Appendix

Proof of Observation 1. Differentiating (4) with respect to *c*,

$$-\left[v+h\right]\chi\frac{d^2\alpha}{df^2}\frac{\partial f}{\partial c}=n\,,$$

and hence, by (1),

$$\frac{\partial f}{\partial c}=\frac{n}{-[v+h]\chi\dfrac{d^2\alpha}{df^2}}<0\,. \tag{A1}$$

Differentiating (4) with respect to n,

$$-\left[v+h\right]\chi\frac{d^2\alpha}{df^2}\frac{\partial f}{dn}=c\,,$$

and hence, by (1),

$$\frac{\partial f}{\partial n}=-\frac{c}{\left[v+h\right]\chi\dfrac{d^2\alpha}{df^2}}<0\,. \tag{A2}$$

Differentiating (4) with respect to χ,

$$-\left[v+h\right]\frac{d\alpha}{df}-\left[v+h\right]\chi\frac{d^2\alpha}{df^2}\frac{\partial f}{\partial\chi}=0\,,$$

and hence, by (1),

$$\frac{\partial f}{\partial\chi}=-\frac{d\alpha/df}{\chi\dfrac{d^2\alpha}{df^2}}>0\,. \tag{A3}$$

By (2), $d\chi/dK>0$, hence $\partial f/\partial K>0$. If $K=0$, then $\chi(K)=0$, hence by (4), $f(n\,|\,K=0)=0$, $\forall n\in[0,1]$. Further, if $K\rightarrow\infty$, $\chi(K)\rightarrow 1$, hence, by (4), $\lim_{K\rightarrow\infty} f(n)=f_{\infty}(n)$. [ ]

Proof of Observation 2. We first prove that $B(n)$ is monotone decreasing in n. Consider n_1 and n_2 such that $n_1 < n_2$. Let user n_1 choose the precautions, $f(n_2)$, associated with user n_2. Since $n_1 < n_2$, her expected net benefit would be

$$\begin{aligned} &v-[v+h]\alpha(f(n_2))\chi - p - n_1 cf(n_2) \\ &\qquad > v-[v+h]\alpha(f(n_2))\chi - p - n_2 cf(n_2) \equiv B(n_2 \mid K), \end{aligned} \tag{A4}$$

By (3), the precautions $f(n_1)$ must provide user n_1 with the maximum expected net benefit, and, in particular,

$$\begin{aligned} B(n_1 \mid K) &= v-[v+h]\alpha(f(n_1))\chi - p - n_1 cf(n_1) \\ &\geq v-[v+h]\alpha(f(n_2))\chi - p - n_1 cf(n_2). \end{aligned} \tag{A5}$$

Hence, by (A4) and (A5), $B(n_1|K) > B(n_2 \mid K)$, which is the result.

Since $B(n)$ is monotone decreasing in n, the demand for the service is characterized as follows. Consider the most sophisticated user, $n = 0$. By (3), her cost of precaution is zero and therefore she will choose the highest precaution, i.e., $f(0) \to \infty$. Under the assumption that $v > p$ and by (1), the most sophisticated user would buy since $B(0) = v - p > 0$. Consider the most naïve user, $n = 1$. If $B(1) \geq 0$, then, $B(n) > 0$ for all $n < 1$ and all other users would buy. However, if $B(1) < 0$, the most naïve user does not buy the service, and there exists some critical level as claimed. [ ]

Proof of Observation 3. Differentiating (5) with respect to c,

$$-[v+h]\chi \frac{d\alpha(f(\hat{n}))}{df}\left[\frac{\partial f(\hat{n})}{\partial c} + \frac{\partial f(\hat{n})}{\partial n}\frac{\partial \hat{n}}{dc}\right] - \hat{n}f(\hat{n}) - cf(\hat{n})\frac{\partial \hat{n}}{dc} - c\hat{n}\left[\frac{\partial f(\hat{n})}{\partial c} + \frac{\partial f(\hat{n})}{\partial n}\frac{\partial \hat{n}}{dc}\right] = 0$$

hence, using (4),

$$\frac{\partial \hat{n}}{\partial c} = -\frac{\hat{n}}{c} < 0, \tag{A6}$$

i.e., the marginal user, $\hat{n}$, is decreasing in *c*.

Differentiating (5) with respect to *p*,

$$-[v+h]\chi\frac{d\alpha(f(\hat{n}))}{df}\left[\frac{\partial f(\hat{n})}{\partial p}+\frac{\partial f(\hat{n})}{\partial n}\frac{\partial \hat{n}}{\partial p}\right]-1-cf(\hat{n})\frac{\partial \hat{n}}{\partial p}-\hat{n}c\left[\frac{\partial f(\hat{n})}{\partial p}+\frac{\partial f(\hat{n})}{\partial n}\frac{\partial \hat{n}}{\partial p}\right]=0,$$

hence, using (4),

$$\frac{\partial \hat{n}}{\partial p}=-\frac{1}{cf(\hat{n})}<0 \tag{A7}$$

Differentiating (5) with respect to K,

$$-[v+h]\chi\frac{d\alpha}{df}\left[\frac{\partial f}{\partial K}+\frac{\partial f}{\partial n}\frac{\partial \hat{n}}{\partial K}\right]-[v+h]\alpha(f(\hat{n}))\frac{d\chi}{dK}-\frac{\partial \hat{n}}{\partial K}cf(\hat{n})-c\hat{n}\left[\frac{\partial f}{\partial K}+\frac{\partial f}{\partial n}\frac{\partial \hat{n}}{\partial K}\right]=0,$$

hence, using (4),

$$\frac{\partial \hat{n}}{\partial K}=-\frac{[v+h]\alpha(f(\hat{n}))}{cf(\hat{n})}\frac{d\chi(K)}{dK}<0 \tag{A8}$$

Further differentiating (A8) with respect to *K*,

$$\frac{\partial^2 \hat{n}}{\partial K^2}=\frac{-[v+h]\alpha(f(\hat{n}))}{cf(\hat{n})}\frac{d^2\chi}{dK^2}-\frac{[v+h]}{cf^2}\frac{d\chi}{dK}\left[\frac{\partial f}{\partial K}+\frac{\partial f}{\partial n}\frac{\partial \hat{n}}{\partial K}\right]\left[f\frac{d\alpha}{df}-\alpha\right]>0, \tag{A9}$$

which follows from (1), (2), (A2), (A3), and (A8). By (A8) and (A9), $\hat{n}$ is decreasing and convex in *K*.

If $K\to 0$, then $\chi(K)\to 0$. Hence by (3), users' expected net benefit, $B(n)\to v-p-ncf(n)$, which is maximized with $f(n)=0$. Thus $B(n)\to v-p$, for all *n*. Since $v>p$, all users buy the service. Accordingly, if $K\to 0$, then $\hat{n}\to 1$.

Now, if $K\to\infty$, then $\chi(K)\to 1$, hence, by (3), users' expected net benefit, $B(n)\to v-[v+h]\alpha(f(n))-p-ncf(n)$. As proved by Observation 2, the most sophisticated user would buy the service, i.e., $B(0\,|\,K\to\infty)>0$. Consider the user with $n=1$. If her expected net benefit, $B(1) \to v - [v+h]\alpha(f(1)) - p - cf(1) \geq 0$, then by Observation 2, $B(n) > 0$ for all *n*. Hence all users will buy the service. Otherwise, if $B(1) < 0$, then there exists some $\hat{n}_0$ such that

$$B(\hat{n}_0) \to v - [v+h]\alpha(f(\hat{n}_0)) - p - \hat{n}_0\, cf(\hat{n}_0) = 0 ,$$

which completes the proof.

Proof of Observation 4. To simplify notation, define

$$A(K) = \int_0^{\hat{n}(K)} \alpha(f(n) \mid K) d\Phi(n) . \qquad \text{(A10)}$$

Since $d\alpha/df < 0$, then $\partial A/\partial f < 0$. Further $\partial \mathrm{A}/\partial \hat{n} > 0$. Substituting (A10) in (8), and then differentiating with respect to η,

$$eA\left[-\frac{d\chi}{dk_i} + [1-\eta]\frac{d^2\chi}{dk_i^2}\frac{\partial k_i}{\partial \eta}\right] = \frac{d^2 c_K}{dk_i^2}\frac{\partial k_i}{\partial \eta} ,$$

which simplifies to

$$\frac{\partial k_i}{\partial \eta} = \frac{e\dfrac{d\chi}{dk_i}A}{e[1-\eta]A\dfrac{d^2\chi}{dk_i^2} - \dfrac{d^2 c_K}{dk_i^2}} < 0 . \qquad \text{(A11)}$$

Similarly, differentiating (8) with respect to $\hat{n}$,

$$e\left[1-\eta\right]\left[\frac{\partial \mathrm{A}}{\partial \hat{n}}\frac{d\chi}{dk_i} + A(\hat{n})\frac{d^2\chi}{dk_i^2}\frac{\partial k_i}{\partial \hat{n}}\right] = \frac{d^2 c_K}{dk_i^2}\frac{\partial k_i}{\partial \hat{n}} ,$$

which simplifies to

$$\frac{\partial k_i}{\partial \hat{n}} = \frac{e\left[1-\eta\right]\dfrac{\partial \mathrm{A}}{\partial \hat{n}}\dfrac{d\chi}{dk_i}}{\dfrac{d^2 c_K}{dk_i^2} - e\left[1-\eta\right]\mathrm{A}(\hat{n})\dfrac{d^2\chi}{dk_i^2}} > 0 . \qquad \text{(A12)}$$

When $\hat{n}=0$, no one buys the service, it doesn't pay for the hackers to attack the service, hence $K=0$. When $\hat{n}=1$, all users buy the service. Since the hacker's expected net benefit, (7), is concave in k_i, there exists $\tilde{k}>0$ that satisfies the first order condition, (8), and maximizes the expected net benefit.

Similarly, we can show that

$$\frac{\partial k_i}{\partial f}=\frac{e[1-\eta]\frac{\partial \mathrm{A}}{\partial f}\frac{d\chi}{dk_i}}{\frac{d^2 c_K}{dk_i^2}-e[1-\eta]\mathrm{A}(\hat{n})\frac{d^2\chi}{dk_i^2}}<0, \tag{A13}$$

and that there exists some $k_0>0$ such that if $f=0$, then $k_i=k_0$, and there exists some $k_\infty>0$ such that if $f\to\infty$, then $k_i=k_\infty$.

Proof of Lemma 1. By Observations 1 and 3 respectively, f is increasing in K and $\hat{n}$ is decreasing in K. Accordingly, $A(K)$ is monotonically decreasing in K, regardless of the user distribution $\Phi(n)$. Further, if $K=0$, then by (2), $\chi=0$, hence all users would choose $f(n)=0$ and, by (3), get $B(n\,|\,K)=v-p$. By assumption, $v-p>0$, hence, if all $k_i=0$, then $K=0$, and $\hat{n}=1$, and so, $A>0$.

With regard to hacker targeting, by Observation 4, k_i is monotonically increasing in A. Further, if $A=0$ (because either $\hat{n}=0$ or $\alpha(f(n))=0$, for all n), then hackers will not target the service, $k_i=0$.

Figure 4.3 depicts $k_i(A)$ and $\mathrm{A}(k)$, which describe the best response functions of the hackers and users, respectively. Since the functions are continuous, they have a non-trivial intersection, say (k_i^*, A^*).

Given hacker targeting $k_1^*,...,k_Z^*$, let $K^* \equiv k_1^* + ... + k_Z^*$, and further, let $\hat{n}(K^*)$ and $f(n \mid K^*)$ be the marginal user and user precautions respectively. Then, by (9), the conditional vulnerability

$$A' = \int_0^{\hat{n}(K^*)} \alpha(f(n) \mid K^*) d\Phi(n) .$$

Now, we claim that $A^* = A'$, and prove the claim by contradiction as follows.

(i) Suppose otherwise that $A' > A^*$. Then, referring to Figure 4.3, the function $k_i(A)$ gives the hacker's best-response k_i'. Since $k_i(A)$ is monotonically increasing in A, we have $k_i' > k_i^*$, and so, $K' \equiv k_1^* + ... + k_i' + ... + k_Z^* > k_1^* + ... + k_i^* + ... + k_Z^* = K^*$. Since $\hat{n}$ is decreasing in K and $f(.)$ is increasing in K, it follows that $\hat{n}(K') < \hat{n}(K^*)$ and $f(n \mid K') > f(n \mid K^*)$, which implies that $A' < A^*$, which contradicts the original assumption.

(ii) Suppose otherwise that $A' < A^*$. Then, referring to Figure 4.3, the function $k(\mathrm{A})$ gives the hacker's best-response k'. Since $k(\mathrm{A})$ is monotonically increasing in A, we have $k_i' < k_i^*$, and so, $K' \equiv k_1^* + ... + k_i' + ... + k_Z^* < k_1^* + ... + k_i^* + ... + k_Z^* = K^*$. Since $\hat{n}$ is decreasing in K and $f(.)$ is increasing in K, it follows that $\hat{n}(K') > \hat{n}(K^*)$ and $f(n \mid K') < f(n \mid K^*)$, which implies that $A' > A^*$, which contradicts the original assumption.

Therefore, we must have $A^* = A'$. In symmetric equilibrium, $k_i^* = k^*$, $i = 1,...,Z$. Hence, there exists a non-trivial equilibrium comprising k^*, $\hat{n}(k^*)$ and $f(\hat{n} \mid k^*)$. [ ]

Proof of Proposition 1. Expand (8) to distinguish between the precaution of end-user n' denoted $f(n')$ and the precautions of all other users, f,

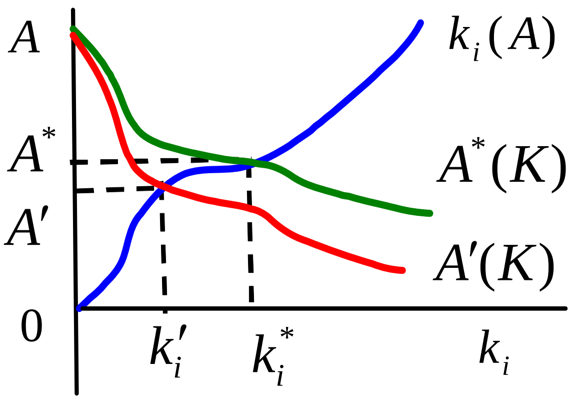
$$e[1-\eta]\frac{d\chi}{dk_i}\left[\int_{[0,n')}\alpha(f(n))d\Phi(n)+\alpha(f(n'))d\Phi(n')+\int_{(n',\hat{n}]}\alpha(f(n))d\Phi(n)\right]=\frac{dc_K}{dk_i}. \qquad \text{(A14)}$$

By (A14), an increase in precautions, f, by all other users except n' would reduce the term in brackets, and hence induce all hackers to reduce targeting, $\Delta k_i < 0$, all i. This would imply $\Delta\chi < 0$, which in (4), shifts down the left-hand side. Therefore, user n' would reduce $f(n')$. [ ]

Proof of Proposition 2. This follows directly from the proof of Table 4.2, by noting that (A16) will hold, and hence $\partial A/\partial c \geq 0$, if c is sufficiently high, and not hold if c is sufficiently low.

Proof of Table 4.2

Figure 4A Increase in price, *p*



<u>User cost of precaution, c</u>

By Observations 1 and 4, an increase in the user cost of precaution, c, directly leads to reduced user precautions, f, and service demand, $\hat{n}$. By (9), these have mixed effects on the users' best-response function, $A(K)$. By (8), the increase in the user cost of precaution has no direct effect on $k(A)$. Accordingly, the net effect on targeting, k, and conditional vulnerability, A, depends on the sign of $\partial A/\partial c$, which is calculated as follows,

$$\frac{\partial A(K)}{\partial c} = \alpha(f(\hat{n}))\frac{\partial \hat{n}}{\partial c}\frac{d\Phi(\hat{n})}{dn} + \int_0^{\hat{n}} \frac{d\alpha}{df}\frac{\partial f(n)}{\partial c}d\Phi(n) \tag{A15}$$

Substituting from (4) and (A1), it follows that $\partial A/\partial c \geq 0$ if and only if

$$-\frac{1}{[v+h]\chi}\int_0^{\hat{n}} \frac{n\cdot d\alpha/df}{d^2\alpha/df^2}d\Phi(n) \geq \frac{\hat{n}\alpha(f(\hat{n}))}{c}\frac{d\Phi(\hat{n})}{dn}$$

or

$$c \geq \frac{[v+h]\chi\alpha(f(\hat{n}))\hat{n}}{-\int_0^{\hat{n}} \frac{n\cdot d\alpha/df}{d^2\alpha/df^2}d\Phi(n)}\frac{d\Phi(\hat{n})}{dn}. \tag{A16}$$

We analyze two cases below.

(i) $\partial A/\partial c \geq 0$. Referring to Figure 4A, an increase in c would lead to a new equilibrium, with higher targeting, $k_i' \leq k_i^*$, higher conditional vulnerability, $A' \leq A^*$, and hence higher effective vulnerability, $\chi(K')A' \leq \chi(K^*)A^*$, where $K' = k_1' + ... + k_Z'$ and $K^* = k_1' + ...k_{i-1}' + k_i^* + k_{i+1}' + ... + k_Z'$. In sum, when $\partial A/\partial c \geq 0$, we must have $dk_i/dc \geq 0$, all *i*, and $dA/dc \geq 0$.

With regard to the marginal user, i.e., service demand,

$$\frac{d\hat{n}}{dc} = \frac{\partial \hat{n}}{\partial c} + \frac{\partial \hat{n}}{\partial K}\frac{dK}{dc} = \frac{\partial \hat{n}}{\partial c} + \frac{\partial \hat{n}}{\partial K}\left[\frac{dk_1}{dc} + ... + \frac{dk_Z}{dc}\right]. \tag{A17}$$

By Observation 3, $\partial \hat{n}/\partial c < 0$ and $\partial \hat{n}/\partial K < 0$, while from above, $dk_i/dc \geq 0$, for all *i*. Hence, substituting in (A17), we have $d\hat{n}/dc < 0$.

Regarding the precautions, from above, $A' \leq A^*$, hence by (9).

$$\frac{dA}{dc} = \alpha(f(\hat{n}))\frac{d\hat{n}}{dc}\frac{d\Phi(\hat{n})}{dn} + \int_0^{\hat{n}} \frac{d\alpha}{df}\frac{df(n)}{dc}d\Phi(n) \geq 0. \tag{A18}$$

Now, $d\hat{n}/dc < 0$, hence, substituting in (A18), it follows that $df/dc < 0$.

(ii) $\partial A/\partial c < 0$. Referring to Figure 4A, an increase in c would lead to a new equilibrium, with lower targeting, $k_i' > k_i^*$, lower conditional vulnerability, $A' > A^*$, and hence lower effective vulnerability, $\chi(K')A' > \chi(K^*)A^*$, where $K' = k_1' + ... + k_Z'$ and $K^* = k_1' + ...k_{i-1}' + k_i^* + k_{i+1}' + ... + k_Z'$. In sum, when $\partial A/\partial c < 0$, we must have $dk_i/dc < 0$, all *i*, and $dA/dc < 0$.

With regard to user precautions,

$$\frac{df}{dc} = \frac{\partial f}{\partial c} + \frac{\partial f}{\partial K}\frac{dK}{dc} = \frac{\partial f}{\partial c} + \frac{\partial f}{\partial K}\left[\frac{dk_1}{dc} + ... + \frac{dk_Z}{dc}\right]. \tag{A19}$$

By Observation 1, $\partial f/\partial c < 0$ and $\partial f/\partial K > 0$, while from above, $dk_i/dc < 0$, for all *i*. Hence, substituting in (A19), we have $df/dc < 0$.

Regarding the marginal user, from above, $A' > A^*$, hence by (9),

$$\frac{dA}{dc} = \alpha(f(\hat{n}))\frac{d\hat{n}}{dc}\frac{d\Phi(\hat{n})}{dn} + \int_0^{\hat{n}} \frac{d\alpha}{df}\frac{df(n)}{dc}d\Phi(n) < 0. \tag{A20}$$

Now, $df/dc < 0$, hence, substituting in (A20), it follows that $d\hat{n}/dc < 0$.

<u>*Enforcement rate,* η*, and hacking cost,* $c_K(.)$</u>

First, consider the effect of an increase in enforcement, η. By Observations 1 and 3, the increase in enforcement has no direct effect on users' precautions or demand $\hat{n}$. Hence, by (9), the best-response function $A(k)$ remains unchanged. By Observation 4, the enforcement increase directly leads hackers to reduce targeting, hence their best-response function, $k_i(A)$, shifts to the left. Accordingly, in the new equilibrium, targeting is lower, $k_i' > k_i^*$, and the conditional vulnerability is higher, $A' < A^*$.

Since the increase in enforcement results in lower targeting, k_i, hence lower hacker effectiveness, $\chi(K)$, but higher conditional vulnerability, A, the impact on the effective user vulnerability, χA, depends on the balance of the effects on hackers and users.

With regard to user precautions,

$$\frac{df}{d\eta} = \frac{\partial f}{\partial \eta} + \frac{\partial f}{\partial K}\frac{dK}{d\eta}. \qquad \text{(A21)}$$

By (4), $\partial f / \partial \eta = 0$, by Observation 1, $\partial f / \partial K > 0$, while from above, $dK / d\eta < 0$. Hence, substituting in (A21), we have $df / d\eta < 0$.

Similarly, with regard to the marginal user, i.e., service demand,

$$\frac{d\hat{n}}{d\eta} = \frac{\partial \hat{n}}{\partial \eta} + \frac{\partial \hat{n}}{\partial K}\frac{dK}{d\eta}. \qquad \text{(A22)}$$

By (5), $\partial \hat{n} / \partial \eta = 0$, by Observation 3, $\partial \hat{n} / \partial K < 0$, while from above, $dK / d\eta < 0$. Hence, substituting in (A22), we have $d\hat{n} / d\eta > 0$, which completes the proof.

The effect of an increase in the targeting cost is similar. For brevity, we omit the proof.

Price, p

By Observation 1, a price increase has no direct effect on user precautions, while, by Observation 3, the price increase directly reduces the demand, $\hat{n}$. Accordingly, by (9), for $k_i > 0$, the best-response function $A(k)$ shifts downward, while, by (9), for $k_i = 0$, $A(0)$ does not change with *p*. By (8), the price increase has no direct effect on $k_i(A)$.

Figure 4A depicts the new equilibrium: the users' best-response function shifts from $A^*(K)$ downward to $A'(K)$, while the hackers' best-response function remains unchanged. In the new equilibrium, targeting is lower, $k_i^* > k_i'$, and the conditional vulnerability is lower, $A^* > A'$.

Given that the increase in price, p, leads to lower targeting, k, it would, by (2) result in lower hacker effectiveness, χ. Thus, the effective user vulnerability, χA, decreases with price, *p*.

With regard to user precautions,

$$\frac{df}{dp} = \frac{\partial f}{\partial p} + \frac{\partial f}{\partial K}\frac{dK}{dp}. \qquad \text{(A23)}$$

By (4) $\partial f / \partial p = 0$, by Observation 1, $\partial f / \partial K > 0$, while from above, $dK / dp < 0$. Hence, substituting in (A23), we have $df / dp < 0$.

Regarding the marginal user, from above, $A^* > A'$, hence, by (9),

$$\frac{dA}{dp} = \alpha(f(\hat{n}))\frac{d\hat{n}}{dp}\frac{d\Phi(\hat{n})}{dn} + \int_0^{\hat{n}} \frac{d\alpha}{df}\frac{df(n)}{dp} d\Phi(n) < 0. \qquad \text{(A24)}$$

From above, $df / dp < 0$, hence substituting in (A24), it follows that $d\hat{n} / dp < 0$, which completes the proof. [ ]

Proof of Proposition 3. By assumption, $\partial A/\partial c > 0$, hence $dK/dc > 0$ and $dW/dc < 0$. By (12) and (14), $|dW/dc| > |dW/d\eta|$ if and only if

$$\int_0^{\hat{n}} nf(n)d\Phi(n) > p\frac{d\Phi(\hat{n})}{dn}\left[\frac{d\hat{n}}{d\eta}+\frac{d\hat{n}}{dc}\right]-\left[v+h\right]\frac{d\chi(K)}{dK}\left[\frac{dK}{d\eta}+\frac{dK}{dc}\right]A, \tag{A25}$$

where

$$\frac{d\hat{n}}{d\eta}=\frac{\partial\hat{n}}{\partial K}\frac{dK}{d\eta}, \tag{A26}$$

$$\frac{d\hat{n}}{dc}=\frac{\partial\hat{n}}{\partial c}+\frac{\partial\hat{n}}{\partial K}\frac{dK}{dc}, \tag{A27}$$

$$\frac{dK}{d\eta}=\frac{\partial K}{\partial\eta}+\frac{\partial K}{\partial A}\frac{dA}{d\eta}, \tag{A28}$$

$$\frac{dA}{d\eta}=\frac{\partial A}{\partial\eta}+\frac{\partial A}{\partial K}\frac{dK}{d\eta}=\frac{\partial A}{\partial K}\frac{dK}{d\eta}. \tag{A29}$$

Substituting from (A6) and (A8) in (A27),

$$\frac{d\hat{n}}{dc}=-\frac{\hat{n}}{c}-\frac{\left[v+h\right]\alpha(f\left(\hat{n}\right))}{cf\left(\hat{n}\right)}\frac{d\chi(K)}{dK}\frac{dK}{dc}. \tag{A30}$$

Further, by differentiating (8) with respect to c,

$$\frac{dk_i}{dc}=\frac{e[1-\eta]\dfrac{d\chi}{dk_i}\dfrac{dA}{dc}}{\dfrac{d^2c_K}{dk_i^2}-e[1-\eta]A\dfrac{d^2\chi}{dk_i^2}}. \tag{A31}$$

In symmetric equilibrium, $k_i^*=k$, $i=1,\ldots,Z$, hence, by substituting from (A31),

$$\frac{dK}{dc}=\frac{dk_1}{dc}+\ldots+\frac{dk_Z}{dc}=Z\frac{dk_i}{dc}=\frac{eZ[1-\eta]\dfrac{d\chi}{dk}\dfrac{dA}{dc}}{\dfrac{d^2c_K}{dk^2}-e[1-\eta]A\dfrac{d^2\chi}{dk^2}}. \tag{A32}$$

Similarly, in symmetric equilibrium, by (A11),

$$\frac{\partial K}{\partial \eta}=\frac{\partial k_1}{\partial \eta}+\ldots+\frac{\partial k_Z}{\partial \eta}=Z\frac{\partial k_i}{\partial \eta}=\frac{-eZ\frac{d\chi}{dk}A}{\frac{d^2c_K}{dk^2}-e[1-\eta]A\frac{d^2\chi}{dk^2}}. \tag{A33}$$

Substituting from (A29) in (A28), and then substituting from (A33), we have

$$\frac{dK}{d\eta}=\frac{\frac{\partial K}{\partial \eta}}{1-\frac{\partial K}{\partial A}\frac{\partial A}{\partial K}}=\frac{\frac{eZ\frac{d\chi}{dk}A}{e[1-\eta]A\frac{d^2\chi}{dk^2}-\frac{d^2c_K}{dk^2}}}{1-\frac{\partial K}{\partial A}\frac{\partial A}{\partial K}}. \tag{A34}$$

By substituting from (A26), (A8) and (A30), the sufficient condition (A25) simplifies to

$$p\frac{d\Phi(\hat{n})}{dn}\frac{\hat{n}}{c}+\int_0^{\hat{n}}nf(n)d\Phi(n)>-\left[v+h\right]\frac{d\chi(K)}{dK}\left[p\frac{d\Phi(\hat{n})}{dn}\frac{\alpha(f(\hat{n}))}{cf\left(\hat{n}\right)}+A\right]\left[\frac{dK}{d\eta}+\frac{dK}{dc}\right]. \tag{A35}$$

Further substituting from (A32) and (A34) in (A35), and then simplifying, we have

$$c\left\{\frac{p\frac{d\Phi(\hat{n})}{dn}\frac{\hat{n}}{c}+\int_0^{\hat{n}}nf(n)d\Phi(n)}{p\frac{d\Phi(\hat{n})}{dn}\frac{\alpha(f\left(\hat{n}\right))}{cf\left(\hat{n}\right)}+A}\right\}\left\{\frac{\frac{d^2c_K}{dk^2}-e[1-\eta]A\frac{d^2\chi}{dk^2}}{eZA\left[v+h\right]\left[\frac{d\chi}{dK}\right]^2}\right\}>\frac{c}{1-\frac{\partial K}{\partial A}\frac{\partial A}{\partial K}}-[1-\eta]\frac{c}{A}\frac{dA}{dc}. \tag{A36}$$

Now,

$$\frac{dA}{dc}=\frac{\partial A}{\partial c}+\frac{\partial A}{\partial K}\frac{dK}{dc}=\frac{\partial A}{\partial c}+\frac{\partial A}{\partial K}\frac{dK}{dA}\frac{dA}{dc},$$

which implies

$$\frac{dA}{dc}=\frac{\frac{\partial A}{\partial c}}{1-\frac{\partial K}{\partial A}\frac{\partial A}{\partial K}}. \tag{A37}$$

Substituting from (5) and (A37) in (A36), then multiplying both sides by $1/\left[1-\eta\right]$, and then substituting from (8), and simplifying, the sufficient condition simplifies to

$$c\left\{\frac{\frac{p\frac{d\Phi(\hat{n})}{dn}}{cf(\hat{n})}\left[\frac{v-p}{c}-\frac{[v+h]\alpha(f(\hat{n}))\chi}{c}\right]+\int_0^{\hat{n}} nf(n)d\Phi(n)}{\frac{p\frac{d\Phi(\hat{n})}{dn}}{cf(\hat{n})}\alpha(f(\hat{n}))+A}\right\}\left\{\frac{\frac{d^2c_K}{dk^2}}{\frac{dc_K}{dk}}+\frac{e[1-\eta]A}{\frac{dc_K}{dk}}\left[-\frac{d^2\chi}{dk^2}\right]\right\}$$

$$>\frac{\left[\frac{c}{1-\eta}-\frac{c}{A}\frac{\partial A}{\partial c}\right]}{1-\frac{\partial K}{\partial A}\frac{\partial A}{\partial K}}Z[v+h]\frac{d\chi}{dK}. \tag{A38}$$

Condition (A38) will be satisfied if the users' benefit relative to the cost of precaution, $[v-p]/c$, and the hackers' expected enjoyment relative to targeting cost, $e[1-\eta]\Big/\frac{dc_K}{dk}$, are sufficiently large. []