \frac{\partial}{\partial S_m}C(S_m, \Lambda) - H\varphi(k_h)\rho(e_h)w - L\varphi(k_l)\rho(e_l)w, \end{aligned}$$

which simplifies to

$$\frac{\partial W}{\partial S_m} = H\left\{-\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m}\ln(1-\varphi(k_h)\rho(e_h))\right\}\left[V_h + pq_h\right]$$
$$-\frac{\partial}{\partial S_m}C(S_m,\Lambda) - H\varphi(k_h)\rho(e_h)w - L\varphi(k_l)\rho(e_l)w. \qquad \text{(A28)}$$

Let τ represent the per-unit charge on seller solicitations. Then, substituting in (18), seller m would maximize profit by choosing S_m according to

$$\frac{\partial \Pi}{\partial S_m} = H\left\{-\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m}\ln(1-\varphi(k_h)\rho(e_h))\frac{S_m}{S_{\sim m}+S_m}\right.$$
$$\left.+\left\{1-\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m}\right\}\frac{S_{\sim m}}{\left[S_{\sim m}+S_m\right]^2}\right\}pq_h - \tau - \frac{\partial}{\partial S_m}C(S_m,\Lambda) = 0. \qquad \text{(A29)}$$

Equating (A28) and (A29),

$$\tau = \underbrace{H\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m}\ln(1-\varphi(k_h)\rho(e_h))V_h}_{\text{negative of increase in high-type consumers' surplus from increased probability of sale}}$$
$$+\underbrace{H\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m}\ln(1-\varphi(k_h)\rho(e_h))\frac{S_{\sim m}}{S_{\sim m}+S_m}pq_h}_{\text{negative of increase in revenue of the other sellers from increased probability of sale}}$$
$$+\underbrace{H\left\{1-\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m}\right\}\frac{S_{\sim m}}{\left[S_{\sim m}+S_m\right]^2}pq_h}_{\text{"stealing" of revenue from other sellers}} \qquad \text{(A30)}$$
$$+\underbrace{H\varphi(k_h)\rho(e_h)w}_{\text{increase in harm to high-type consumers}} + \underbrace{L\varphi(k_l)\rho(e_l)w}_{\text{increase in harm to low-type consumers}}.$$

It is clear that τ is decreasing in the first term and increasing in the last two terms. To analyze the second and third terms on the right-hand side of (A30), we use Lemma 1,

$$\varphi(k_h)\rho(e_h)\frac{1}{\left[S_{\sim m}+S_m\right]\varphi(k_h)\rho(e_h)}\left\{1-\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m}\right\}$$
$$=\varphi(k_h)\rho(e_h)\left\{\sum_{z=0}^{S_{\sim m}+S_m-1}\frac{1}{z+1}\binom{S_{\sim m}+S_m-1}{z}\left[\varphi(k_h)\rho(e_h)\right]^z\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m-1-z}\right\} \qquad \text{(A31)}$$
$$>\varphi(k_h)\rho(e_h)\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m-1},$$

since $[1-\varphi(k_h)\rho(e_h)]^{S_{\sim m}+S_m-1}$ is just the first term in the summation. Now, by (1) and (5), $\varphi(k_h)\rho(e_h)<1$. Hence, by the Taylor expansion,

$$\begin{aligned}\ln(1-\varphi(k_h)\rho(e_h)) &\cong -\varphi(k_h)\rho(e_h)-\frac{[\varphi(k_h)\rho(e_h)]^2}{2}-\frac{[\varphi(k_h)\rho(e_h)]^3}{3}-\ldots\\ &> -\varphi(k_h)\rho(e_h)-[\varphi(k_h)\rho(e_h)]^2-[\varphi(k_h)\rho(e_h)]^3-\ldots \qquad \text{(A32)}\\ &= -\frac{\varphi(k_h)\rho(e_h)}{1-\varphi(k_h)\rho(e_h)},\end{aligned}$$

and so, (A31) simplifies to

$$\begin{aligned}&\frac{1}{S_{\sim m}+S_m}\left\{1-[1-\varphi(k_h)\rho(e_h)]^{S_{\sim m}+S_m}\right\}\\ &> [1-\varphi(k_h)\rho(e_h)]^{S_{\sim m}+S_m}\frac{\varphi(k_h)\rho(e_h)}{1-\varphi(k_h)\rho(e_h)} > -[1-\varphi(k_h)\rho(e_h)]^{S_{\sim m}+S_m}\ln(1-\varphi(k_h)\rho(e_h)),\end{aligned} \qquad \text{(A33)}$$

which implies that the sum of the second and third terms on the right-hand side of (A30) is positive. Thus, τ is increasing in the demand that sellers take from one another.

It remains to prove that the optimal charge is positive. We analyze two cases:

Case (i): $U_h(k_j,e_j)$ is maximized at $k_j=0$ and $e_j=0$. Consider seller *m*'s solicitations. In equilibrium, $\partial\Pi/\partial S_m=0$, and hence by (18),

$$\begin{aligned}&H\left\{1-[1-\varphi(k_h)\rho(e_h)]^{S_{\sim m}+S_m}\right\}\frac{S_{\sim m}}{[S_{\sim m}+S_m]^2}pq_h\\ &=H[1-\varphi(k_h)\rho(e_h)]^{S_{\sim m}+S_m}\ln(1-\varphi(k_h)\rho(e_h))\frac{S_m}{S_{\sim m}+S_m}pq_h+\frac{\partial}{\partial S_m}C(S_m,\Lambda).\end{aligned} \qquad \text{(A34)}$$

Using (A34), the second and third terms on the right-hand side of (A30) add up to

$$\tau' \equiv H[1-\varphi(k_h)\rho(e_h)]^{S_{\sim m}+S_m}\ln(1-\varphi(k_h)\rho(e_h))\,pq_h+\frac{\partial}{\partial S_m}C(S_m,\Lambda). \qquad \text{(A35)}$$

Now, by (A33), the second and third terms on the right-hand side of (A30) are positive; hence, $\tau'>0$, and so,

$$H\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m}\ln(1-\varphi(k_h)\rho(e_h)) > -\frac{1}{pq_h}\cdot\frac{\partial}{\partial S_m}C(S_m,\Lambda). \qquad \text{(A36)}$$

Using (A36), the first, fourth and fifth terms on the right-hand side of (A30) add up to

$$\begin{aligned}\tau'' &\equiv H\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m}\ln(1-\varphi(k_h)\rho(e_h))V_h + H\varphi(k_h)\rho(e_h)w + L\varphi(k_l)\rho(e_l)w \\ &> -\frac{V_h}{pq_h}\frac{\partial}{\partial S_m}C(S_m,\Lambda) + H\varphi(k_h)\rho(e_h)w + L\varphi(k_l)\rho(e_l)w.\end{aligned} \qquad \text{(A37)}$$

Consider whether

$$\underbrace{\left\{H\,\varphi(k_h)\rho(e_h) + L\varphi(k_l)\rho(e_l)\right\}}_{\text{Effective proportion of consumers receiving solicitations}}\frac{w}{V_h} > \frac{1}{pq_h}\cdot\frac{\partial}{\partial S_m}C(S_m,\Lambda). \qquad \text{(A38)}$$

By (2), (4), (5), (10), (11), (13), and (14), both high- and low-type consumers would choose finite efforts in concealment and deflection, and hence the effective proportion of consumers receiving solicitations, i.e., the term in braces on the left-hand side of (A38), would be positive. By the Profitability Condition, the marginal cost of solicitation, $\partial C(S_m,\Lambda)/\partial S_m$, is sufficiently small relative to the seller's incremental margin, pq_h. Accordingly, (A38) holds and so, by (A37), $\tau'' > 0$. Hence, by (A30) and (A35), the optimal charge, $\tau = \tau' + \tau'' > 0$.

Case (ii): $U_h(k_j, e_j)$ is maximized at some $k_j^* > 0$ or $e_j^* > 0$. In this case, using the Taylor expansion (A32), (A3) implies

$$\begin{aligned}\varphi(k_j)\rho(e_j)w &> \varphi(k_j)\rho(e_j)\left[1-\varphi(k_j)\rho(e_j)\right]^{S_1+\ldots+S_N-1}V_h \\ &> -\left[1-\varphi(k_j)\rho(e_j)\right]^{S_1+\ldots+S_N}\ln(1-\varphi(k_j)\rho(e_j))V_h,\end{aligned} \qquad \text{(A39)}$$

which implies that the sum of the first and fourth terms in (A30) is positive. Since, by (A33), the sum of the second and third terms is always positive, and the fifth term is positive too, the optimal charge, $\tau > 0$.

□

Proof of Proposition 5. Let *h* denote one high-type consumer and $j \neq h$ index the other $[H-1]$ high-type consumers, and *l* denote one low-type consumer and $i \neq l$ index the other $[L-1]$ low-type consumers.

Summing (9) over all high-type consumers and (12) over all low-type consumers, in symmetric equilibrium, consumer welfare is

$$\begin{aligned}\Psi = &\left\{1-\left[1-\varphi(k_h)\rho(e_h)\right]^{S_1+\ldots+S_N}\right\}V_h + [H-1]\left\{1-\left[1-\varphi(k_j)\rho(e_j)\right]^{S_1+\ldots+S_N}\right\}V_h \\ &-\left[S_1+\ldots+S_N\right]\varphi(k_h)\rho(e_h)w - C_K(k_h) - C_E(e_h) \\ &-\left[S_1+\ldots+S_N\right][H-1]\varphi(k_h)\rho(e_h)w - [H-1]C_K(k_h) - [H-1]C_E(e_h) \\ &-\left[S_1+\ldots+S_N\right]\varphi(k_l)\rho(e_l)w - C_K(k_l) - C_E(e_l) \\ &-\left[S_1+\ldots+S_N\right][L-1]\varphi(k_l)\rho(e_l)w - [L-1]C_K(k_l) - [L-1]C_E(e_l).\end{aligned} \quad \text{(A40)}$$

Differentiating (A40) with respect to a high-type consumer *h*'s effort in deflection, e_h, and a low-type consumer *l*'s effort in deflection, e_l, yields (11) and (14), and so

$$\frac{\partial \Psi}{\partial e_h} = \frac{\partial U_h}{\partial e_h} \quad \text{and} \quad \frac{\partial \Psi}{\partial e_l} = \frac{\partial U_l}{\partial e_l}. \quad \text{(A41)}$$

Now, differentiating (A40) with respect to a high-type consumer *h*'s effort in concealment, k_h, and arranging terms,

$$\begin{aligned}\frac{\partial \Psi}{\partial k_h} = &\left[S_1+\ldots+S_N\right]\left\{\left[1-\varphi(k_h)\rho(e_h)\right]^{S_1+\ldots+S_N-1}V_h - w\right\}\rho(e_h)\frac{d}{dk_h}\varphi(k_h) - \frac{d}{dk_h}C_K(k_h) \\ &+\underbrace{\left[S_1+\ldots+S_N\right][H-1]\left\{\left[1-\varphi(k_j)\rho(e_j)\right]^{S_1+\ldots+S_N-1}V_h - w\right\}\rho(e_j)\frac{d}{dk_h}\varphi(k_j)}_{\text{Net increase in utility to other high-type consumers}} \\ &\underbrace{-\left[S_1+\ldots+S_N\right]L\rho(e_l)w\frac{d}{dk_h}\varphi(k_l)}_{\text{Increase in harm to low-type consumers}}.\end{aligned} \quad \text{(A42)}$$

Similarly, differentiating (A40) with respect to a low-type consumer *l*'s effort in concealment, k_l, and arranging terms,

$$\begin{aligned}\frac{\partial\Psi}{\partial k_l} = &-[S_1 + \ldots + S_N]\rho(e_l)w\frac{d}{dk_l}\varphi(k_l) - \frac{d}{dk_l}C_K(k_l)\\ &+\underbrace{[S_1+\ldots+S_N]H\left\{[1-\varphi(k_h)\rho(e_h)]^{S_1+\ldots+S_N-1}V_h - w\right\}\rho(e_h)\frac{d}{dk_l}\varphi(k_h)}_{\text{Net increase in utility to high-type consumers}} \\ &-\underbrace{[S_1+\ldots+S_N][L-1]\rho(e_i)w\frac{d}{dk_l}\varphi(k_i).}_{\text{Increase in harm to other low-type consumers}}\end{aligned} \quad \text{(A43)}$$

We separate the proof into two cases.

Case (i): $U_h(k_j, e_j)$ is maximized at $k_j = 0$. This implies that at $k_j = 0$, $\partial U_h / \partial k_j \le 0$. By (4), at $k_j = 0$, $dC_K / dk_j = 0$. Hence, by (10), $\partial U_h / \partial k_j \le 0$ implies that for any e_j,

$$[1-\varphi(k_j)\rho(e_j)]^{S_1+\ldots+S_N-1}V_h \ge w, \quad \text{(A44)}$$

since, by (A2), $d\varphi / dk_j < 0$. Substituting (A44) into (11), and using (3), $\partial U_h / \partial e_j \le 0$, and so, high-type consumers would also choose zero effort in deflection, $e_h = 0$. Hence, for high-type consumers, the result holds trivially.

It remains to consider the low-type consumers. Substituting (A15) in (A43), and, in symmetric equilibrium, $k_i = k_l$ and $e_i = e_l$, we have

$$\begin{aligned}\frac{\partial\Psi}{\partial k_l} = &-[S_1 + \ldots + S_N]\rho(e_l)w\frac{d}{dk_l}\varphi(k_l) - \frac{d}{dk_l}C_K(k_l)\\ &-\frac{S_1+\ldots+S_N}{\Lambda}H\varphi(k_h)\rho(e_h)[1-\varphi(k_h)\rho(e_h)]^{S_1+\ldots+S_N-1}V_h\frac{d\alpha}{dk_l} \\ &+\frac{S_1+\ldots+S_N}{\Lambda}\Big[H\rho(e_h)\varphi(k_h)+[L-1]\rho(e_l)\varphi(k_l)\Big]w\frac{d\alpha}{dk_l}.\end{aligned} \quad \text{(A45)}$$

By (13), the first two terms on the right-hand side of (A45) add to zero. Hence, (A45) simplifies to

$$\begin{aligned}\frac{\partial\Psi}{\partial k_l} = &-\frac{S_1+\ldots+S_N}{\Lambda}H\varphi(k_h)\rho(e_h)[1-\varphi(k_h)\rho(e_h)]^{S_1+\ldots+S_N-1}V_h\frac{d\alpha}{dk_l} \\ &+\frac{S_1+\ldots+S_N}{\Lambda}\Big[H\rho(e_h)\varphi(k_h)+[L-1]\rho(e_l)\varphi(k_l)\Big]w\frac{d\alpha}{dk_l}.\end{aligned} \quad \text{(A46)}$$

Now, in equilibrium, $\partial \Pi / \partial S_m = 0$, and hence by (18),

$$\begin{aligned}\frac{\partial}{\partial S_m} C(S_m, \Lambda) = &-H\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m} \ln(1-\varphi(k_h)\rho(e_h)) \frac{S_m}{S_{\sim m}+S_m} pq_h \\ &+ H\left\{1-\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m}\right\} \frac{S_{\sim m}}{[S_{\sim m}+S_m]^2} pq_h .\end{aligned} \tag{A47}$$

Using the Taylor expansion, (A32), the first term on the right-hand side of (A47) simplifies to

$$\begin{aligned}&-H\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m} \ln(1-\varphi(k_h)\rho(e_h)) \frac{S_m}{S_{\sim m}+S_m} pq_h \\ &= H\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m} \left\{\varphi(k_h)\rho(e_h) + \frac{[\varphi(k_h)\rho(e_h)]^2}{2} + \ldots\right\} \frac{S_m}{S_{\sim m}+S_m} pq_h \\ &> H\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m} \varphi(k_h)\rho(e_h) \frac{S_m}{S_{\sim m}+S_m} pq_h \\ &= H\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m-1}\left[1-\varphi(k_h)\rho(e_h)\right]\varphi(k_h)\rho(e_h) \frac{S_m}{S_{\sim m}+S_m} pq_h \\ &= H\varphi(k_h)\rho(e_h)\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m-1} \frac{S_m}{S_{\sim m}+S_m} pq_h \\ &\qquad - H\left[\varphi(k_h)\rho(e_h)\right]^2\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m-1} \frac{S_m}{S_{\sim m}+S_m} pq_h ,\end{aligned} \tag{A48}$$

and using Lemma 1, the second term on the right-hand side of (A47) simplifies to

$$\begin{aligned}&H\left\{1-\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m}\right\} \frac{S_{\sim m}}{[S_{\sim m}+S_m]^2} pq_h \\ &= H\varphi(k_h)\rho(e_h)\left\{\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m-1}\right. \\ &\qquad + \frac{S_{\sim m}+S_m-1}{2}\left[\varphi(k_h)\rho(e_h)\right]\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m-2} \\ &\qquad + \frac{[S_{\sim m}+S_m-1][S_{\sim m}+S_m-2]}{6}\left[\varphi(k_h)\rho(e_h)\right]^2\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m-3} \\ &\qquad \left. + \ldots\right\} \frac{S_{\sim m}}{S_{\sim m}+S_m} pq_h \\ &> H\varphi(k_h)\rho(e_h)\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m-1} \frac{S_{\sim m}}{S_{\sim m}+S_m} pq_h \\ &\qquad + H\frac{S_{\sim m}+S_m-1}{2}\left[\varphi(k_h)\rho(e_h)\right]^2\left[1-\varphi(k_h)\rho(e_h)\right]^{S_{\sim m}+S_m-2} \frac{S_{\sim m}}{S_{\sim m}+S_m} pq_h .\end{aligned} \tag{A49}$$

In equilibrium, we must have $S_m < S_{\sim m}$ and $[S_{\sim m} + S_m - 1]/2 \geq 1$. Since $0 \leq \varphi(k_h)\rho(e_h) \leq 1$, $[1-\varphi(k_h)\rho(e_h)]^{S_{\sim m}+S_m-2} \geq [1-\varphi(k_h)\rho(e_h)]^{S_{\sim m}+S_m-1}$. Accordingly, in absolute value, the second term on the right hand side of (A49) exceeds the second term on the right hand side of (A48). Thus, adding (A48) and (A49), and then substituting in (A47),

$$\frac{\partial}{\partial S_m} C(S_m, \Lambda) > H\varphi(k_h)\rho(e_h)[1-\varphi(k_h)\rho(e_h)]^{S_{\sim m}+S_m-1} pq_h. \qquad \text{(A50)}$$

By similar reasoning as around (A38),

$$\left[H\varphi(k_h)\rho(e_h) + [L-1]\varphi(k_l)\rho(e_l)\right]\frac{w}{V_h} > \frac{1}{pq_h}\cdot\frac{\partial}{\partial S_m} C(S_m, \Lambda), \qquad \text{(A51)}$$

since, by the Profitability Condition, the term on the right hand side of (A51) is sufficiently small. Accordingly, applying (A50) and (A51) to (A45), $\partial\Psi/\partial k_l < 0$. Now, by (14) and (A41), $\partial\Psi/\partial e_l = 0$. Thus, $\partial\Psi/\partial k_l < 0 = \partial\Psi/\partial e_l$, which is the result.

Case (ii): $U_h(k_j, e_j)$ is maximized at some $k_j^* > 0$. This implies that in equilibrium, $\partial U_h/\partial k_j = 0$, and so, the first two terms on the right-hand side of (A42) sum to 0. By (A3) and (A15), the third and fourth terms on the right-hand side of (A42) are both negative. Accordingly, $\partial\Psi/\partial k_h < 0$. If the high-type consumer chooses positive effort in deflection, by (11) and (A41), $\partial\Psi/\partial e_h = 0$, and thus, $\partial\Psi/\partial k_h < \partial\Psi/\partial e_h$. If, however, the high-type consumer chooses zero effort in deflection, then $\partial\Psi/\partial e_h = 0$ the result is trivial.

Finally, by (13), $U_l(k_i, e_i)$ is always maximized at some $k_i^* > 0$. Thus, by similar reasoning as above, $\partial\Psi/\partial k_l < 0$, and so, by (14) and (A41), $\partial\Psi/\partial k_l < \partial\Psi/\partial e_l$. □

Empirical implications: High-type consumers choose positive efforts in concealment and deflection

The case where high-type consumers choose positive efforts in concealment and deflection divides into two sub-cases, depending on the direction in which sellers' solicitations respond to consumer efforts in concealment and deflection.

If the net response of sellers' solicitation is negative, the empirical implications are as presented in Table A1.

On variable	Effect of an increase in								
	V_h	pq_h	H	L	c_K	c_E	w	c	N
S	+	+	–	–	–	–	?	–	?
k_l	+	+	–	–	–	?	?	–	?
e_l	+	+	–	–	?	–	?	–	?
k_h	–	+	–	–	–	?	?	–	?
e_h	–	+	–	–	?	–	?	–	?

Table A1

If the net response of sellers' solicitation is positive, the empirical implications are as presented in Table A2.

On variable	Effect of an increase in								
	V_h	pq_h	H	L	c_K	c_E	w	c	N
S	+	+	?	?	+	+	+	–	?
k_l	+	+	?	?	?	+	+	–	?
e_l	+	+	?	?	+	?	+	–	?
k_h	–	+	?	?	?	+	+	–	?
e_h	–	+	?	?	+	?	+	–	?

Table A2

Proof of Extensions.

(iii) Low-type consumers' demand. Similar to (A12) and the discussion around (A13), by differentiating (23), it is clear that $\partial^2\Pi/\partial e_h \partial S_m < 0$ and $\partial^2\Pi/\partial e_l \partial S_m < 0$, which prove that sellers' solicitation is a strategic substitute with both consumer types' efforts in deflection.

Now, by similar reasoning as leading to (A16),

$$\begin{aligned}
&\frac{\partial^2\Pi}{\partial k_h \partial S_m} \\
&= \lambda[1-\varphi(k_h)\rho(e_h)]^{S_{\sim m}+S_m-1}\left[S_m \ln(1-\varphi(k_h)\rho(e_h))+1\right]\rho(e_h)pq_h \\
&\quad -\frac{L}{\Lambda}\varphi(k_l)\frac{d\alpha}{dk_h}[1-\varphi(k_l)\rho(e_l)]^{S_{\sim m}+S_m-1}\left[S_m \ln(1-\varphi(k_l)\rho(e_l))+1\right]\rho(e_l)pq_l \\
&\quad -\frac{\partial^2}{\partial\Lambda\partial S_m}C(S_m,\Lambda)\frac{d\alpha}{dk_h},
\end{aligned} \tag{A52}$$

where λ was defined in (A17). By (A13) and (A17), the first and third terms on the right hand side of (A52) are negative, whereas the second term is positive. If $q_l(p)$ is sufficiently small, then the first term on the right hand side of (A52) dominates the second term, and hence $\partial^2\Pi/\partial k_h \partial S_m < 0$. Similar derivations show that $\partial^2\Pi/\partial k_l \partial S_m > 0$ when $q_l(p)$ is small. These prove the results of Proposition 2 with regard to consumers' efforts in concealment.

If, however, $q_l(p)$ is large, then, by similar derivation as above, it is straightforward to show that $\partial^2\Pi/\partial k_l \partial S_m < 0$. That is, sellers' solicitation is also a strategic substitute with low-type consumers' effort in concealment. [ ]

(iv) Pricing. In symmetric equilibrium, $k_j = k_h$ and $e_j = e_h$, for all j = 1,…, H, and $S_y = S$ and $F_y = F$, for all y = 1,…, N. Hence, by (26), seller m's revenue at any price p is

$$\widetilde{R}_m(p) = H\left\{1-[1-\varphi(k_h)\rho(e_h)]^{S_m}\right\}\left\{1-\{1-[1-\varphi(k_h)\rho(e_h)]^{S}\}F(p)\right\}^{N-1} pq_h(p). \tag{A53}$$

and the corresponding profit is $\tilde{\Pi}_m(p) = \tilde{R}_m(p) - C(S_m, \Lambda)$. In a randomized-strategy equilibrium, seller *m* must receive the same revenue, $\overline{R}_m$, and profit, $\overline{\Pi}_m$, at all prices in the support $[\underline{p}, \overline{p}]$. Equal revenue implies that

$$\overline{R}_m = H\left\{1 - \left[1 - \varphi(k_h)\rho(e_h)\right]^{S_m}\right\}\left\{1 - \left\{1 - \left[1 - \varphi(k_h)\rho(e_h)\right]^{S}\right\}F(p)\right\}^{N-1} pq_h(p), \quad \text{(A54)}$$

and hence

$$F(p) = \frac{1}{1 - \left[1 - \varphi(k_h)\rho(e_h)\right]^S}\left\{1 - \left\{\frac{\overline{R}_m}{Hpq_h(p)\{1 - \left[1 - \varphi(k_h)\rho(e_h)\right]^{S_m}\}}\right\}^{\frac{1}{N-1}}\right\}. \quad \text{(A55)}$$

Since there is no mass point in symmetric pricing equilibrium (Varian 1980; Narasimhan 1988; McAfee 1994), $F(\overline{p}) = 1$. Substituting in (A54),

$$\overline{R}_m = H\left\{1 - \left[1 - \varphi(k_h)\rho(e_h)\right]^{S_m}\right\}\left[1 - \varphi(k_h)\rho(e_h)\right]^{[N-1]S} \overline{p}q_h(\overline{p}). \quad \text{(A56)}$$

Substituting (A56) in (A55), the equilibrium price distribution is

$$F(p) = \frac{1}{1 - \left[1 - \varphi(k_h)\rho(e_h)\right]^S}\left\{1 - \left[1 - \varphi(k_h)\rho(e_h)\right]^S\left[\frac{\overline{p}q_h(\overline{p})}{pq_h(p)}\right]^{\frac{1}{N-1}}\right\}. \quad \text{(A57)}$$

By (A56), seller *m*'s profit is

$$\Pi(S_m) = H\left\{1 - \left[1 - \varphi(k_h)\rho(e_h)\right]^{S_m}\right\}\left[1 - \varphi(k_h)\rho(e_h)\right]^{[N-1]S} \overline{p}q_h(\overline{p}) - C(S_m, \Lambda). \quad \text{(A58)}$$

The first order condition is

$$\frac{\partial \Pi}{\partial S_m} = -H\left[1 - \varphi(k_h)\rho(e_h)\right]^{[N-1]S + S_m} \ln(1 - \varphi(k_h)\rho(e_h))\overline{p}q_h(\overline{p}) - \frac{\partial}{\partial S_m}C(S_m, \Lambda). \quad \text{(A59)}$$

Differentiating (A59) with respect to any particular high-type consumer's effort in deflection, e_h,

$$\frac{\partial^2\Pi}{\partial e_h \partial S_m} = \left[1-\varphi(k_h)\rho(e_h)\right]^{[N-1]S+S_m-1}\left\{\left\{[N-1]S+S_m\right\}\ln(1-\varphi(k_h)\rho(e_h))+1\right\}\varphi(k_h)\frac{d\rho}{de_h}\bar{p}q_h(\bar{p}). \tag{A60}$$

By similar reasoning as leading to (A13),

$$\left\{[N-1]S+S_m\right\}\ln(1-\varphi(k_h)\rho(e_h))+1>0, \tag{A61}$$

and hence by (5), $\partial^2\Pi/\partial e_h\partial S_m<0$. Further, by (A59), it is obvious that $\partial^2\Pi/\partial e_l\partial S_m=0$. Hence, sellers' solicitation is a strategic substitute with high-type consumers' effort in deflection and independent of low-type consumers' effort in deflection.

Now, differentiating (A59) with respect to any particular high-type consumer's effort in concealment, k_h,

$$\frac{\partial^2\Pi}{\partial k_h \partial S_m} = \lambda\left[1-\varphi(k_h)\rho(e_h)\right]^{[N-1]S+S_m-1}\left\{\left\{[N-1]S+S_m\right\}\ln(1-\varphi(k_h)\rho(e_h))+1\right\}\rho(e_h)\bar{p}q_h(\bar{p})$$
$$-\frac{\partial^2}{\partial\Lambda\,\partial S_m}C(S_m,\Lambda)\frac{d\alpha}{dk_h}, \tag{A62}$$

where $\lambda<0$ as in (A17). By (A61) and the discussion after (A16), $\partial^2\Pi/\partial k_h\partial S_m<0$, which proves that seller solicitation is a strategic substitute with high-type consumers' effort in concealment. Similarly, by (A61) and the discussion after (A18), if $\partial^2 C(S_m,\Lambda)/\partial\Lambda\partial S_m$ is sufficiently small, then $\partial^2\Pi/\partial k_l\partial S_m>0$, which proves that sellers' solicitation is a strategic complement with low-type consumers' effort in concealment. This completes the proof of Proposition 2 with randomized pricing.

Finally, by inspecting (A21), it is obvious that, since Proposition 3 only concerns the direct privacy harm imposed on consumers by sellers' solicitations, the result applies to the setting with randomized pricing. □