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RESERVATIONS: CUSTOMER INSURANCE IN THE MARKETING OF CAPACITY

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This paper develops a simple model of pricing and choice of capacity in a market where risk-averse customers are *ex-ante* uncertain about their valuation for use of the capacity. As unused capacity has no salvage value, the seller maximizes profit by economizing on capacity through over-booking. Thus each customer faces both uncertainty in his own valuation as well as uncertainty about availability of capacity. Because he is risk-averse, the customer seeks insurance.

Provision of insurance is complicated because each customer's realized valuation is private information. Since the seller does not know each customer's type, she must elicit the information. The most profitable pricing strategy takes the form of a reservation that induces each customer to act according to his private information: those with high valuation for use of the capacity exercise while those with lower valuation do not exercise. In this way, the reservation provides partial insurance against the customer's uncertainty in valuation (privately observed contingency) and full insurance against unavailability of capacity (observable by both seller and customer).
(Capacity; Insurance; Pricing)

1. Introduction

Whenever risk-averse individuals face uncertainty, they have demand for insurance. For many centuries, people have bought insurance against injury and death, and property-owners have bought insurance against theft or damage to their property by fire. More recently, markets have developed to provide insurance against fluctuations in the prices of commodities and common stock. Such insurance is commonly provided through options to buy (call options) and options to sell (put options).

In marketing too, options to buy are ubiquitous: an airline seat reservation is an option for the customer to take up a seat at a fixed price on a specified flight, which option must be exercised no later than a set time. Likewise, reservations of hotel rooms, rental cars, and restaurant tables are options to buy. This paper seeks to explain the wide use of reservations in the marketing of capacity.

Capacity such as airline seats, hotel rooms, rental cars, and restaurant tables have one important element in common—if at the end of the day, they are unused, their use cannot be taken into inventory. The perishability cuts two ways: neither the seller nor the buyer can store the capacity. I shall argue that these examples share another crucial property: the customer is not certain about his value for the capacity until close to the time that he intends to use it. Since the capacity is perishable, the seller will try to avoid excess capacity, and indeed, will maximize profits by over-booking.

Thus, the customer faces two sources of uncertainty: first, his own random value for

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the capacity, and secondly, the availability of capacity at the time of use. A risk-averse customer will seek insurance. The provision of such insurance, however, is complicated because unlike changes in the market price of a commodity or common stock, the customer's personal value for the capacity is a contingency that is difficult for third parties to verify.

The objective of this paper is to characterize the pricing strategy and choice of capacity that maximize the seller's expected profit. It will be shown that the pricing strategy takes the form of a reservation that the seller offers at no charge, in contrast to financial options. The reservation is designed so that the customer exercises it only if his value for the capacity is high. It provides each customer with full insurance against unavailability of capacity but only partial insurance against uncertainty in valuation.

In my framework, provision of insurance is difficult because the insured contingency is private information. The problem addressed in Rothschild and Stiglitz (1976) differs in a crucial way: in their setting, each individual's innate probability of loss is private information, but the amount of the loss if incurred (the insured contingency) can be observed by both insurer and insured. Hence the seller must design a menu of insurance contracts to induce self-selection by the individuals at the time of contracting.

Moorthy (1984), and Gerstner and Hess (1987) also apply the self-selection approach to analyze the problem of a seller faced with customers who have different valuations for a product, and where each customer's valuation is private information. The common thread in the self-selection analyses is that one party is uninformed about the *ex-ante* type of the other.

The issues in my paper are more closely related to analyses of the principal-agent problem. In Basu, Lal, Srinivasan, and Staelin (1985), a risk-neutral party (sales manager) cannot observe the effort exerted by a risk-averse party (salesperson). The optimal contract between the parties bases the salesperson's compensation on observable sales, and balances inducement of effort against insurance against risk.[1]

Lal and Staelin (1986) combine the principal-agent and self-selection approaches to generalize Basu et al. In this setting, salespersons are *ex-ante* heterogeneous in their selling ability, and, once employed, each salesperson's effort is unobservable. Hence, the sales manager is uniformed about the (pre-contract) type of the salesperson, as well as the post-contract action that the salesperson will choose. Lal and Staelin show that it is optimal for the sales manager to offer a menu of compensation contracts, each of which provides the compromise between incentive for effort and insurance appropriate for a particular type of salesperson.

In the present paper, the parties, seller and customers, have symmetric information *ex-ante*: it is only after they have contracted that the customer receives private information about his valuation of the seller's capacity. So the profit-maximizing contract provides incentive for customers to choose to exercise their reservation or not according to their new information, i.e., the contract induces self-selection after the contract is signed.

In other related work, Pasternack (1985) analyzes the optimal return policy for the manufacturer of a perishable product as a newspaper who has a single distributor and where the objective is to maximize total profit of the manufacturer and distributor. In Pasternack's model, the uncertainty concerns total demand and both members of the channel are risk-neutral, whereas the focus of the present paper will be on the nature of pricing strategies where the composition of demand is uncertain and where the customers are risk-averse.

2. Model

To be concrete, consider the following situation. A man in Los Angeles would like to visit a friend in San Francisco over the next long weekend, but, alas, his friend will not

[1] See Grossman and Hart (1983) for a definitive analysis of the principal-agent problem.

know whether she will be free until the Monday of that week. The man would prefer to travel to San Francisco by the Friday 6 PM flight on a particular airline. So let the capacity under consideration be seats on a particular scheduled airline flight.

There are n such customers, each with identical risk-averse Neumann-Morgenstern utility function, $U(\cdot)$, where $U(0) = 0$, $U'(\cdot) > 0$, and $U''(\cdot) < 0$. At time 1, all customers are subject to the same uncertainty about a visit to San Francisco. Let the customers' expected utility, seller's expected revenue, and other variables at time 1 be called the *ex-ante* customer utility, seller's revenue, *etc.*[2] At time 1, each customer must choose one of two plans—either visit the San Francisco friend or spend the weekend in some alternative activity that involves no uncertainty. Let $U(\bar{v})$ be the customer's utility from the non-contingent alternative net of any price that must be paid.[3]

At time 2, each customer will learn whether his friend will be free, but by then, the noncontingent alternative yielding $U(\bar{v})$ will no longer be available. Suppose that the customer's friend will be free. If a seat on the preferred flight is available, the customer will derive value v_{ha} from the seat. This value is measured before subtracting any price that he must pay for travel. If seats are unavailable, the customer must travel by some less desired means and will receive value v_{hu}, which is measured net of any price that must be paid for the alternative means of travel. Quite naturally, $v_{hu} < v_{ha}$.

Suppose that his friend will not be free. Assume that the customer receives value v_l from the best alternative way to spend the weekend available *after* hearing from the San Francisco friend. For simplicity, I assume that this value does not depend on whether the customer can obtain a seat on the particular flight. If the customer travels to San Francisco, he will enjoy v_l less any price paid, while if he does not, he will enjoy v_l net of any price paid for the alternative.[4]

I assume that the customer strictly prefers his friend to be free in the sense that

ASSUMPTION 1. $v_l < v_{hu}$, and thus, $v_l < v_{hu} < v_{ha}$. Further, I assume that the *ex-ante* noncontingent alternative is relatively attractive in the sense that

ASSUMPTION 2. $v_{hu} < \bar{v} < v_{ha}$.

Call customers whose friends turn out to be free, type-h (high-valuation) customers, and the remainder, type-l (low-valuation) customers. Assume that customers' types, h and l, are independently and identically distributed, l with probability $\lambda \in (0, 1)$, and h with probability $(1 - \lambda)$. Each customer's type is private information and, in particular, information that other customers and the seller cannot obtain. By Assumptions 1 and 2,

$$\lambda U(v_l) + (1 - \lambda) U(v_{hu}) < U(\bar{v}). \tag{1}$$

Besides the (potential) customers, there is an unlimited number of "speculators." Speculators do not really seek to fly, but their presence constrains the seller from giving rebates to those who do not wish to fly. They have the same utility function $U(\cdot)$ as customers. At time 1 each speculator has a noncontingent alternative that yields utility $U(\bar{v})$. At time 2, each speculator's value of a seat is v_l less any price that must be paid; this valuation is not subject to *ex-ante* uncertainty. If the speculator does not get a seat, he obtains value v_l net of any price that must be paid for the alternative activity available at time 2. Any person who purchases a product of the seller will be called a 'purchaser,' whether he be customer or speculator.

[2] *Ex-ante* because information is released to the customers only at the following stage.

[3] In principal-agent models, a similar assumption specifies the agent's opportunity cost of working for the principal.

[4] To simplify notation for activities other than taking the particular flight, I adopt the convention of measuring the customer's value from these activities *net* of any price that must be paid. All values from the particular flight are measured *gross* of any price paid.

The seller is risk-neutral, and faces a two-stage problem: first at time 0, she must select a capacity k to offer. Let the cost of providing k units of capacity be $C(k)$, with $C(0) = 0$, and assume that $C(\cdot)$ be sufficiently small that

ASSUMPTION 3.

$$C(n) < (1-\lambda)n\left\{v_{ha} - U^{-1}\left[\frac{U(\bar{v}) - \lambda U(v_l)}{1-\lambda}\right]\right\}.$$

This ensures that it will be profitable for the seller to be in business and offer some capacity.[5] Let the marginal cost of capacity, $MC(k)$, be nondecreasing in k.

Given the capacity k, the seller must choose a pricing strategy. Assume that the marginal cost of use of capacity is zero, and that the capacity has no salvage value—any seats unsold after the flight has taken off are worthless. Hence the seller will maximize profit by setting prices to maximize expected revenue.

In practice, many airlines adjust their ticket prices at various times up to the time of the flight, for instance, some offer cheap stand-by fares just before takeoff. Other airlines commit in advance to fares and will not vary them at any time. To allow for both possibilities, I shall consider both (a) pricing strategies in which the seller can commit to time-2 prices at time 1 when customers must choose between planning to visit San Francisco and the non-contingent alternative, and (b) pricing strategies in which the seller cannot commit in advance and can set time-2 prices only at time 2.[6]

If the seller can set time-2 prices only at time 2, she essentially sells by Spot Sale, illustrated in Figure 1. By assumption, at time 1, the seller does nothing. Next at time 2, the seller sets a price p to any purchaser who wishes to take a seat on the flight. At that time, each customer and speculator must decide whether to buy. If the number of purchasers seeking seats is larger than the available capacity, the available seats are allocated by lottery. Those purchasers who do not obtain seats pay nothing.

If the seller can set time-2 prices at time 1, she has two alternatives. One is a *Firm Advance Order*, shown in Figure 2. At time 1, the seller offers a contract that specifies that at time 2, the holder pay p_u if a seat is unavailable or pay p_a if a seat is available.[7] Each customer and speculator must decide whether to purchase an order at time 1. At time 2, if the number of order-holders exceeds the available capacity, the available seats are allocated by lottery. The firm advance order includes as a special case, $p_u = p_a$, i.e., a contract where the customer simply pays a fixed price regardless of whether he obtains a seat.

Alternatively, with pre-commitment, the seller could offer a *Reservation* (Figure 3). At time 1, the seller offers a contract that specifies that the holder has the right to exercise at time 2 under the condition that the holder pay p_d if he does not exercise, or pay p_{eu} if he exercises but a seat is unavailable, or pay p_{ea} if he exercises and a seat is available.[8] At time 2, each customer and speculator must decide whether to exercise. If the number of reservation-holders who exercise exceeds the available capacity, the available seats are allocated by lottery. The reservation includes as a special case, $p_d = p_{eu} = 0$, where at time 1, the seller commits to charge the customer at time 2 p_{ea} only if the customer wishes to fly and a seat is available, and to charge (or pay) the customer nothing otherwise.

I assume that, at each decision point, the seller will choose so as to maximize profit from that point onward,[9] and further that the seller will not renege on any terms of a

[5] This is a technical assumption to ensure that costs are less than expected revenue.

[6] The issue of commitment arises also in pricing of durable goods; for instance, some lithographers destroy their plates in order to credibly commit not to sell at lower prices in subsequent periods. See Moorthy (1988).

[7] The price p_u may be negative.

[8] The price p_{eu} might be negative.

[9] This is the condition of sequential rationality (Kreps and Wilson 1982).

contract with a purchaser. In addition, I assume that the purchasers do not haggle with the seller and that purchasers may not re-sell seats. For convenience, I adopt the protocol that if a pricing strategy leaves the customer indifferent between two alternatives, the customer will decide according to the seller's preference. In practice, the seller could adjust the prices slightly to provide the customer with a strict preference.

In the following section, I will show that (i) spot sales and firm advance orders do not maximize the seller's expected profit, (ii) the profit-maximizing strategy is a reservation under which customers who find that they have high valuation exercise while those with low valuation do not exercise, and (iii) the seller will not sell to speculators. In §4, I characterize the capacity that maximizes expected profit, and in §5, analyze the implications of changes in demand for the pricing strategy and capacity. §6 concludes the paper with remarks on directions for future research.

3. Pricing Strategy

3.1. *Spot Sales*

Let the seller offer capacity k through spot sales (see Figure 1). At time 2, suppose a customer learns that he is type l. Then his satisfaction will be v_l whether he obtains a seat or not. Since he must pay p for a seat, he will prefer not to buy. Suppose that he learns that he is type h. Then his utility if seats are unavailable will be $U(v_{hu})$, while his utility if a seat is available will be $U(v_{ha} - p)$.

At the opening of time 2, the seller has no contract with the customer, hence, by the assumption of sequential rationality, she will set p to maximize profit from that point onward. The seller will set p so that the type-h customer just prefers to buy a seat, i.e., so that $U(v_{hu}) = U(v_{ha} - p)$, which implies $p = v_{ha} - v_{hu} > 0$. Then his utility will be $U(v_{hu})$ regardless of whether he obtains a seat, hence the customer's *ex-ante* expected utility will be

$$\lambda U(v_l) + (1 - \lambda) U(v_{hu}) < U(\bar{v}),$$

by (1). Because the seller extracts all the consumer surplus from type-h customers at time 2, each customer will choose the noncontingent alternative at time 1, thereby precluding himself from even considering the option of flying at time 2.

Consider spot sales to speculators: the seller's revenue will be p from every speculator who obtains a seat, hence revenue will be positive only if $p \geq 0$. A speculator's utility if

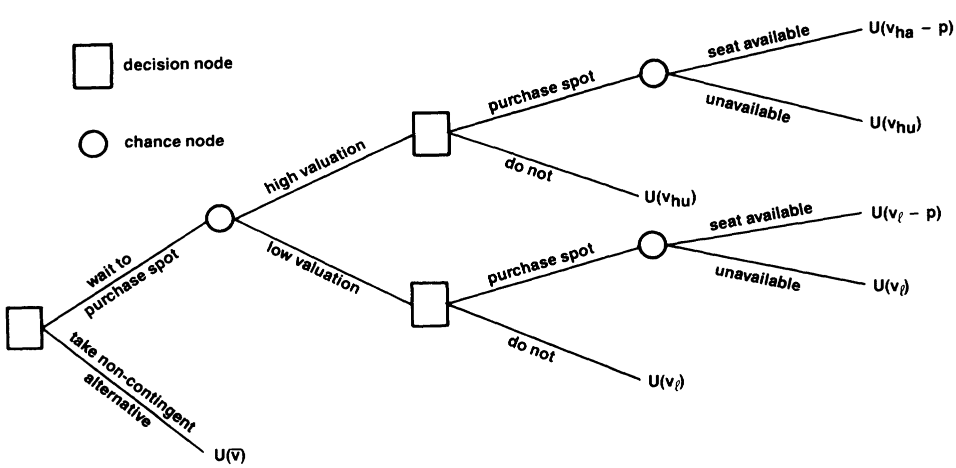

FIGURE 1. Customer's Utility with Spot Purchase.

he buys a seat is $U(v_l - p) \leq U(v_l)$, since $p \geq 0$. But $U(v_l)$ is the speculator's utility if he does not buy, hence he will not buy from spot sales. Therefore, neither customers nor speculators will buy through spot sales, and the seller's revenue from spot sales will be zero.

3.2. *Firm Advance Orders*

Let the seller offer capacity k through firm advance orders at time 1. I will show that, even ignoring the constraint imposed by the presence of speculators, the firm advance order that yields the seller most expected profit from customers provides less profit than a suitably-designed reservation. I then show that the seller will not profit from selling firm advance orders to speculators.

Since all customers are *ex-ante* identical, if one customer chooses to purchase a firm advance order, all customers will do so. Each customer's utility will depend on his information at time 2 and whether a seat is available (see Figure 2). Suppose that he finds that he is type-h: if seats are unavailable, he must pay price p_u, and will have utility $U(v_{hu} - p_u)$, while if a seat is available, he must pay price p_a and will have utility $U(v_{ha} - p_a)$.

Since all customers have orders, the total demand for seats is n. The capacity is k, hence the probability that a seat will be available for the customer is k/n. Thus, the expected utility of a type-h customer will be

$$\left(1 - \frac{k}{n}\right)U(v_{hu} - p_u) + \frac{k}{n}U(v_{ha} - p_a). \tag{2}$$

Similarly, the expected utility of a type-l customer will be

$$\left(1 - \frac{k}{n}\right)U(v_l - p_u) + \frac{k}{n}U(v_l - p_a). \tag{3}$$

At time 1, the customer must decide whether to buy a firm advance order on the basis of his *ex-ante* expected utility

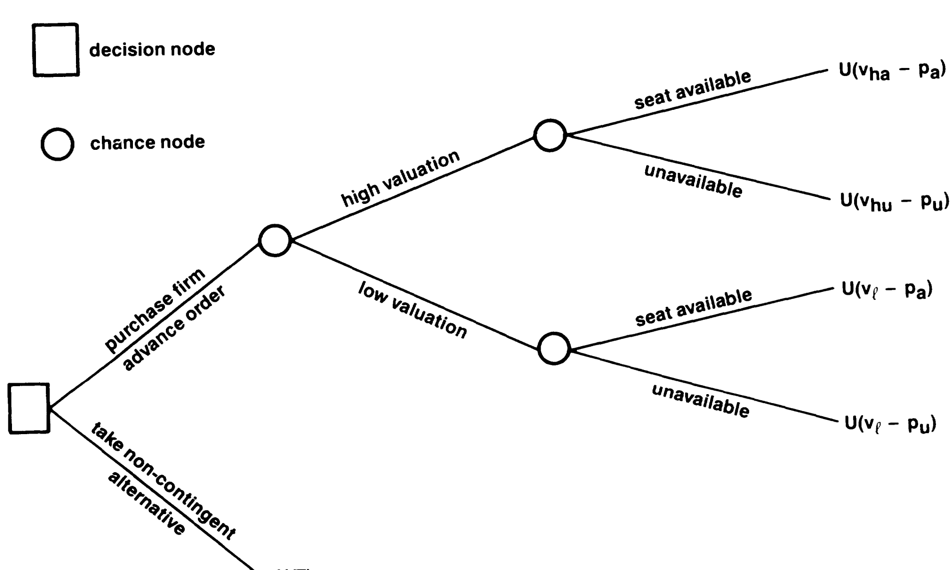

FIGURE 2. Customer's Utility with Firm Advance Order.

$$\lambda\left[\left(1-\frac{k}{n}\right)U(v_l-p_u)+\frac{k}{n}U(v_l-p_a)\right] + (1-\lambda)\left[\left(1-\frac{k}{n}\right)U(v_{hu}-p_u)+\frac{k}{n}U(v_{ha}-p_a)\right].$$

He will choose the order over the non-contingent alternative if the *ex-ante* expected utility from the order is at least $U(\bar{v})$.

The seller will receive p_a from each of k customers and p_u from each of the remaining $(n-k)$ customers, hence the seller's *ex-ante* expected revenue from customers is $R(k) = p_u(n-k)+p_ak$, therefore, with respect to the customers, the seller seeks a pricing strategy, (p_u, p_a), to maximize

$$R(k) = p_u(n-k)+p_ak, \tag{4}$$

subject to the individual rationality constraint

$$\lambda\left[\left(1-\frac{k}{n}\right)U(v_l-p_u)+\frac{k}{n}U(v_l-p_a)\right] + (1-\lambda)\left[\left(1-\frac{k}{n}\right)U(v_{hu}-p_u)+\frac{k}{n}U(v_{ha}-p_a)\right] \geq U(\bar{v}). \tag{5}$$

Under the firm advance order, the type-l customer is subject to risk because his utility depends on whether he obtains a seat. The following proposition shows that there exists a reservation contract that yields the seller higher expected profit than the most profitable firm advance order. Under the reservation, the type-l customer does not exercise and hence avoids risk. This reduction in risk raises the customer's *ex-ante* expected utility, which in turn allows the seller to extract larger profit. The same results hold, *a fortiori*, when the constraint imposed by the speculators is added to the problem.

It remains to check that the seller will not profit from sales of firm advance orders to speculators. Consider a speculator who buys a firm advance order. Let him obtain a seat with probability $\hat{\alpha}$, then the seller's *ex-ante* expected revenue from each speculator will be $\hat{\alpha}p_a + (1-\hat{\alpha})p_u$. This must be nonnegative, otherwise the seller will not offer the product. The speculator's *ex-ante* expected utility will be $\hat{\alpha}U(v_l-p_a)+(1-\hat{\alpha})U(v_l - p_u) < U[v_l-\hat{\alpha}p_a-(1-\hat{\alpha})p_u] \leq U(v_l)$ since $U(\cdot)$ is concave. Now by Assumptions 1 and 2, $U(v_l) < U(\bar{v})$, hence the speculators will not buy firm advance orders and the seller's revenue will be zero.

PROPOSITION 1. *A strategy of selling through firm advance orders will not maximize expected profit.*[10]

3.3. *Reservations*

Let the seller offer capacity k through reservations at time 1. Since all customers are *ex-ante* identical, if one customer chooses to purchase a reservation, all customers will do so. The customer's utility will depend on his information at time 2, and whether he exercises and if so, whether a seat is available (see Figure 3). The prices that the seller sets for the reservation can induce four possible exercise strategies: (i) both types l and h exercise, (ii) both types do not exercise, (iii) type-l customers exercise and type-h do not, and (iv) type-l customers do not exercise and type-h do exercise.

[10] The proofs of this and subsequent results are presented in the Appendix. The reader will find it convenient to read the proof of Proposition 1 together with the proof of Proposition 2.

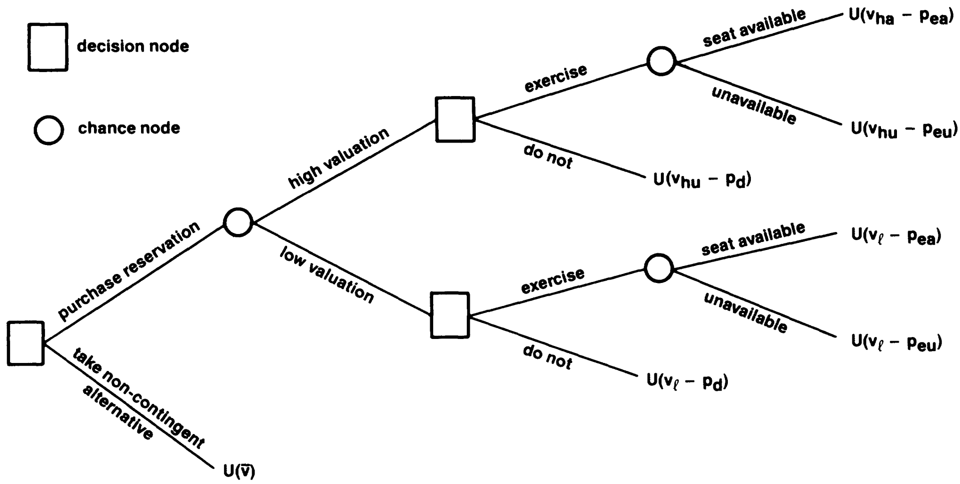

FIGURE 3. Customer's Utility with Reservation.

I shall show that pricing such that the customer exercises according to strategies (i), (ii), or (iii) will not maximize expected profit. (i) If the prices lead both types of customer to exercise, the reservation is equivalent to a firm advance order. Proposition 1 proves that this does not maximize expected profit. (ii) If the prices lead both types of customer not to exercise their reservations, the customer does not ever enjoy the airline seat, hence each customer will choose the noncontingent alternative at time 1, so the strategy yields the seller zero revenue. (iii) If the prices lead type-l customer to exercise and type-h not to exercise, each customer avoids exercising when having a seat would increase his satisfaction, and exercises when a seat does not increase his satisfaction. Hence, at time 1, the customer will choose the noncontingent alternative, and the seller's revenue will be zero.[11]

In the remaining case, the seller prices the reservation so that type-l customers do not exercise and type-h do exercise. The seller influences the customers' exercise decisions through her pricing strategy. But the decisions that the seller expects the customers to take must maximize the customers' utility. Consider a customer who finds that he is of high type. By assumption, he exercises his reservation. If seats are unavailable, he must pay price p_{eu}, and will have utility $U(v_{hu} - p_{eu})$. If a seat is available, he must pay price p_{ea} and will have utility $U(v_{ha} - p_{ea})$. Whether a seat is available will depend on how many of the other $(n - 1)$ customers exercise, i.e., how many are type-h.[12]

Let H_{n-1} be the realized number of type-h customers among the other $(n-1)$ customers. If the total number of customers of type h (himself included) is no greater than the available capacity, $H_{n-1} + 1 \le k$, i.e., $H_{n-1} < k$, the customer will obtain a seat with certainty. If, however, the total number of customers of type h exceeds the available capacity, $H_{n-1} \ge k$, a lottery will be held, and he will obtain a seat with probability $k/(H_{n-1} + 1)$. Hence the probability that a seat will be available for him is

$$\alpha(k) \stackrel{\text{def}}{=} \sum_{i=0}^{n-1} \min\left\{1, \frac{k}{i+1}\right\} \Pr(H_{n-1} = i). \tag{6}$$

Thus, for a type-h customer, expected utility if he exercises his reservation is $(1 - \alpha)U(v_{hu}$

[11] These arguments are stated formally in the proof of Proposition 2.

[12] I show below that speculators will not buy the reservation, hence only customers can possibly exercise.

$- p_{eu}) + \alpha U(v_{ha} - p_{ea})$, while expected utility if he does not exercise is $U(v_{hu} - p_d)$. Type h will exercise if

$$(1 - \alpha)U(v_{hu} - p_{eu}) + \alpha U(v_{ha} - p_{ea}) \geq U(v_{hu} - p_d). \tag{7}$$

Consider a customer who finds that he is type-l. The probability that a seat will be available if he should exercise his reservation will depend on the number of type-h customers among the other $(n - 1)$ customers. The probability is

$$\sum_{i=0}^{n-1} \min\left\{1, \frac{k}{i+1}\right\} \Pr(H_{n-1} = i),$$

which, by (6), is equal to α. Hence type l will not exercise if

$$(1 - \alpha)U(v_l - p_{eu}) + \alpha U(v_l - p_{ea}) \leq U(v_l - p_d). \tag{8}$$

At time 1, each customer must choose between a reservation and the noncontingent alternative. He will choose the reservation if the *ex-ante* expected utility that it provides

$$\lambda U(v_l - p_d) + (1 - \lambda)(1 - \alpha)U(v_{hu} - p_{eu}) + (1 - \lambda)\alpha U(v_{ha} - p_{ea}) \geq U(\bar{v}). \tag{9}$$

The seller's revenue from the customers will depend on how many customers exercise and, of those, how many obtain seats. Let H_n represent the realized number of type-h customers from the population of size n. By assumption, all the customers of type h will exercise their reservations. If $H_n \leq k$, all will obtain seats, and the seller will receive revenue of $p_{ea}H_n$ from type-h customers. If, however, $H_n > k$, only k of the type-h customers will obtain seats. The seller will receive price p_{ea} from each customer who obtains a seat, and price p_{eu} from the remaining $(H_n - k)$. Hence the seller's revenue from type-h customers will be $p_{eu}(H_n - k) + p_{ea}k$. In both cases, the seller's revenue from type-l customers will be $p_d(n - H_n)$. Thus the seller's *ex-ante* expected revenue from customers is

$$R(k) \stackrel{\text{def}}{=} p_d \cdot \mathscr{E}(n - H_n) + p_{eu} \cdot \mathscr{E}[\max\{0, H_n - k\}] + p_{ea} \cdot \mathscr{E}[\min\{k, H_n\}], \tag{10}$$

where $\mathscr{E}(\cdot)$ denotes the expectation operator.

The seller's final consideration is how to deal with the speculators. If a speculator purchases a reservation and does not exercise, his price will be p_d. If $p_d < 0$, every speculator will purchase a reservation and not exercise, so as to collect a payment $(-p_d)$ from the seller. With a large number of speculators, such speculation would cause the seller massive losses. To avoid this, the seller must impose the constraint that

$$p_d \geq 0. \tag{11}$$

Suppose that the speculator exercises, and obtains a seat with probability $\hat{\alpha}$, then the seller's *ex-ante* expected revenue from each speculator will be $\hat{\alpha}p_{ea} + (1 - \hat{\alpha})p_{eu}$. This must be nonnegative, otherwise the seller will not offer the product. Then the speculator's *ex-ante* expected utility will be $\hat{\alpha}U(v_l - p_{ea}) + (1 - \hat{\alpha})U(v_l - p_{eu}) < U[v_l - \hat{\alpha}p_{ea} - (1 - \hat{\alpha})p_{eu}] \leq U(v_l)$ since $U(\cdot)$ is concave. Now by Assumptions 1 and 2, $U(v_l) < U(\bar{v})$, hence if it is profitable for the seller to sell to speculators, the speculators will prefer the noncontingent alternative.

To summarize, the seller seeks a pricing strategy, (p_d, p_{eu}, p_{ea}), to maximize *ex-ante* expected revenue (10) subject to the individual rationality constraint (9) and the self-selection constraints for type h, (7), and for type l, (8), and the constraint to exclude speculators, (11). Proposition 2 characterizes the solution to the problem.

PROPOSITION 2. *The profit-maximizing pricing strategy is a reservation with prices given by*

$$p_d = 0, \tag{12}$$

$$p_{eu} = p_{ea} - (v_{ha} - v_{hu}), \quad \text{and} \tag{13}$$

$$p_{ea} = v_{ha} - U^{-1}\left[\frac{U(\bar{v}) - \lambda U(v_l)}{1 - \lambda}\right]. \tag{14}$$

Proposition 2 proves that the price of the reservation itself, i.e., the price for customers who do not exercise, $p_d = 0$. To explain: the profit-maximizing strategy seeks to insure the risk-averse customer by compensating him when his satisfaction is low (type l) and taking from him when his satisfaction is higher (type h). Since each customer's type is private information, the seller must induce customers to reveal their type credibly. Her mechanism is to require those seeking compensation not to exercise their reservations. Then only the type-l customers will refrain from exercising. Hence the strategy compensates by reducing the price to customers who do not exercise. This price, however, may not be reduced below 0 because of the speculators (constraint (11)), thus at the solution, $p_d = 0$.

By (13), under the profit-maximizing pricing strategy,

$$v_{ha} - p_{ea} = v_{hu} - p_{eu}. \tag{15}$$

Substituting from (12) and (15) in (9),

$$\lambda U(v_l) + (1 - \lambda) U(v_{hu} - p_{eu}) \geq U(\bar{v}), \tag{16}$$

which, by (1) implies that $p_{eu} < 0$, i.e., the seller compensates customers who exercise but for whom seats are unavailable. So if the type-h customer exercises, he will be fully insured against capacity being unavailable (equation (15)).

The expected utility of a type-l customer is $U(v_l)$, whereas the expected utility of a type-h customer is

$$U(v_{hu} - p_{eu}) \geq U(v_{hu}) > U(v_l).$$

Thus, the customer receives higher expected utility when he is type h, i.e., he receives only partial insurance against uncertainty in his satisfaction.

If each customer's realized valuation were common knowledge to customer and seller, the seller would maximize profit by insuring each customer fully against both uncertainty in satisfaction and uncertainty in the availability of capacity. Because customers' valuations are private information, the seller can provide only partial insurance only against uncertainty in customer valuation. Equation (14) implies that the customer's *ex-ante* expected utility will be

$$\lambda U(v_l) + (1 - \lambda) U(v_{ha} - p_{ea}) = U(\bar{v}), \tag{17}$$

i.e., the seller extracts all of the customer's consumer surplus at time 1.[13]

In a special case of a reservation, $p_d = p_{eu} = 0$, i.e., at time 1, the seller commits to charge the customer at time 2 p_{ea} only if the customer wishes to fly and a seat is available, and to charge (or pay) the customer nothing otherwise. Proposition 2 implies that this special case does not maximize profit.

The pricing strategy of Proposition 2 may be implemented in two ways. In the first, the seller collects p_{ea} at the time of reservation, and promises to refund p_{ea} if the customer does not exercise, i.e., cancels the reservation, or refund $(p_{ea} - p_{eu})$ if the customer exercises but capacity is not available. This method assures the seller that the customer has sufficient funds to cover the fare and is practised by some hotels and most airlines.

In the second method, the seller collects nothing at the time of reservation ($p_d = 0$),

[13] In practice, the seller could lower p_{ea} slightly so that the customer would strictly prefer to buy the reservation.

and promises to charge the customer p_{ea} if he exercises and obtains a seat, or pay the customer $(-p_{eu})$ if he exercises and does not obtain a seat. Compared with the first method, this method involves one fewer transfer of funds between seller and customer. It is used by some hotels.

4. Capacity

At time 0, the seller must choose capacity in the light of the prices that she will set subsequently at time 1. Since all customers are *ex-ante* identical, and each customer has demand for at most one seat, the prices specified in Proposition 2 depend only on each customer's satisfaction in the various eventualities and the probability that he is type-h. In particular, the prices do not vary with the capacity, k, or the total number of customers, n. Substituting from Proposition 2 in (10), at time 0, the seller's expected revenue is

$$R(k) = p_{eu} \cdot \mathscr{E}[\max\{0, H_n - k\}] + p_{ea} \cdot \mathscr{E}[\min\{k, H_n\}]. \tag{10'}$$

Now $\mathscr{E}[\min\{k, H_n\}] + \mathscr{E}[\max\{0, H_n - k\}] = \mathscr{E}[\min\{k, H_n\} + \max\{0, H_n - k\}] = \mathscr{E}[H_n] = (1 - \lambda)n$, hence

$$\begin{aligned} R(k) &= p_{ea}(1 - \lambda)n + (p_{eu} - p_{ea}) \cdot \mathscr{E}[\max\{0, H_n - k\}], \\ &= p_{ea}(1 - \lambda)n - (v_{ha} - v_{hu}) \cdot \mathscr{E}[\max\{0, H_n - k\}], \end{aligned}$$

by substituting from (15).

Thus, marginal revenue

$$\begin{aligned} MR(k) &\overset{\text{def}}{=} R(k) - R(k-1) \\ &= -(v_{ha} - v_{hu}) \cdot \mathscr{E}[\max\{0, H_n - k\}] \\ &\qquad + (v_{ha} - v_{hu})\mathscr{E}[\max\{0, H_n - k + 1\}] \\ &= (v_{ha} - v_{hu}) \cdot \sum_{j=0}^{n} [-\max\{0, j - k\} + \max\{0, j - k + 1\}] \Pr(H_n = j) \\ &= (v_{ha} - v_{hu}) \cdot \sum_{j=k}^{n} [-(j - k) + (j - k + 1)] \Pr(H_n = j), \quad \text{i.e.,} \end{aligned}$$

$$MR(k) = (v_{ha} - v_{hu}) \cdot \sum_{j=k}^{n} \Pr(H_n = j), \tag{18}$$

which is monotone decreasing in capacity, k.

From (10′), if the seller has zero capacity, her expected profit will be

$$R(0) - C(0) = p_{eu}\mathscr{E}(H_n) < 0, \tag{19}$$

since $p_{eu} < 0$. This may be interpreted as a *fixed cost* of offering reservations. By (10′),

$$R(n) - C(n) = p_{ea}(1 - \lambda)n - C(n) > 0,$$

by (14) and Assumption 3. This shows that the seller will offer positive capacity. Since $MR(\cdot)$ is decreasing in k and $MC(\cdot)$ is increasing, the capacity that maximizes expected profit may be characterized as follows:

PROPOSITION 3. *The profit-maximizing capacity, k^*, is given by*

$$MR(k^*) \geq MC(k^*) \quad \textit{and} \quad MR(k^* + 1) \leq MC(k^* + 1).$$

Now the total number of customers is n, and under the profit-maximizing pricing strategy, all n customers place reservations. The number of reservation-holders, n, should be distinguished from the number of customers who wish to exercise after individual valuations are realized, H_n. The seller chooses capacity k to maximize expected profit. If $H_n > k$, the seller must compensate $(H_n - k)$ customers for whom there are no seats available. If $H_n < k$, the seller is left with unsold capacity which has no salvage value.

Proposition 3 indicates that if $MR(n) = (v_{ha} - v_{hu})(1 - \lambda)^n < MC(n)$, the profit-maximizing capacity $k^* < n$. The difference $(n - k^*)$ may be interpreted as the extent of over-booking. The seller over-books, even though she must compensate every type-h customer who exercises but must be turned away, because the number that must be compensated is max $\{0, H_n - k^*\}$, and not $(n - k^*)$.

COROLLARY. *If* $MR(n) < MC(n)$, *the seller will over-book.*

Would the seller prefer to hold aside some capacity to sell to customers who do not purchase reservations? These seats would then be sold by spot sale. The earlier argument that proved that the seller would not sell by spot sales[14] shows that the answer is negative.

Would the seller prefer to market some capacity through firm advance orders? Proposition 1 applies to any subcapacity k' offered through firm advance orders. It shows that the seller will raise revenue by offering those k' seats through reservations.

In reality, airlines adjust their capacity in the various classes in light of demand information received in the interim. What capacity would the seller choose if she could set capacity after the customers decide whether to exercise? Suppose that the seller may adjust capacity within the limits $(\underline{k}, \bar{k})$.[15] In the context of the model of this paper, the ability of the seller to adjust $k \in (\underline{k}, \bar{k})$ *ex-post* does not affect the profit-maximizing pricing strategy. From (13), the incremental revenue from selling a seat to a type-h customer with certainty is $p_{ea} - p_{eu} = v_{ha} - v_{hu}$. Provided that $v_{ha} - v_{hu} > MC(k)$, the profit-maximizing capacity will be

$$k = \begin{cases} \underline{k} & \text{if} \quad H_n \leq \underline{k}, \\ H_n & \text{if} \quad \underline{k} < H_n < \bar{k}, \\ \bar{k} & \text{if} \quad H_n \geq \bar{k}. \end{cases}$$

5. Managerial Implications of Changes in Demand

A key question for managers of service businesses is how to adjust their pricing strategy and capacity in response to changes in customer satisfaction, the distribution of satisfactions, and the total population. The following table summarizes the profit-maximizing adjustments.[16] That $p_d = 0$ is driven by the need to curb speculation, hence p_d should not be adjusted.

Adjustments to Pricing Strategy and Capacity

	Effect of increase in					
	v_l	v_{hu}	v_{ha}	$\bar{v}$	λ	n
p_d	nil	nil	nil	nil	nil	nil
p_{eu}	higher	higher	nil	lower	lower	nil
p_{ea}	higher	nil	higher	lower	lower	nil
k	nil	smaller	larger	nil	smaller	larger

[14] §3.1.

[15] If there were no lower limit $\underline{k}$, we would not observe empty seats on flights, while if there were no upper limit $\bar{k}$, we would not observe passengers with confirmed reservations being 'bumped'.

[16] For brevity, the proofs of the results in the table will be omitted.

One interesting managerial implication is that the seller should offer smaller capacity if the utility of a type-h customer for whom a seat is unavailable, v_{hu}, is higher. When v_{hu} is higher, the customer's need for insurance is reduced, hence the seller should raise p_{eu}, i.e., *reduce* the compensation to customers who are denied boarding (equation (13)). Since the penalty to the seller for insufficient capacity will be lower, the seller can raise profit by increasing the extent of over-booking, i.e., reducing capacity. By enlarging total capacity, the emergence of competing airlines may reduce the customer's loss of utility from being denied a seat on the preferred flight. The model therefore predicts that increased competition will lead sellers to *reduce* compensation for denied boarding and reduce capacity.

The model also implies that if the number of customers, n, is higher, the seller should set a larger capacity. As argued earlier, however, the seller should not adjust the prices p_{eu} and p_{ea} because the additional customers are *ex-ante* identical to the original ones.

At times like Thanksgiving and Christmas, air travellers are generally less *ex-ante* uncertain about wanting to fly. In terms of the model, λ will be lower or $(1 - \lambda)$ higher. The model implies that the seller should jack up the price p_{ea}, reduce the compensation $(-p_{eu})$, and offer more capacity.

6. Concluding Remarks

Where customers' valuation for use of capacity is *ex-ante* uncertain and unused capacity has no salvage value, a seller will maximize profit by over-booking her capacity. Thus each customer faces both uncertainty in his own valuation as well as uncertainty about availability of capacity. If the customer is risk-averse, he seeks insurance, and if the seller is risk-neutral, she is the natural provider of insurance.

The provision of insurance, however, is complicated because each customer's realized valuation is private information. Since the seller does not know each customer's type, she must elicit the information. The profit-maximizing pricing strategy takes the form of a reservation that induces customers with high valuation for use of the capacity to exercise and those with lower valuation not to exercise. In this way, the reservation provides partial insurance against the customer's uncertainty in valuation (privately observed contingency) and full insurance against unavailability of capacity (observable by both seller and customer).

This paper explains a puzzle in marketing that reservations, which are a valuable product, are systematically provided free of charge by airlines, hotels, and car rental agencies. The reason is that the price of the reservation is also the payment by the customer in the event that his satisfaction is low. A full-information insurance contract would call for the seller to compensate the customer in that event. But in the absence of full information, such a contract would draw speculators to reserve for the sake of the compensation. The best feasible is not to charge the customer for the reservation.

The model presented here is very simplified and subject to a number of limitations. The first is Assumption 1, that $v_{hu} > v_l$, which provides that the customer receive greater satisfaction when he would like to use the capacity but cannot than when he does not wish to use the capacity. A number of situations fit this assumption: I have given one to illustrate the model; another example is where the customers are New York bankers who with some probability will have a profitable deal to close in the Chicago the following week—even if they cannot get the preferred flight, they can take an alternative, close the deal, and be better off than with no business at all. Nevertheless, there may be situations where it is more reasonable to suppose that $v_{hu} \leq v_l$.

A second limitation is the assumption that the customer has only two possible valuations. In reality, there may be a number of possible valuations, e.g., depending on the size of the deal that is to be concluded. In such a setting, the seller may increase profit by setting a schedule of exercise prices, with higher prices offering higher priority of service from the fixed capacity.[17]

There are three additional directions for future research. The first is to consider risk-aversion on the part of the seller. It is fair to assume that the seller is risk-neutral when the seller is owned by a large number of shareholders with well-diversified portfolios and the customers' valuations are independently distributed. If, however, the customers' valuations are closely correlated, the risk that the seller bears is much larger—and the seller may well not be neutral with regard to the larger risk. For example, in the market for hotel rooms in ski resorts, customers' demand will depend on the weather and their exercise decisions will be closely correlated. If the seller is not risk-neutral, she will insist on a reservation fee or cancellation penalty in order to place part of the risk on the customer.

A second direction is to consider the effect of *ex-ante* heterogeneity in the degree to which customers face uncertainty about their valuation. For instance, the customer population may consist of executives who are relatively more uncertain, and retirees or students who can plan ahead and commit in advance to a particular flight. The second group does not value insurance against uncertainty in their valuation, because it is very unlikely that they will cancel, hence a firm advance order is as good as a reservation.

Now suppose that the first group of customers derives a greater satisfaction from travelling if they realize a demand to travel. Then the seller would like to charge them higher prices for seats. In this setting, a natural means by which to segment the market is to offer two products—reservations with no cancellation penalty and firm advance orders (with 100% cancellation penalty)—with the price for available seat higher in the reservation. In practice, airlines commonly offer discount tickets that carry cancellation penalties of 25–100%.

A third direction is to model the incentive to speculate in more detail. Some speculators may suffer less than customers from being denied a seat. For instance, college students who reserve seats around Thanksgiving are happy to fly if they get a seat, but are even happier to be 'bumped' and obtain compensation. Moreover, in practice, airlines try to bid for passengers to give up confirmed reservations before employing lotteries. The presence of such speculators and airline practices may well limit the extent to which the seller can provide customers with full insurance against unavailable seats.

The analysis of this paper should be regarded only as a first step towards an understanding of marketing strategies that provide customers with insurance. A detailed enquiry into the various alternative assumptions should yield further insight into the varied reservation practices actually observed in retail and industrial marketing, for instance, such questions as why airline shuttle services, most cinemas, and some restaurants refuse to permit reservations, why most professional football games and concerts sell out but most baseball games do not, and why options to lease property and to manufacture aircraft are never sold separately from firm orders.[18]

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[17] Harris and Raviv (1981) analyze the optimal schedule of priority prices for the allocation of a fixed capacity.

[18] This paper was received in July 1987 and has been with the author 5 months for 4 revisions. Accepted by Subrata K. Sen.

Appendix

PROOF OF PROPOSITION 1.

- First, I shall show that under a revenue-maximizing firm advance order,

$$v_{hu} - p_u \leq v_{ha} - p_a. \tag{20}$$

Suppose otherwise, that

$$v_{hu} - p_u > v_{ha} - p_a. \tag{$*$}$$

Then $p_u < p_a - (v_{ha} - v_{hu}) < p_a$, hence

$$v_l - p_u > v_l - p_a. \tag{$**$}$$

Consider a variation $\delta p_u > 0$ and $\delta p_a < 0$ such that

$$\left(1 - \frac{k}{n}\right)\delta p_{eu} + \frac{k}{n}\,\delta p_{ea} = 0.$$

Substituting in (4), the effect on the seller's revenue,

$$\delta R(k) = \delta p_{eu}(n-k) - \delta p_{ea}k = \left[\left(1 - \frac{k}{n}\right)p_u + \frac{k}{n}\,p_a\right]n = 0.$$

Substituting in (5), the effect on the customer's *ex-ante* expected utility is

$$\begin{aligned}\delta \mathscr{E} U = -\lambda\left[\left(1 - \frac{k}{n}\right)U'(v_l - p_u)\delta p_{eu} + \frac{k}{n}\,U'(v_l - p_a)\delta p_{ea}\right]&\\ - (1-\lambda)\left[\left(1 - \frac{k}{n}\right)U'(v_{hu} - p_u)\delta p_{eu} + \frac{k}{n}\,U'(v_{ha} - p_a)\delta p_{ea}\right]&\\ = \{\lambda[U'(v_l - p_u) - U'(v_l - p_a)] + (1-\lambda)[U'(v_{hu} - p_u) - U'(v_{ha} - p_a)]\}\,\frac{k}{n}\,\delta p_{ea}.&\end{aligned}$$

Since $U(\cdot)$ is concave, ($*$) and ($**$) imply that $U'(v_{hu} - p_u) < U'(v_{ha} - p_a)$, and $U'(v_l - p_u) < U'(v_l - p_a)$, hence $\delta \mathscr{E} U > 0$.

Thus the variation provides the seller with equal *ex-ante* expected revenue, and the customer with strictly higher expected utility. Now the variation may be adjusted slightly to increase both revenue and utility, hence the original strategy is dominated.

• Similarly, it may be shown that under a revenue-maximizing firm advance order,

$$p_u < p_a. \tag{21}$$

For brevity, I will sketch the proof. Suppose otherwise that $p_u \geq p_a$, in which case

$$v_l - p_u \leq v_l - p_a \quad \text{and} \quad v_{hu} - p_u \leq v_{hu} - p_a < v_{ha} - p_a.$$

Consider a variation $\delta p_{eu} < 0$ and $\delta p_{ea} > 0$ such that

$$\left(1 - \frac{k}{n}\right)\delta p_{eu} + \frac{k}{n}\,\delta p_{ea} = 0,$$

then it may be shown that this will leave the seller's revenue unchanged, but provided that $\lambda < 1$, will increase the customer's *ex-ante* expected utility.

• Finally, I show that the revenue-maximizing firm advance order is dominated by a reservation under which the type-h customers exercise and the type-l do not exercise. From (4), the *ex-ante* expected revenue from the firm advance order is

$$R(k) = (n-k)p_u + kp_a.$$

Now $R(k) \geq 0$, otherwise, the firm advance order clearly does not maximize revenue.

Let

$$p_d \stackrel{\text{def}}{=} \left(1 - \frac{k}{n}\right)p_u + \frac{k}{n}\,p_a. \tag{22}$$

Since $R(k) \geq 0$, it follows that $p_d \geq 0$. By (21) and (22),

$$p_u < p_d < p_a. \tag{23}$$

Substituting from (23) in (2), the expected utility of the type-h customer is

$$\left(1 - \frac{k}{n}\right)U(v_{hu} - p_u) + \frac{k}{n}\,U(v_{ha} - p_a) > U(v_{hu} - p_u) > U(v_{hu} - p_d) \tag{$\dagger$}$$

by substituting from (20) and (23). Now $U(\cdot)$ is concave, hence from (3), the expected utility of type l is

$$\left(1 - \frac{k}{n}\right)U(v_l - p_u) + \frac{k}{n}\,U(v_l - p_a) < U\left[\left(1 - \frac{k}{n}\right)(v_l - p_u) + \frac{k}{n}\,(v_l - p_a)\right] = U(v_l - p_d), \tag{$\dagger\dagger$}$$

by (22).

Consider a reservation contract (p_d, p_u, p_a) where p_d is the price if the customer does not exercise, p_u is the price if he exercises but seats are unavailable, and p_a is the price if he exercises and a seat is available. By (†) and (††), this will lead the type-h customers to exercise, and the type-l customers not to exercise, and it will provide both types with strictly higher expected utility than the firm advance order. Hence the reservation provides the customer with strictly higher *ex-ante* expected utility.

Suppose that the number of type-h customers is H_n. Then $(n - H_n)$ customers will not exercise and must each pay p_d. H_n customers will exercise. If $H_n \le k$, the seller's revenue from the reservation will be

$$p_d(n - H_n) + p_a H_n > p_d n = (n - k)p_u + kp_a,$$

by substituting from (23) and (22). Hence, if $H_n \le k$, the alternative strategy will provide higher revenue. If $H_n > k$, k of the type-h customers will be allocated seats and pay p_a while the remaining $(H_n - k)$ will be denied boarding and must pay p_u. Then the alternative strategy will provide the seller with revenue

$$p_d(n - H_n) + p_u(H_n - k) + p_a k > p_u(n - k) + p_a k,$$

by substituting from (23). In both cases, the alternative strategy provides strictly higher expected utility and strictly higher revenue than the firm advance order, therefore it dominates. ■

LEMMA. *Under a reservation that leads type-h customers to exercise and type-l not to exercise,*

$$(1 - \lambda)\alpha n = \mathscr{E}[\min\{k, H_n\}], \quad \text{and} \tag{24}$$

$$(1 - \lambda)(1 - \alpha)n = \mathscr{E}[\max\{0, H_n - k\}]. \tag{25}$$

PROOF. By the definition of α, equation (6),

$$\begin{aligned}(1 - \lambda)\alpha n &= \sum_{i=0}^{n-1} \min\left\{1, \frac{k}{i+1}\right\} \frac{(n-1)!}{i!(n-1-i)!}(1-\lambda)^{i+1}\lambda^{n-1-i} n \\ &= \sum_{i=0}^{n-1} \min\left\{1, \frac{k}{i+1}\right\}(i+1)\frac{n!}{(i+1)!(n-1-i)!}(1-\lambda)^{i+1}\lambda^{n-(i+1)} \\ &= \sum_{j=1}^{n} \min\left\{1, \frac{k}{j}\right\} j \binom{n}{j}(1-\lambda)^j \lambda^{n-j} \\ &= \sum_{j=1}^{n} \min\{j, k\} \Pr(H_n = j) \\ &= \sum_{j=0}^{n} \min\{j, k\} \Pr(H_n = j) \overset{\text{def}}{=} \mathscr{E}[\min\{k, H_n\}],\end{aligned}$$

since the 0th term is 0. This proves (24).

Now $\mathscr{E}[\min\{k, H_n\}] + \mathscr{E}[\max\{0, H_n - k\}] = \mathscr{E}[\min\{k, H_n\} + \max\{0, H_n - k\}] = \mathscr{E}[H_n] = (1 - \lambda)n$, hence, by (24), $\mathscr{E}[\max\{0, H_n - k\}] = (1 - \lambda)n - (1 - \lambda)\alpha n$, which proves (25). ■

PROOF OF PROPOSITION 2.

- A reservation under which both types l and h exercise is equivalent to a firm advance order, hence by Proposition 1, such a reservation is dominated.
- Next, consider a reservation under which both types do not exercise. If a type-l customer does not exercise his reservation, he will receive utility $U(v_l - p_d)$. If a type-h customer does not exercise his reservation, he will receive utility $U(v_{hu} - p_d)$. Hence, the customer's *ex-ante* expected utility from the contract will be $\lambda U(v_l - p_d) + (1 - \lambda)U(v_{hu} - p_d) \le \lambda U(v_l) + (1 - \lambda)U(v_{hu}) < U(\bar{v})$, by substituting from (11) and (1). Thus the customer will not buy this reservation.
- Next, I show that a reservation that leads type-h customers not to exercise and type-l to exercise yields zero *ex-ante* expected revenue. Let a customer who exercises his reservation obtain a seat with probability $\hat{\alpha}$. Then

$$(1 - \hat{\alpha})U(v_{hu} - p_{eu}) + \hat{\alpha}U(v_{ha} - p_{ea}) \le U(v_{hu} - p_d), \tag{***}$$

so that the type-h customer will choose not to exercise, and

$$(1 - \hat{\alpha})U(v_l - p_{eu}) + \hat{\alpha}U(v_l - p_{ea}) \ge U(v_l - p_d),$$

so that the type-l customer will choose to exercise.

Thus the *ex-ante* expected utility of the customer will be

$$\begin{aligned}\lambda[(1 - \hat{\alpha})U(v_l - p_{eu}) + \hat{\alpha}U(v_l - p_{ea})] + (1 - \lambda)U(v_{hu} - p_d) &< \lambda[(1 - \hat{\alpha})U(v_{hu} - p_{eu}) \\ + \hat{\alpha}U(v_{ha} - p_{ea})] + (1 - \lambda)U(v_{hu} - p_d) \le \lambda U(v_{hu} - p_d) &+ (1 - \lambda)U(v_{hu} - p_d) = U(v_{hu} - p_d),\end{aligned}$$

since $v_l < v_{hu}$ and $v_l < v_{ha}$, and by substituting from (***). By (11) and (1), $U(v_{hu} - p_d) \leq U(v_{hu}) < U(\bar{v})$, therefore, the customer will not buy the reservation.

• By the preceding steps, the solution must be a reservation that leads type-h customers to exercise and type-l not to exercise. To characterize the profit-maximizing prices, I ask which of the constraints from (7)–(11) bind at the solution.

By self-selection constraint for type h, (7), the expected utility of type h is at least $U(v_{hu} - p_d)$. The expected utility of type l is $U(v_l - p_d) < U(v_{hu} - p_d)$, since $v_l < v_{hu}$, hence the expected utility of type l is strictly less than that of type h. As the customer is risk-averse, his *ex-ante* expected utility would be increased by a variation to lower the expected utility of type h and increase the expected utility of type l.

Such a variation potentially would be constrained by either (7) or (11). Suppose that (7) binds: then the expected utility of type h would be $U(v_{hu} - p_d)$, hence the customer's *ex-ante* expected utility would be

$$\lambda U(v_l - p_d) + (1 - \lambda)U(v_{hu} - p_d) < U(\bar{v})$$

by (11) and (1). Thus, if constraint (7) binds, the customer will not buy the reservation, hence it cannot bind and constraint (10) must bind, i.e.,

$$p_d = 0. \tag{12}$$

Substituting from (12) and the lemma in (10), *ex-ante* expected revenue is

$$R(k) = p_{eu}(1 - \lambda)(1 - \alpha)n + p_{ea}(1 - \lambda)\alpha n. \tag{10'}$$

Substituting from (12) in (8), the self-selection constraint for type l is

$$(1 - \alpha)U(v_l - p_{eu}) + \alpha U(v_l - p_{ea}) \leq U(v_l). \tag{8'}$$

Suppose that this constraint were binding: since $U(\cdot)$ is concave,

$$U[v_l - (1 - \alpha)p_{eu} - \alpha p_{ea}] > (1 - \alpha)U(v_l - p_{eu}) + \alpha U(v_l - p_{ea}) = U(v_l),$$

hence $(1 - \alpha)p_{eu} + \alpha p_{ea} < 0$, which, by (10′), implies that $R(k) < 0$. Thus, if constraint (8′) binds, the seller's revenue will be negative, hence the constraint cannot bind.

Since constraints (7) and (8′) do not bind, the seller can relax constraint (9) by maximizing the expected utility of type h. To do so, she should set p_{eu} and p_{ea} so that type h receives equal utility whether a seat is available or not, i.e.,

$$v_{hu} - p_{eu} = v_{ha} - p_{ea}, \tag{15}$$

which proves (13). Substituting from (12) and (15) in (9), *ex-ante* expected utility will be $\lambda U(v_l) + (1 - \lambda)U(v_{ha} - p_{ea})$. To maximize the seller's *ex-ante* expected revenue, the price p_{ea} should be set to extract all consumer surplus at time 1,

$$\lambda U(v_l) + (1 - \lambda)U(v_{ha} - p_{ea}) = U(\bar{v}), \tag{17}$$

which proves (14). ■

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