Online Appendix

A. Proofs of results

Proof of Proposition 1:

Positively correlated marginal and total benefits

Set up the vendor's maximization as a Lagrangian,

$$L(T,p,\lambda) = T + \int_{\underline{s}}^{\bar{s}} [p-c]q(T,p,s)dG(s) + \lambda\left[\int_{\underline{s}}^{\bar{s}} u(x(T,p,s))dG(s) - \bar{u}\right], \qquad \text{(A1)}$$

where λ is the Lagrange multiplier ("shadow price") of the buyer's individual rationality constraint (12). Now $\lambda > 0$, since any relaxation of the buyer's individual rationality constraint would allow the vendor to raise the entry fee and increase profit. Since there is no direct constraint on the sign of T or p, the problem has an interior solution.

The first order condition with respect to T,

$$\begin{aligned}\frac{\partial L(T,p,\lambda)}{\partial T} &= 1 + \lambda\int_{\underline{s}}^{\bar{s}} u'(x(T,p,s))x_T(T,p,s)dG(s) \\ &= \int_{\underline{s}}^{\bar{s}} [1-\lambda u'(x(T,p,s))]dG(s) = 0\end{aligned} \qquad \text{(A2)}$$

using (7). Further, the first order condition with respect to p,

$$\begin{aligned}\frac{\partial L(T,p,\lambda)}{\partial p} &= \int_{\underline{s}}^{\bar{s}} \left[q(T,p,s) + [p-c]q_p(T,p,s)\right]dG(s) \\ &\quad + \lambda\int_{\underline{s}}^{\bar{s}} u'(x(T,p,s))x_p(T,p,s)dG(s) = 0.\end{aligned} \qquad \text{(A3)}$$

From (7) and (8), we have

$$x_p(T,p,s) = b_q(q(T,p,s),s)q_p(T,p,s) - q(T,p,s) - pq_p(T,p,s) = -q(T,p,s). \qquad \text{(A4)}$$

Hence (A3) simplifies to

$$\begin{aligned}\frac{\partial L(T,p,\lambda)}{\partial p} &= \int_{\underline{s}}^{\bar{s}} \left[q(T,p,s) + [p-c]q_p(T,p,s)\right]dG(s) - \lambda\int_{\underline{s}}^{\bar{s}} u'(x(T,p,s))q(T,p,s)dG(s) \\ &= \int_{\underline{s}}^{\bar{s}} q(T,p,s)[1-\lambda u'(x(T,p,s)]dG(s) + [p-c]\int_{\underline{s}}^{\bar{s}} q_p(T,p,s)dG(s) = 0.\end{aligned} \qquad \text{(A5)}$$

We claim that $1-\lambda u'(x(T,p,s))$ is strictly increasing in s. To show this, consider the derivative,

$$\lambda \frac{\partial}{\partial s} u'(x(T,p,s)) = \lambda u''(x(T,p,s)) \cdot b_s(q(T,p,s),s) < 0, \tag{A6}$$

since, by assumption, $u''(.) < 0$, and using (1). This proves the claim.

Using (A6), the first order condition (A2) implies that there exists $s^0 \in (\underline{s}, \bar{s})$ such that

$$1 - \lambda u'(x(T,p,s)) \qquad \begin{cases} < 0 & \text{if } s < s^0 \\ = 0 & \text{if } s = s^0 \\ > 0 & \text{if } s > s^0 \end{cases} \tag{A7}$$

Meanwhile, differentiating (7) partially with respect to *s*, and re-arranging, we have

$$q_s(T,p,s) = -\frac{b_{qs}(q(T,p,s),s)}{b_{qq}(q(T,p,s),s)} > 0, \tag{A8}$$

since, by (2), $b_{qs}(q,s) > 0$, and by assumption, $b_{qq}(q,s) < 0$. Hence given (T,p), the quantity demanded increases with *s*.

Referring to the first term on the right-hand side of (A5), we have

$$\begin{aligned}
&\int_{\underline{s}}^{\bar{s}} q(T,p,s)[1-\lambda u'(x(T,p,s))]dG(s) \\
&\quad = \int_{\underline{s}}^{s^0} q(T,p,s)[1-\lambda u'(x(T,p,s))]dG(s) + \int_{s^0}^{\bar{s}} q(T,p,s)[1-\lambda u'(x(T,p,s))]dG(s) \\
&\quad > \int_{\underline{s}}^{s^0} q(T,p,s^0)[1-\lambda u'(x(T,p,s))]dG(s) + \int_{s^0}^{\bar{s}} q(T,p,s^0)[1-\lambda u'(x(T,p,s))]dG(s) \\
&\quad = \int_{\underline{s}}^{\bar{s}} q(T,p,s^0)[1-\lambda u'(x(T,p,s))]dG(s) = 0,
\end{aligned} \tag{A9}$$

where the inequality uses (A7) and (A8), and the final equality uses (A2). By (A9) and (A5), we have

$$[p-c]\int_{\underline{s}}^{\bar{s}} q_p(T,p,s)dG(s) < 0. \tag{A10}$$

Differentiating (7) partially with respect to *p*, and re-arranging,

$$q_p(T,p,s) = \frac{1}{b_{qq}(q(T,p,s),s)} < 0, \tag{A11}$$

since, by assumption, $b_{qq}(q,s) < 0$. Applying (A11) to (A10), it follows that $p^* > c$.

Negatively correlated marginal and total benefits

The proof is identical to that with positively correlated marginal and total benefits up to (A7).

Differentiating (7) partially with respect to s, and re-arranging, we have

$$q_s(T,p,s) = -\frac{b_{qs}(q(T,p,s),s)}{b_{qq}(q(T,p,s),s)} < 0, \tag{A12}$$

since, by (3), $b_{qs}(q,s) < 0$, and by assumption, $b_{qq}(q,s) < 0$. Hence given (T,p), the quantity demanded decreases with s.

Referring to the first term on the right-hand side of (A5), we have

$$\begin{aligned}
&\int_{\underline{s}}^{\bar{s}} q(T,p,s)[1-\lambda u'(x(T,p,s))]dG(s) \\
&\quad = \int_{\underline{s}}^{s^0} q(T,p,s)[1-\lambda u'(x(T,p,s))]dG(s) + \int_{s^0}^{\bar{s}} q(T,p,s)[1-\lambda u'(x(T,p,s))]dG(s) \\
&\quad < \int_{\underline{s}}^{s^0} q(T,p,s^0)[1-\lambda u'(x(T,p,s))]dG(s) + \int_{s^0}^{\bar{s}} q(T,p,s^0)[1-\lambda u'(x(T,p,s))]dG(s) \\
&\quad = \int_{\underline{s}}^{\bar{s}} q(T,p,s^0)[1-\lambda u'(x(T,p,s))]dG(s) = 0,
\end{aligned} \tag{A13}$$

where the inequality uses (A7) and (A12), and the final equality uses (A2). By (A13) and (A5), we have

$$[p-c]\int_{\underline{s}}^{\bar{s}} q_p(T,p,s)dG(s) > 0. \tag{A14}$$

Applying (A11) to (A14), it follows that $p^* < c$.

Entry fee

Suppose, otherwise, that $T < 0$. Then, substituting in (10),

$$\begin{aligned}
x(T,p,s) &\equiv b(q(T,p,s),s) + I - T - pq(T,p,s) \\
&> b(q(T,p,s),s) + I - pq(T,p,s).
\end{aligned} \tag{A15}$$

Now,

$$b(q(T,p,s),s) - pq(T,p,s) \geq b(0,s), \tag{A16}$$

otherwise the buyer would choose to consume nothing ex post, which means that the vendor's expected profit would be just T, which, by assumption is negative, hence cannot be profit-maximizing. Using (A16), (A15) simplifies to $x(T, p, s) > b(0, s) + I$, which implies that

$$\int_{\underline{s}}^{\bar{s}} u(x(T, p, s))dG(s) > \int_{\underline{s}}^{\bar{s}} u(b(0, s) + I)dG(s) \equiv \bar{u}\,, \tag{A17}$$

using (12). Thus, $T < 0$ implies that the individual rationality constraint does not bind, which is a contradiction, since, at maximum profit, the individual rationality constraint must bind. Accordingly, we have $T \geq 0$.

Differentiating the individual rationality constraint (12),

$$\int_{\underline{s}}^{\bar{s}} u'(x(T, p, s))\left[x_p(T, p, s)dp + x_T(T, p, s)dT\right]dG(s) = 0\,. \tag{A18}$$

By (A4), $x_p(T, p, s) = -q(T, p, s)$, while by (4), (8), and (7),

$x_T(T, p, s) = b_q q_T - 1 - pq_T = -1$. Accordingly, (A18) simplifies to

$$\frac{dT}{dp} = -\frac{\int_{\underline{s}}^{\bar{s}} qu'(x(T, p, s)dG(s)}{\int_{\underline{s}}^{\bar{s}} u'(x(T, p, s)dG(s)} < 0\,, \tag{A19}$$

which proves that the entry fee is decreasing in the usage charge. [ ]

Proof of Lemma 2: By (7) and (9), the buyer's ex post consumption of the service does not depend on her degree of risk aversion and, given that she subscribes, her consumption does not depend on the entry fee. Hence, the vendor's profit as a function of p and α is

$$\pi(p, T(p, \alpha)) = T(p, \alpha) + \int_{\underline{s}}^{\bar{s}} [p - c]q(p, s)dG(s)\,. \tag{A20}$$

Twice differentiating,

$$\frac{\partial^2 \pi}{\partial p \partial \alpha} = \frac{\partial^2 T}{\partial p \partial \alpha} + \frac{\partial^2}{\partial p \partial \alpha}\int_{\underline{s}}^{\bar{s}} [p - c]q(p, s)dG(s) = \frac{\partial^2 T}{\partial p \partial \alpha}\,, \tag{A21}$$

since the integral does not vary with buyer risk aversion, α.

At profit maximum, the buyers' individual rationality constraint, (12), would bind. Substituting from (15) in (12), buyers' individual rationality constraint simplifies to

$$\int_{\underline{s}}^{\bar{s}} e^{-\alpha x(T,p,s)} dG(s) = \int_{\underline{s}}^{\bar{s}} e^{-\alpha[b(0,s)+I]} dG(s) . \tag{A22}$$

Substituting from (10) into (A22), and solving for T, we have

$$e^{\alpha T} = \frac{\int_{\underline{s}}^{\bar{s}} e^{-\alpha b(0,s)} dG(s)}{\int_{\underline{s}}^{\bar{s}} e^{-\alpha[b(q,s)-pq]} dG(s)},$$

or

$$\alpha T = \ln\left(\int_{\underline{s}}^{\bar{s}} e^{-\alpha b(0,s)} dG(s)\right) - \ln\left(\int_{\underline{s}}^{\bar{s}} e^{-\alpha[b(q,s)-pq]} dG(s)\right).$$

Differentiating with respect to p,

$$\frac{\partial T}{\partial p} = -\frac{\int_{\underline{s}}^{\bar{s}} q e^{-\alpha[b(q,s)-pq]} dG(s)}{\int_{\underline{s}}^{\bar{s}} e^{-\alpha[b(q,s)-pq]} dG(s)} < 0 .$$

Further, differentiating with respect to α,

$$\begin{aligned}\frac{\partial^2 T}{\partial p \partial \alpha} = {} & \frac{\int_{\underline{s}}^{\bar{s}} q[b(q,s)-pq] e^{-\alpha[b(q,s)-pq]} dG(s) \int_{\underline{s}}^{\bar{s}} e^{-\alpha[b(q,s)-pq]} dG(s)}{\left[\int_{\underline{s}}^{\bar{s}} e^{-\alpha[b(q,s)-pq]} dG(s)\right]^2} \\ & - \frac{\int_{\underline{s}}^{\bar{s}} q e^{-\alpha[b(q,s)-pq]} dG(s) \int_{\underline{s}}^{\bar{s}} [b(q,s)-pq] e^{-\alpha[b(q,s)-pq]} dG(s)}{\left[\int_{\underline{s}}^{\bar{s}} e^{-\alpha[b(q,s)-pq]} dG(s)\right]^2}.\end{aligned} \tag{A23}$$

Defining,

$$B(s) = b(q(p,s),s) - pq(p,s) , \tag{A24}$$

then (A23) simplifies to

$$\frac{\partial^2 T}{\partial p \partial \alpha} = \frac{E(qBe^{-\alpha B})E(e^{-\alpha B}) - E(qe^{-\alpha B})E(Be^{-\alpha B})}{[E(e^{-\alpha B})]^2} , \tag{A25}$$

where the expectation, $E(.)$, is with respect to the state, s.

Consider the numerator of the right-hand side of (A25):

$$Z \equiv E(qBe^{-\alpha B})E(e^{-\alpha B}) - E(qe^{-\alpha B})E(Be^{-\alpha B})$$
$$= E\left(qBe^{-\alpha B}E(e^{-\alpha B})\right) - E\left(Be^{-\alpha B}E(qe^{-\alpha B})\right) \qquad \text{(A26)}$$
$$= E\left(Be^{-\alpha B}\left(qE(e^{-\alpha B}) - E(qe^{-\alpha B})\right)\right).$$

We claim that $Z > 0$. If for all *s*, $qE(e^{-\alpha B}) - E(qe^{-\alpha B}) > 0$, we have $Z > 0$, since the buyer will always enjoy non-negative surplus, or, referring to (A23), $B(q,s) \geq 0$, otherwise, she will choose not to consume. Otherwise, if $qE(e^{-\alpha B}) - E(qe^{-\alpha B}) > 0$ does not always hold, then there exists some $\tilde{s} \in [\underline{s}, \bar{s}]$ such that $qE(e^{-\alpha B}) - E(qe^{-\alpha B}) < 0$ if and only if $s < \tilde{s}$, and $qE(e^{-\alpha B}) - E(qe^{-\alpha B}) > 0$ if and only if $s > \tilde{s}$. Note, also, that, by (A23),

$$\frac{dB}{ds} = \frac{\partial b}{\partial q}\frac{\partial q}{\partial s} + \frac{\partial b}{\partial s} - p\frac{\partial q}{\partial s} = \frac{\partial b}{\partial s} > 0, \qquad \text{(A27)}$$

after substituting from (7) and using (1). Hence, $B(s) \leq B(\tilde{s})$ for $s < \tilde{s}$, and $B(s) \geq B(\tilde{s})$ for $s > \tilde{s}$. Accordingly,

$$E\left(Be^{-\alpha B}\left(qE(e^{-\alpha B}) - E(qe^{-\alpha B})\right)\right)$$
$$= \int_{\underline{s}}^{\tilde{s}} \underbrace{B}_{\leq B(\tilde{s})} e^{-\alpha B}\underbrace{\left[qE(e^{-\alpha B}) - E(qe^{-\alpha B})\right]}_{<0} dG(s) + \int_{\tilde{s}}^{\bar{s}} \underbrace{B}_{\geq B(\tilde{s})} e^{-\alpha B}\underbrace{\left[qE(e^{-\alpha B}) - E(qe^{-\alpha B})\right]}_{>0} dG(s)$$
$$> B(\tilde{s})\int_{\underline{s}}^{\tilde{s}} e^{-\alpha B}\left(qE(e^{-\alpha B}) - E(qe^{-\alpha B})\right)dG(s) + B(\tilde{s})\int_{\tilde{s}}^{\bar{s}} e^{-\alpha B}\left(qE(e^{-\alpha B}) - E(qe^{-\alpha B})\right)dG(s)$$
$$= B(\tilde{s})E\left(e^{-\alpha B}\left(qE(e^{-\alpha B}) - E(qe^{-\alpha B})\right)\right) = B(\tilde{s})\left[E\left(e^{-\alpha B}qE(e^{-\alpha B})\right) - E\left(e^{-\alpha B}E(qe^{-\alpha B})\right)\right] = 0,$$

which proves $Z > 0$.

By (A25) and (A26), since $Z > 0$, we have $\partial^2 T / \partial p \partial \alpha > 0$, hence, by (A21), the vendor's profit is supermodular in *p* and *α*. [ ]

Proof of Proposition 2: Let the profit-maximizing usage charge be $p = p(\alpha)$. Given $\alpha_1 > \alpha_2$, we must show that $p(\alpha_1) \geq p(\alpha_2)$. Consider any usage charge, $p < p(\alpha_2)$. By Lemma 3, the vendor's profit function is supermodular in *p* and *α*. Applying Amir (2005), Theorem 1, we infer that

$$\pi\left(p(\alpha_2), \alpha_1\right) - \pi\left(p, \alpha_1\right) \geq \pi\left(p(\alpha_2), \alpha_2\right) - \pi\left(p, \alpha_2\right) \geq 0.$$

This proves that if the buyers' degree of risk aversion is $\alpha_1 > \alpha_2$, a usage charge $p < p(\alpha_2)$ does not maximize profit. Accordingly, $p(\alpha_1) \geq p(\alpha_2)$, which proves that the profit-maximizing usage charge, p, is non-decreasing in the buyers' degree of risk aversion, α. By Proposition 1, if the usage charge, p, is higher, then the entry fee, T, must be lower, which completes the proof. [ ]

Proof of Proposition 3: Set up the vendor's problem as a Lagrangian,

$$L(T, p, \mu) = \int_{\underline{s}}^{\bar{s}} u(b(q(T, p, s), s) + m(T, p, s))dG(s) + \mu[T + \int_{\underline{s}}^{\bar{s}} [p - c]q(T, p, s)dG(s)], \tag{A28}$$

where μ is the Lagrange multiplier for constraint (17). Note that $\mu > 0$ because buyers can achieve strictly higher utility when the constraint is slightly relaxed.

The first order condition with respect to T is

$$\begin{aligned}\frac{\partial L(T, p, \mu)}{\partial T} &= \int_{\underline{s}}^{\bar{s}} u'(b(q(T, p, s), s) + m(T, p, s))m_T(T, p, s)dG(s) + \mu \\ &= \mu - \int_{\underline{s}}^{\bar{s}} u'(b(q(T, p, s), s) + m(T, p, s))dG(s) = 0,\end{aligned} \tag{A29}$$

where we make use of (9), and, from (8), $m_T(T, p, s) = -1$. Further, the first order condition with respect to p is

$$\begin{aligned}\frac{\partial L(T, p, \lambda)}{\partial p} &= \int_{\underline{s}}^{\bar{s}} u'(b(q(T, p, s), s) + m(T, p, s))(-q(T, p, s))dG(s) \\ &+ \mu\int_{\underline{s}}^{\bar{s}} q(T, p, s) + (p - c)q_p(T, p, s)dG(s) = 0,\end{aligned} \tag{A30}$$

where we make use of (7), and, from (8), $m_p(T, p, s) = -pq_p(T, p, s) - q(T, p, s)$.

Now (A29) and (A30) are identical to (A2) and (A3) with the substitution $\mu = \frac{1}{\lambda}$. Since we established Proposition 1 using only (A2) and (A3), the same analysis applies here. By (17), if $p > c$, then $T < 0$, and if $p < c$, then $T > 0$, which completes the proof. [ ]

B. Additional results: Distribution of demand

How should the pricing strategy be adjusted to shifts in the distribution of demand (marginal benefit)? It might seem intuitive that: (i) if the marginal benefit is higher in the sense of first-order stochastic dominance, then, the usage charge should be higher, and (ii) if the uncertainty is larger, then the difference between the usage charge and marginal cost should be larger, since buyers would want more insurance.

The difficulty with (i) is that the usage charge should be set according to the distribution of demand and the buyer's degree of risk aversion. How the pricing strategy should be adjusted to an increase in demand depends on its impact on the distribution of ex post net benefits and the buyer's degree of risk aversion. Below, we provide an example in which an increase in demand in the sense of first-order stochastic dominance is associated with a lower usage charge.

There are two difficulties with (ii). One is that the usage charge should be set according to the entire distribution, $G(.)$, rather than the degree of uncertainty, however characterized. Two "equally uncertain" distributions could lead to different profit-maximizing usage charges. The other difficulty is that the buyer chooses her consumption according to the realized state, hence, any change in the distribution of uncertainty would induce changes in consumption, and so, induce corresponding changes in her ex post net benefit and ex ante utility.[1]

Below, we illustrate how an increase in uncertainty would be associated with a larger difference between the usage charge and marginal cost, and hence, a higher usage charge.

[1] By contrast, in investments, the random variable of concern is the investor's wealth, which is itself the source of uncertainty (Levy 1992).

However, the managerial significance of this finding is somewhat limited by the reliance on a condition that might be difficult to interpret in practice. Accordingly, we do not highlight the finding as a proposition.[2]

From (A5), we have, at profit maximum

$$\lambda = \frac{1}{E(u'(x))} \,. \tag{B1}$$

Further, (A8) implies that

$$[p-c]E(q_p) = \lambda E(q \cdot u'(x)) - E(q) \,.$$

Re-arranging and substituting from (B1), the profit-maximizing margin is characterized by

$$p - c = \frac{\mathrm{Cov}(q, u'(x))}{E(q_p) \cdot E(u'(x))} \tag{B2}$$

By (A11), $E(q_p) < 0$. If $b_{qs}(q,s) > 0$, q increases with s, while $u'(.)$ decreases with s, and so, $\mathrm{Cov}(q, u'(x)) < 0$, and hence, $p > c$ Similarly, if $b_{qs}(q,s) < 0$, then $\mathrm{Cov}(q, u'(x)) > 0$ and $p < c$.

Suppose that the state s is distributed according to

$$s = \begin{cases} a & \text{with probability} \quad k \\ -a & \text{with probability} \quad 1-k \end{cases}, \tag{B3}$$

with $a > 0$ and $k \in (0,1)$. Then, we have

$$E(q) = k \cdot q(p,a) + (1-k) \cdot q(p,-a) \,,$$

$$E(u'(x)) = k \cdot u'(x(p,a)) + (1-k) \cdot u'(x(p,-a)) \,,$$

and

[2] Hayes (1987) encountered similar difficulties. Rather than specifying the change in distribution in terms of the source of uncertainty, she specified the change in terms of the net benefit, what she called "utility", which is *endogenous*. In addition, her result (Proposition 2) depended on a specific technical condition, for which she gave no interpretation and no managerial application.

$$\begin{aligned}\mathrm{Cov}(q, u'(x)) &= E[(q - Eq)\cdot(u'(x) - Eu'(x))] \\ &= k(1-k)[q(p,a) - q(p,-a)][u'(x(p,a)) - u'(x(p,-a))]\end{aligned}$$

Substituting in (B2), the profit-maximizing price, *p*, is characterized by

$$\begin{aligned} p(a) - c &= \frac{k[1-k]}{E(q_p)}\cdot\frac{[q(p,a)-q(p,-a)][u'(x(p,a))-u'(x(p,-a))]}{ku'(x(p,a))+[1-k]u'(x(p,-a))} \\ &= -\frac{k[1-k]}{E(q_p)}[q(p,a)-q(p,-a)]\frac{\frac{u'(x(p,-a))}{u'(x(p,a))}-1}{[1-k]\frac{u'(x(p,-a))}{u'(x(p,a))}+k} \qquad \text{(B4)} \\ &= -\frac{k}{E(q_p)}[q(p,a)-q(p,-a)]\left[1-\frac{\frac{1}{1-k}}{\frac{u'(x(p,-a))}{u'(x(p,a))}+\frac{k}{1-k}}\right] > 0. \end{aligned}$$

We first show that an increase in the distribution of demand in the sense of first-order stochastic dominance may lead to a lower profit-maximizing usage charge. Suppose that distribution increases to

$$s = \begin{cases} a+\delta & \text{with probability} \quad k \\ -a+\delta & \text{with probability} \quad 1-k \end{cases}, \qquad \text{(B5)}$$

which is an increase in the sense of first-order stochastic dominance. Let $b = \sqrt{q(s+a)}$, which satisfies (1) and implies that $b_{qs} > 0$, and $u(x) = -e^{-rx}$.

Using (B4),

$$p(a+\delta) - c = -\frac{k}{E(q_p(\delta))}[q(p,a+\delta)-q(p,-a+\delta)]\left[1-\frac{\frac{1}{1-k}}{\frac{u'(x(p,-a+\delta))}{u'(x(p,a+\delta))}+\frac{k}{1-k}}\right]. \qquad \text{(B6)}$$

Since $b = \sqrt{q(s+a)}$, we have

$$q(p,s) = \frac{s+a}{4p^2},$$

and so,

$$q(p,a) - q(p,-a) = q(p,a+\delta) - q(p,-a+\delta). \qquad \text{(B7)}$$

Further, we have

$$q_p(p,s) = -\frac{s+a}{2p^3},$$

and, so, for any $\delta > 0$,

$$E(q_p(\delta)) = -\frac{a+E(s(\delta))}{2p^3} = -\frac{\delta + 2ak}{2p^3} < E(q_p(0)) < 0. \tag{B8}$$

Since $u(x) = -e^{-rx}$, then $u'(x) = re^{-rx}$, and so,

$$\frac{u'(x(p,-a+\delta))}{u'(x(p,a+\delta))} = \frac{e^{-rx(p,-a+\delta)}}{e^{-rx(p,a+\delta)}} = \exp\left(-rx(p,-a+\delta) + rx(p,a+\delta)\right). \tag{B9}$$

By (10), we have

$$x(p,s) = b(q,s) - pq + I - T = \frac{s}{4p} + I - T$$

hence,

$$x(p,a+\delta) - x(p,-a+\delta) = x(p,a) - x(p,-a). \tag{B10}$$

By (B9), this implies

$$\frac{u'(x(p,-a+\delta))}{u'(x(p,a+\delta))} = \frac{u'(x(p,-a))}{u'(x(p,a))}. \tag{B11}$$

Thus, substituting from (B7), (B8), and (B11) in (B6), we conclude that $p(a) > p(a+\delta) > 0$, for small δ. Hence an increase in demand leads to a lower usage charge.

Next, we show that, under particular conditions, the profit-maximizing usage charge increases with the degree of uncertainty. Assume that the slope of the ex post demand (marginal benefit) curve, q_p, is constant, therefore the ex post deadweight loss depends on the margin, $p-c$, but not the state, s. Further, assume that the probability $k = 1/2$. Then, an increase in a would amount to a mean-preserving spread of the distribution of demand. Now, $q(p,a) - q(p,-a)$ is increasing in a. Since $u''(.) < 0$ and $x_s > 0$,

$$\frac{u'(x(p,-a))}{u'(x(p,a))}$$

is also increasing in a. Hence, given p, the absolute value of the right-hand side of (B4) is increasing with a. Therefore, if the parameter, a, is larger, the profit-maximizing margin, $p(a)-c$, and usage charge, p, will be higher. Hence, the increase in uncertainty leads to a higher usage charge.

When $b_{qs}(q,s)>0$, as Figure B1 shows, the argument is valid as long as the slope of $\text{Cov}(q,u'(x))/[E(q_p)\cdot E(u'(x))]$ as a function of p is less than 1 (including the slope being negative). Note that the function tends to be decreasing in p, since large p would make it approach zero. If the slope exceeds 1, then an upward shift of the $\text{Cov}(q,u'(x))/[E(q_p)\cdot E(u'(x))]$ curve would result in the profit-maximizing usage charge being lower, as shown in Figure B2.

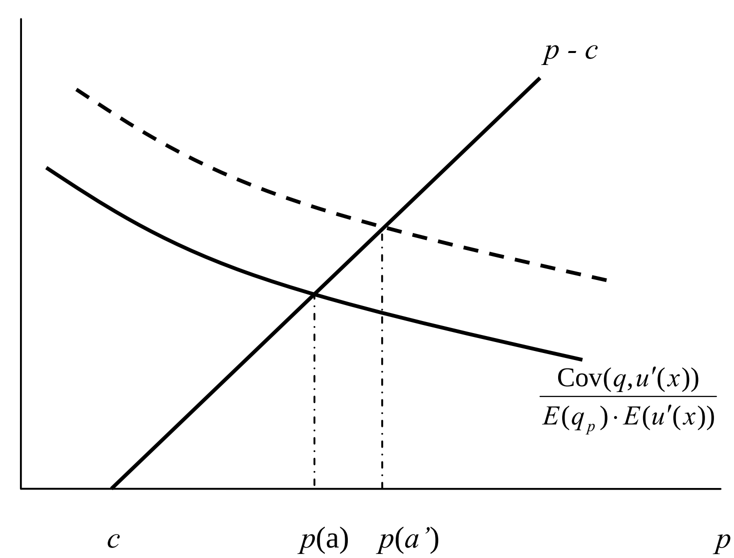

Figure B1

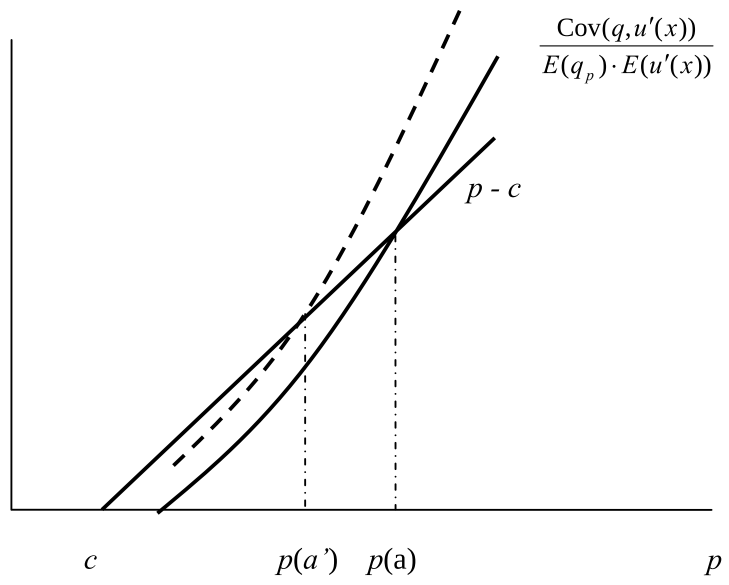

Figure B2

When $b_{qs}(q,s) < 0$, the dashed curve in the figures would be on the other side of the solid curve. The argument for this case is symmetric to that for the case of $b_{qs}(q,s) > 0$.