On the Sequence of Customer Acquisitions and Pricing in the Model of Sales

Kai-Lung Hui,* Sang-Yong T. Lee,† and I.P.L. Png††

Abstract

The basic models of price dispersion due to Butters (1977) and Varian (1980), further developed by Narasimhan (1988), have been applied to a broad range of issues in industrial organization and marketing. Much of the previous research assumes that the captive and switcher segments are exogenously symmetric or asymmetric. We embed the Varian (1980) model in a broader setting that considers how captive/switcher customer segments are determined through *customer acquisition*. The customer segments are symmetric if sellers acquire customers simultaneously with setting prices. If customers are acquired before prices are set, then the equilibrium customer segments are asymmetric unless customer acquisitions are randomized and the marginal cost of customer acquisition increases at a sufficiently slow rate. If prices are set before customers are acquired, then the equilibrium customer segments are almost certainly asymmetric. With constant returns to scale, the customer segments are symmetric *only if* sellers acquire customers simultaneously with setting prices.

* City University of Hong Kong; † Hanyang University; †† National University of Singapore. Corresponding author: Ivan Png, Department of Information Systems, National University of Singapore, 3 Science Drive 2, Singapore 117543, Tel: +65 6516-6807; http://www.comp.nus.edu.sg/~ipng/. This is an extensively revised version of an earlier paper with Il Horn Hann, “Sales and Promotions: A More General Model”. We thank seminar participants at Boston University, INSEAD, Tsinghua University, Chinese University of Hong Kong, and the University of Bern, Ioana Chioveanu, Bai Chong En, Yuxin Chen, and especially John Morgan for helpful advice and suggestions.

1. Introduction

When two or more sellers of an identical product compete on price for consumers, who are variously captives of particular sellers ("captives") or comparison shoppers ("switchers"), the equilibrium outcome is randomized pricing (Butters 1977; Salop and Stiglitz 1977; Rosenthal 1980; Varian 1980; Narasimhan 1988; Baye et al. 1992). Randomized pricing has been interpreted as sales or price promotions, and has been fundamental in explaining empirical observations of price dispersion (Villas-Boas 1995; Baye and Morgan 2004b; Hosken and Reiffen 2004; Baye et al. 2006).

The basic models of randomized pricing due to Butters (1977), Varian (1980), and Narasimhan (1988) have been applied to a broad range of issues in industrial organization and marketing.[1] Many of the studies have proceeded from the assumption that the captive and switcher segments are given exogenously. The customer segments are assumed either to be symmetric across the competing sellers (following Butters (1977) and Varian (1980)) or asymmetric (following Narasimhan (1988)).

Realistically, the captive and switcher segments are the outcome of expenditures by sellers on *customer acquisition*. Conventional stores choose location, so acquiring consumers in the geographical neighborhood; manufacturers advertise on TV and other mass media, so acquiring consumers who see the advertisements. Internet retailers pay for search links, so acquiring consumers who click on the links.

---

[1] The issues have included price promotion (Raju et al. 1990; Robert and Stahl 1993; Baye and Morgan 2001; Chen et al. 2002; Iyer and Pazgal 2003; Ghose et al. 2002; Chen and Hitt 2004; Moscarini and Ottaviani 2004), price matching (Png and Hirshleifer 1987; Corts 1996; Chen et al. 2001; Moorthy and Winter 2002), advertising and branding (Meurer and Stahl 1994; Baye and Morgan 2004; Dukes 2004; Chioveanu 2007), and other aspects of business and marketing strategy (Deneckere et al. 1992; McAfee 1994; McGahan and Ghemawat 1994; Lal and Villas-Boas 1998; Roy 2000; Chen et al. 2001; Chen and Iyer 2002; Hong et al. 2002; Morgan and Sefton 2003; Manduchi 2004). As of August 22, 2007, the Social Science Citation Index reported the following citation numbers: Butters (1977) – 196; Varian (1980) – 245; Narasimhan (1988) – 91.

We address the following question in this paper – given that competing sellers decide on both customer acquisition and pricing, under what circumstances are customer segments symmetric vis-à-vis asymmetric across the sellers?

We consider three scenarios with different sequences of seller actions – where sellers acquire customers *before*, *simultaneous* with, and *after* setting prices. We start from a point at which all potential consumers and all sellers are identical. Specifically, consumers have identical *ex ante* information about the sellers and prices, and they have no preferences for any particular sellers. Each seller incurs a cost to acquire potential consumers. Given sellers' expenditures, consumers *endogenously* divide into captive and switcher segments, depending on whether they are acquired by just one seller or multiple sellers.

We show that the customer segments are symmetric across sellers if sellers acquire consumers and set prices *simultaneously* (and the converse is also true if the cost of customer acquisitions exhibits constant returns to scale). If sellers acquire consumers *before* setting prices, then the *realized* customer segments are asymmetric unless customer acquisitions are randomized and the marginal cost of customer acquisition increases at a sufficiently slow rate. These results provide a foundation for the previous research which exogenously assumed symmetric or asymmetric customer segments.

Finally, we analyze a related scenario, where sellers acquire consumers *after* setting prices. We show that, in this scenario, the customer segments, which are direct functions of the *realized* prices set by sellers, are almost certainly asymmetric across sellers.[2]

2. Related Work

[2] To our knowledge, this is the first attempt to study the scenario where sellers acquire customers after setting prices. Although this scenario is not directly relevant to the research that assumes that customer segments are given when sellers compete on price, it fits many market settings and strategies, such as the acquisition of consumers by "authorized retailers", and manufacturer-suggested retail prices (MSRP), and hence, by itself, is an interesting scenario to investigate.

Several studies in the price dispersion literature have considered how the captive/switcher customer segments are formed. In particular, Butters (1977) considered a scenario where competing sellers simultaneously invest in advertising and set prices. He characterized advertising and sales price distributions. However, it is not clear whether realized customer segments would be symmetric or asymmetric.

Robert and Stahl (1993) studied competition where sellers set advertising and prices simultaneously, and then, in a subsequent stage, consumers can search among sellers whose advertisements they have not received. They find a unique symmetric equilibrium in this scenario with consumer search.

McAfee (1994) was the first to analyze customer acquisition. In an important contribution, he modeled competition between multiple sellers who first invest in "availability" to secure customers, and then set prices. Consumers can get the item only if reached by a seller's promotion. Consumers reached by only one seller are captive to that seller, while those reached by multiple sellers are switchers. Although all sellers and consumers were *ex ante* identical, the unique equilibrium among those involving continuous pricing strategies was asymmetric.[3]

Chioveanu (2007) also studied competition between multiple sellers over two stages, but focusing on *persuasive* advertising. In the first stage, sellers advertise to persuade consumers, and then, in the second stage, sellers set prices. Consumers reached by only one seller are captive to that seller, while those *not reached* by any seller are switchers. She shows that there exist multiple asymmetric equilibria, which is in line with the results of Baye et al. (1992).

[3] Chen and Iyer (2002) considered a two-seller setting where sellers "address" consumers before setting prices. They conclude that the equilibrium could be symmetric or asymmetric. Chen and Iyer's results, however, are founded on a linear horizontal differentiation model, and hence may not generalize to more than two sellers.

These studies have not addressed a fundamental question: under what circumstances are the captive/switcher customer segments symmetric vis-a-vis asymmetric? In particular, how does the symmetry vis-à-vis asymmetry of customer segments depend on the sequence of seller actions (customer acquisition before, simultaneous with, or after pricing)?

3. Setting

The market has n sellers, who independently invest effort to *acquire* potential consumers and market some item. The sellers' acquisition *rates* (or "mind shares") are denoted by α_i, where $i = 1, \ldots, n$, and $0 \leq \alpha_i \leq 1$.[4] Each consumer derives a benefit v from one unit of the item. A potential consumer can buy the item only if acquired by a seller, and, in particular, she does not seek out sellers.[5]

Depending on the context, the form of "customer acquisition" may vary. For instance, it may be choosing the location of a retail store, so acquiring consumers in the geographical neighborhood; or it may be advertising in the mass media, so acquiring consumers who see the advertisements; or it may be telemarketing, so acquiring consumers who listen to the sales "pitch"; or it may be bidding for search links, so acquiring consumers who click on the links.

Let seller i's cost of customer acquisition be $C(\alpha_i)$, where

$$C(\alpha_i) \geq 0\,,\ C'(\alpha_i) > 0\,, \text{ and } C''(\alpha_i) > 0 \tag{1}$$

[4] If there are L potential consumers, the acquisition rate or mind share can be interpreted as the actual number of consumers acquired, A_i, divided by the total market potential, $\alpha_i = A_i / L$.

[5] Essentially, we are assuming that consumers are not aware of the existence of the item. Many directly marketed products and services (e.g., membership-based resort accommodation packages, houseware sets, body builders, health supplements, etc.) are new to consumers, and hence consumers would not enquire about prices without first being "acquired" by some seller. The key difference between our setting and search-theoretic models of price dispersion (Baye et al. 2006) is that in our setting, consumers are not allowed to search for prices.

for all $\alpha_i \in [0,1]$.[6] We further assume away any fixed cost, i.e., $C(0)=0$, and that sellers do have an incentive to acquire consumers, i.e., $C'(0) < v$.

A potential consumer who is acquired by just one seller will pay up to *v* for the item. A consumer who is acquired by more than one seller will be a "switcher" and buy from the seller offering the lowest price, or buy with equal probability from each of the several sellers offering the same lowest price.

Let $F_i(p)$, with support S_i, where $\underline{p}_i = \inf(S_i)$ and $\hat{p}_i = \sup(S_i)$, represent seller *i*'s pricing strategy, and let $\underline{p} = \min(\underline{p}_i)$.

Consider seller *i*. At any price *p*, seller *i* would sell to a consumer that it acquires unless that consumer is acquired by another seller, *and* the other seller sets a lower price than seller *i*. The probability that any consumer is also acquired by another seller *j* is α_j, while the probability that seller *j*'s price is lower than *p* is $F_j(p)$.[7] Accordingly, taking account of all competing sellers, the probability that a consumer that seller *i* acquires will buy from seller *i* at price *p* is

$$\prod_{j \neq i} [1 - \alpha_j F_j(p)].$$

Hence, seller *i*'s expected revenue at any price *p* is

$$R_i(p) = p\alpha_i \prod_{j \neq i} [1 - \alpha_j F_j(p)]. \tag{2}$$

[6] Another possible interpretation of the cost of customer acquisition is sellers' investments to reduce costs that consumers incur to switch products (Klemperer 1987). This interpretation is not captured in models (e.g., Baye and Morgan 2004; Chioveanu 2007) in which sellers need *not* incur any cost in selling to the switcher segment of consumers.

[7] In our setting, sellers acquire consumers stochastically, with each consumer having an equal probability of being acquired (Butters 1977; Grossman and Shapiro 1984; McAfee 1994). Hence, the number of times that any particular consumer is acquired follows a binomial distribution, and the number of consumers acquired by each seller has a simple closed form.

To facilitate subsequent analysis, we consolidate in Lemma 1 various results due to McAfee (1994).[8] These generalize the findings of Varian (1980), Narasimhan (1988), and Baye et al. (1992) to a scenario where the captive and switcher segments are *endogenously* determined by sellers' customer acquisitions. It is key to characterizing the pricing outcomes.

Lemma 1 (McAfee 1994). Suppose that either customer acquisitions take place before price setting, or that customer acquisitions are deterministic. Then:

(a) There is no pure-strategy equilibrium in pricing;

(b) The supports of the equilibrium pricing strategies are intervals with no gaps;

(c) The equilibrium pricing strategies do not include any mass points in $[\underline{p}, v)$; at most one seller (call it seller *m*) who acquires strictly more consumers than all other sellers may have a mass point, which must be at *v*;

(d) The equilibrium pricing strategies of all sellers have the same infimum, $\underline{p}$, and

$$\underline{p} = v\prod_{i \neq m}[1-\alpha_i]; \tag{3}$$

(e) The supports of the equilibrium pricing strategies of at least two sellers have the same supremum at *v*. Generally, a seller that acquires more consumers would have a higher supremum, i.e., if $\alpha_i \geq \alpha_j$, then $\hat{p}_i \geq \hat{p}_j$;

(f) For every seller *i*, expected revenue is

$$R_i = \underline{p}\alpha_i = v\alpha_i \prod_{j \neq m}[1-\alpha_j]. \tag{4}$$

Lemma 1 can be qualitatively explained as follows. In any equilibrium that involves deterministic customer acquisitions, sellers will randomize prices. Generally, a seller that acquires more consumers tends to set a higher price, because it would have more captive

[8] Our Lemma 1 consolidates equations (1) to (4), and Lemmata 1 and 2 of McAfee (1994). For brevity, we omit the proof.

customers relative to other sellers. These customers receive only one offer, and hence the seller can make more profit by charging a higher price to them.

We consider two alternative scenarios which are both common in the price dispersion literature. In one scenario, sellers acquire consumers and set prices *simultaneously*. This scenario describes mass advertising through TV and newspapers, and also, direct marketing – advertisements in these media often include prices.

In the other scenario, sellers acquire consumers *before* setting prices. This scenario fits conventional retailing – sellers fix locations to acquire consumers before setting prices. It also fits markets in which sellers use loyalty programs to lock in potential consumers.

4. Simultaneous Customer Acquisition and Pricing

In mass advertising and direct marketing, prices are often embedded within the promotions sent to consumers. Obviously, in this scenario, sellers cannot condition prices on customer acquisition. Proposition 1 helps to clarify the sellers' customer acquisition strategy:

Proposition 1. In the scenario where sellers acquire consumers and set prices simultaneously, customer acquisition is deterministic.

Intuitively, by (2), the revenue of a seller is determined by the extent to which the consumers that it acquires are also acquired by other sellers and the pricing strategies of these other sellers. Even if other sellers randomize acquisitions, the seller's *probability* of selling to any particular consumer that it acquires does not depend on its own level of acquisition. Accordingly, the seller's revenue is a linear function of its own acquisition. Now, the cost of acquisition is convex, hence the expected profit is concave, and so, it would have a unique maximum. Therefore, the seller would not randomize customer acquisition.

Given that the equilibrium acquisitions are deterministic, the pricing results in Lemma 1 apply. Our next result shows that the equilibrium is unique and *symmetric*.

Proposition 2. In the scenario where sellers acquire consumers and set prices simultaneously, the unique equilibrium comprises acquisitions

$$v[1-\alpha_s]^{n-1} = C'(\alpha_s), \tag{5}$$

and the pricing strategy

$$F_s(p) = \frac{1}{\alpha_s}\left\{1-[1-\alpha_s]\left[\frac{v}{p}\right]^{\frac{1}{n-1}}\right\}, \tag{6}$$

on the support $[\underline{p}, v]$, where

$$\underline{p} = [1-\alpha_s]^{n-1} v. \tag{7}$$

For the simultaneous scenario where sellers embed prices into advertising, Butters (1977) derived advertising and sales price distributions, but did not address whether the realized customer segments would be symmetric. Robert and Stahl (1993) have shown that the equilibrium is unique and symmetric. Their result is, however, applicable only to settings in which consumers are aware of the marketed item and actively search for sellers' offers.

In scenarios with no consumer search, much previous research (e.g., Varian 1980; Grossman and Shapiro 1984; Png and Hirshleifer 1987; Baye and Morgan 2001; Iyer and Pazgal 2003; Soberman 2004) simply assumed that the customer segments were symmetric. To our knowledge, Proposition 2 is novel in showing that, when sellers acquire consumers and set prices simultaneously, the equilibrium is unique and symmetric. Accordingly, it provides a fundamental justification of the various analyses that focus exclusively on symmetric outcomes.

5. Customer Acquisition before Pricing

In conventional retailing, sellers acquire consumers by choosing retail locations before setting prices. Airlines, hotels, supermarkets, gasoline retailers, and many other businesses use loyalty programs to acquire and lock in consumers. In these situations, sellers acquire customers before setting prices.

In this section, we study the scenario where sellers first acquire customers and then set prices. We start with the second (pricing) stage. At this stage, each seller knows the number of consumers that other sellers have acquired. Hence, the results in Lemma 1 apply – for any combination of first-stage outcomes, α_1, α_2, …, α_n, a seller's expected revenue is given by (4), and at most one seller would have a mass point at v in its equilibrium pricing strategy.

The equilibrium outcome in this scenario would depend on whether sellers randomize customer acquisitions. Provided that sellers do not randomize customer acquisitions, McAfee (1994) showed that the equilibrium is unique and *asymmetric*. For ease of exposition, let the sellers be labeled in decreasing order of customer acquisitions in the first stage, that is, such that $\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_n$.

Then, by McAfee's (1994) Theorem 3, in the first stage, each seller $i = 2, \ldots, n$ would acquire α^* customers, where

$$v[1-2\alpha^*][1-\alpha^*]^{n-2} = C'(\alpha^*), \qquad (8)$$

and seller 1 would acquire α^{**} customers, where

$$v[1-\alpha^*]^{n-1} = C'(\alpha^{**}). \qquad (9)$$

In the second stage, each seller $i = 2, \ldots, n$ would set prices according to the distribution

$$F(p) = \frac{1}{\alpha^*}\left\{1-[1-\alpha^*]\left[\frac{v}{p}\right]^{\frac{1}{n-1}}\right\}, \qquad (10)$$

on the support $[\underline{p}, v)$, where

$$\underline{p} = [1-\alpha^*]^{n-1} v, \qquad (11)$$

and seller 1 would set prices according to $F_1(p) = [\alpha^* / \alpha^{**}]F(p)$ on the support $[\underline{p}, v)$, with a mass point of weight $1 - [\alpha^* / \alpha^{**}]$ at price *v*.

McAfee (1994, page 29) noted that this equilibrium was unique among those in which sellers' pricing strategies have interval supports. In this equilibrium, although sellers are *ex ante* identical, one seller acquires more consumers and sets higher prices than all other sellers, and it earns a larger profit.[9]

It is instructive to observe that, in this equilibrium with deterministic acquisitions, sellers charge higher prices than the in scenario of acquisitions simultaneous with pricing. This is because, in the first stage, sellers acquir different numbers of consumers. Hence, in the second stage, the seller which acquires more consumers would have more captive customers, and so, would have less incentive to compete on price.

As noted by McAfee (1994, page 31), in this scenario of customer acquisition before pricing, there may exist a symmetric equilibrium with *randomized* customer acquisition. For illustrative purposes, we derive such an equilibrium for the case of two sellers.

Suppose that there are two sellers, *i* and *j*, who adopt the identical acquisition strategy, $G(\alpha)$, with support $[\underline{\alpha}, \hat{\alpha}]$, where $G(\alpha)$ denotes the probability that a seller acquires not more than α consumers. Then, by Lemma 1, seller *i*'s expected profit from choosing α_i is

$$\pi(\alpha_i) = \left\{ \int_{\alpha_j > \alpha_i} [1-\alpha_i] dG(\alpha_j) + \int_0^{\alpha_i} [1-\alpha_j] dG(\alpha_j) \right\} v\alpha_i - C(\alpha_i). \tag{12}$$

In the Appendix, we show that the equilibrium customer acquisition strategy is characterized by

$$v\alpha_i^2[1 - G(\alpha_i)] = \pi(\underline{\alpha}) - [\alpha_i C'(\alpha_i) - C(\alpha_i)], \tag{13}$$

[9] Please refer to McAfee (1994, pages 40-41) for the proof of the equilibrium. For completeness, we expand McAfee's result to characterize the pricing strategy.

where $\pi(\underline{\alpha})$ is a constant that denotes the sellers' equilibrium profit. Note that, if $C'(1)-C(1)<\pi(\underline{\alpha})$, then the equilibrium customer acquisition strategy includes a mass point at $\hat{\alpha}=1$. Otherwise, if $C'(1)-C(1)\geq\pi(\underline{\alpha})$, we have $\hat{\alpha}C'(\hat{\alpha})-C(\hat{\alpha})=\pi(\underline{\alpha})$, and the equilibrium customer acquisition strategy is continuous throughout.

As we show above, when sellers randomize customer acquisitions, the equilibrium strategies are symmetric. However, the *realized* customer segments are almost certainly asymmetric if the marginal cost of customer acquisition increases sufficiently quickly that $C'(1)-C(1)>\pi(\underline{\alpha})$. Otherwise, with positive probability (the product of the two mass points) that the *realized* customer segments will be symmetric, and both sellers acquire *all* consumers in the market (i.e., $\alpha = 1$ for both sellers).

6. Pricing before Customer Acquisition

In distributing mobile telephony, banking, insurance, and brokerage services, the brand owners typically fix prices and then acquire consumers through direct and indirect channels. Manufacturer-suggested retail prices (MSRP) are common in marketing of books and automobiles. The fastest-growing segment of the advertising market today is Internet search advertising. In all of these situations, sellers set prices before acquiring consumers. Obviously, Lemma 1 does not apply, as it conditions prices on deterministic customer acquisitions.

In this section, we derive some basic results for the scenario where sellers first set prices and then acquire consumers. Although this scenario is not directly related to research that studies how price dispersion is related to *a priori* customer segments, it fits many real-life marketing settings, and hence is interesting for that reason.

We start with the second (customer acquisition) stage, where *n* sellers independently acquire consumers given a set of *realized* prices. Let sellers be labeled in decreasing order of prices, i.e., such that $p_1 \geq p_2 \geq ... \geq p_n$. We first clarify the customer acquisition strategy. For essentially the same reason as underlying Proposition 1, customer acquisition is deterministic.

Proposition 3. In the scenario where sellers set prices before acquiring consumers, customer acquisition is deterministic.

Since customer acquisition is deterministic, the outcome in the second (customer acquisition) stage is surprisingly simple. We characterize it in Proposition 4.

Proposition 4. In the scenario where sellers set prices before acquiring consumers, given any *realized* prices $p_1 \geq p_2 \geq ... \geq p_n$, at the second stage, seller n would choose α_n according to

$$p_n = C'(\alpha_n), \tag{14}$$

and other sellers $i < n$ would choose α_i according to

$$p_i \prod_{j=i+1}^{n} [1-\alpha_j] = C'(\alpha_i), \tag{15}$$

where α_j can be obtained by applying (14) and (15) recursively to all sellers $j > i$.

Going back to the first stage, we first show that there is no symmetric pure strategy equilibrium in pricing. The proof is straightforward, and we present only a sketch of the argument. Suppose that, in equilibrium, all sellers choose the same price. Then, a seller can undercut the others' prices by a small $\varepsilon > 0$. This would lead to a second-order loss in price, but a first-order gain in sales, and hence raise its profit, which violates the supposition. By a similar argument, if pricing is randomized, at most one seller will have a mass point in its pricing strategy.

We next show that there is also no asymmetric pure-strategy equilibrium in pricing. Suppose that such an equilibrium exists, and that $p_1 > p_2 > ... > p_n$. Referring to (15), seller i can increase profit by raising p_i to just below p_{i-1}. Similarly, every seller would want to raise price to just below the next higher price. The limit is that all sellers' prices approach the maximum, v, which we have already shown not to be an equilibrium. Hence, there is no pure-strategy equilibrium with every seller choosing a different price.

Let seller i choose its price, p_i, according to the distribution, $F_i(\cdot)$. By the structure of (14) and (15) and the arguments presented above, it is straightforward to show that the sellers would randomize prices over the support $[\underline{p}, v]$, F_i is continuous on $[\underline{p}, v)$, and at most one seller can have a mass point at v.

By the arguments sketched above, in equilibrium, at most one seller can have a mass point in its pricing strategy, and hence the *realized* prices after the first (pricing) stage are almost certainly asymmetric. By Lemma 2, and, specifically, (14) and (15), the equilibrium acquisitions are directly related to the first-stage prices, and hence they are almost certainly asymmetric as well. In the Appendix, we sketch the heuristics to compute the equilibrium pricing strategy, $F_i(\cdot)$, in the case of two sellers.

7. Constant Returns to Scale

The preceding analysis imposed little structure on the cost of customer acquisition. Here, we draw sharper implications by using the logarithmic cost function,

$$C(\alpha) = -\theta \ln(1-\alpha), \qquad (16)$$

where θ is a positive constant. This cost function characterizes *constant returns to scale* in the sense that the cost of acquiring $[\alpha_i + \alpha_j]$ customers is the same whether they are acquired by one seller or two separate sellers (McAfee 1994, page 31).[10]

[10] With L being the total number of potential consumers, and if we let c be the unit cost of each acquisition (e.g., collecting a consumer's address and sending out the promotion), then with $\theta = -c/\ln(1-1/L)$, (16) simplifies to the well-recognized cost function used by Butters (1977) and Grossman and Shapiro (1984). This parameterization is particularly noteworthy, because it can be constructed from the assumption that sellers draw consumers stochastically. The detailed derivations are shown in the Appendix. See also Butters (1977, page 468) and Grossman and Shapiro (1984, page 66).

The results in the scenario of simultaneous customer acquisition and pricing and the scenario of deterministic customer acquisition before pricing do not change with the specification of $C(\cdot)$. In the scenario of randomized customer acquisition before pricing, the logarithmic cost function implies that $\hat{\alpha} < 1$, and so, by (13), the customer acquisition function, $G(\cdot)$, does not contain any mass points. Accordingly, in equilibrium, the *realized* customer segments are almost certainly asymmetric. Hence, with constant returns to scale, we obtain the stronger result that the realized customer segments are symmetric *only if* customer acquisition and pricing are simultaneous.

Next, we turn to the scenario of customer acquisition after pricing. As discussed in Section 6, regardless of the cost function, the customer segments are almost certainly asymmetric, because they are determined by the *realized* prices after the first stage. Below, we illustrate this relationship in the case of constant to returns scale.

Consider the second (customer acquisition) stage. Again, let the sellers be indexed according to the realized prices after the first stage, $p_1 \geq p_2 \geq ... \geq p_n$. Substituting from (16) in (14) and (15) recursively for all sellers, seller i would choose acquisition rate

$$\alpha_i = 1 - \frac{p_{i+1}}{p_i}, \tag{17}$$

where $p_{n+1} \equiv \theta$. Further, because any seller who charges a higher price would not affect seller i's equilibrium sales, seller i's equilibrium profit is simply

$$\pi_i(\alpha_i) = \alpha_i p_i \prod_{j=i+1}^{n} [1-\alpha_j] - C(\alpha_i). \tag{18}$$

Substituting from (16), and applying (14) and (15) recursively, (18) becomes

$$\pi_i = \theta \left[\frac{p_i}{p_{i+1}} - 1 - \ln\left(\frac{p_i}{p_{i+1}} \right) \right]. \tag{19}$$

With constant returns to scale (logarithmic cost), in the second (pricing) stage, the pricing strategy takes a very simple form. The next lower price serves as a sufficient statistic for the prices of all competing sellers. Obviously, sellers that set higher prices are irrelevant as consumers would not buy from them. As for sellers setting lower prices, by recursively applying (14) and (15), the competitive effect of lower prices can be subsumed into the next lower price, p_{i+1}, alone. So, by (17), each seller would choose its customer acquisition rate according to the "gap" between its own price and the next lower price.

Finally, $d\alpha_i / dp_i \geq 0$, and hence, the higher the price that a seller charges in the first stage, the more consumers it acquires in the second stage. Because pricing is randomized in stage 1 and not more than one seller can have a mass point in its equilibrium pricing strategy, the equilibrium customer segments are almost certainly asymmetric.

8. Concluding Remarks

In a parsimonious setting which encompasses both pricing and customer acquisitions, and which generalizes the well known settings of Butters (1977), Varian (1980), Narasimhan (1988), and Baye et al. (1992), we characterize the *realized* outcomes in terms of the captive / switcher customer segments. We find that:

- If price setting and customer acquisitions are *simultaneous*, the equilibrium is unique and the customer segments are *symmetric* (and the converse is also true if the cost of customer acquisitions exhibits constant returns to scale).
- If consumers are acquired *before* prices are set, the equilibrium customer segments are *asymmetric* unless customer acquisitions are randomized *and* the marginal cost of customer acquisition increases at a sufficiently slow rate.
- If customers are acquired *after* prices are set, the equilibrium customer segments are almost certainly *asymmetric*.

These findings provide a fundamental rationale for previous analyses that exogenously assumed customer segments to be symmetric (Varian 1980; Rosenthal 1980; Grossman and Shapiro 1984; Png and Hirshleifer 1987; Meurer and Stahl 1994; Villas-Boas 1995; Baye and Morgan 2001 and 2004a; Iyer and Pazgal 2003; Soberman 2004) or asymmetric (e.g., Narasimhan 1988; McGahan and Ghemawat 1994).[11]

Besides the contribution to the pure analytics of price competition among multiple sellers of an identical product for consumers who vary in loyalty to particular sellers, our results also shed light on the dynamics of customer acquisitions, be it interpreted as mass media advertising, seller availability, direct marketing, consumer addressability, targetability, or investments to reduce consumer switching costs.

Following McAfee (1994), our analysis can easily be extended to allow consumers to have elastic demand. Let k be the marginal cost of producing the item and $q(p)$ be the individual consumer's demand at price p. Then, in the key equations like (3) and (4), each seller equilibrates between the contribution margin, $\phi = [p-k]q(p)$, at the various prices, rather than directly equilibrating between the prices. With this change, all the major results continue to apply.

The immediate direction for future research is to provide a general characterization of the randomized strategies adopted by sellers in the scenarios of pricing before/after customer acquisitions. It would be helpful to know if, with more than two sellers, they would adopt symmetric strategies in such scenarios.

The next direction would be to compare social welfare across the three scenarios. The results would address the important policy question of whether retailers should be required to

[11] Another justification for focusing on symmetric equilibria is experiments showing that subjects tend to focus on symmetric rather than asymmetric behavior (Van Huyck et al. 1990; Battalio et al. 2003).

include prices in advertising, or whether they should be allowed to first acquire consumers and then set prices.[12]

Finally, it would be useful to take account of consumer search (Butters 1977; Stahl 1989; Robert and Stahl 1993; Banks and Moorthy 1999; Anderson and de Palma 2003; Baye and Morgan 2004a). The equilibrium outcome in the various scenarios would then depend on both sellers' investments in customer acquisitions and consumers' investments in searching for sellers. Robert and Stahl (1993) show that if consumers search after sellers set advertising and prices simultaneously, the unique equilibrium is symmetric.[13] The outcomes when sellers acquire consumers before or after setting prices, and then consumers search, remain an open question.

References

Anderson, Simon P., and André de Palma, "Price Dispersion," Department of Economics, University of Virginia, December 2003.

Banks, Jeffrey, and Sridhar Moorthy, "A Model of Price Promotions with Consumer Search," *International Journal of Industrial Organization*, Vol. 17, No. 3, April 1999, 371-398.

Battalio, Raymond, Larry Samuelson, and John Van Huyck, "Optimization Incentives and Coordination Failure in Laboratory Stag Hunt Games," *Econometrica*, Vol. 69, No. 3, May 2001, 749-764.

Baye, Michael R., Dan Kovenock, and Casper G. de Vries, "It Takes Two to Tango: Equilibria in a Model of Sales," *Games and Economic Behavior*, Vol. 4, No. 4, October 1992, 493-510.

Baye, Michael R., and John Morgan, "Information Gatekeepers on the Internet and the Competitiveness of Homogeneous Product Markets," *American Economic Review*, Vol. 91, No. 3, June 2001, 454-474.

Baye, Michael R., and John Morgan, "Brand Awareness and Price Competition in Online Markets," Indiana University and University of California, Berkeley, February 2004a.

[12] With constant returns to scale in customer acquisition, we can show that social welfare in the scenario of simultaneous pricing and customer acquisition weakly dominates that in the scenario of pricing before customer acquisition. How social welfare in the scenario of customer acquisition before pricing compares with the other two scenarios remains an open question. The challenge lies in characterizing the *n*-seller randomized-strategy equilibrium in such a scenario.

[13] The symmetric outcome of Robert and Stahl is consistent with our findings in Section 4. Hence, at least in the scenario of simultaneous customer acquisition and pricing, it seems that introducing consumer search may not change the structure of the equilibrium outcome.

Baye, Michael R. and John Morgan, “Price dispersion in the lab and on the Internet: theory and evidence”, *Rand Journal of Economics*, Vol. 35 No. 3, Fall 2004b, 449-466.

Baye, Michael R., John Morgan, and Patrick Scholten, “Information, Search, and Price Dispersion”, in Terrence Hendershott, ed., *Handbook in Information Systems*, Elsevier, 2006, 323-375.

Butters, Gerald R., “Equilibrium Distributions of Sales and Advertising Prices,” *Review of Economic Studies*, Vol. 44, No. 3, 1977, 465-91.

Carlson, John A., and R. Preston McAfee “Discrete Equilibrium Price Dispersion,” *Journal of Political Economy*, Vol. 91, No. 3, June 1983, 480-493.

Chen, Pei-Yu, and Lorin M. Hitt, “A Model of Price Dispersion in Internet-Enabled Markets,” Graduate School of Industrial Administration, Carnegie Mellon University, March 2004.

Chen, Yuxin, and Ganesh Iyer, “Consumer Addressability and Customized Pricing,” *Marketing Science*, Vol. 21, No. 2, Spring 2002, 197-208.

Chen, Yuxin, Ganesh Iyer, and V. Padmanabhan, “Referral Infomediaries,” *Marketing Science*, Vol. 21, No. 4, 2002, 23-45.

Chen, Yuxin, Chakravarthi Narasimhan, and Z. John Zhang, “Individual Marketing with Imperfect Targetability,” *Marketing Science*, Vol. 20, No. 1, Winter 2001a, 23-41.

Chen, Yuxin, Chakravarthi Narasimhan, and Z. John Zhang, “Consumer Heterogeneity and Competitive Price-Matching Guarantees,” *Marketing Science*, Vol. 20, No. 3, 2001b, 300-314.

Chioveanu, Ioana, “Advertising, Brand Loyalty and Pricing,” University of Alicante, Alicante, Spain, June 2007.

Corts, Kenneth S., “On the competitive effects of price-matching policies,” *International Journal of Industrial Organization*, Vol. 15, No. 3, May 1997, 283-299.

Deneckere, Raymond J., Dan Kovenock, and Robert Lee, “A Model of Price Leadership Based on Consumer Loyalty”, *Journal of Industrial Economics*, Vol. 40 No. 2, 1992, 147-56.

Dukes A., “The advertising market in a product oligopoly,” *Journal of Industrial Economics*, Vol. 52, No. 3, September 2004, 327-348.

Ghose, Anindya, Tridas Mukhopadhyay, and Uday Rajan, “Strategic Benefits of Referral Services,” *Review of Marketing Science Working Papers*, Vol. 2, No. 2, 2002, Article 2.

Grossman, Gene M., and Carl Shapiro, “Informative Advertising with Differentiated Products,” *Review of Economic Studies*, Vol. 51, No. 1, January 1984, 63-81.

Hong, Pilky, R. Preston McAfee, and Ashish Nayyar, “Equilibrium Price Dispersion with Consumer Inventories,” *Journal of Economic Theory*, Vol. 105, No. 2, August 2002, 503-517.

Hosken, Daniel and David Reiffen, “Patterns of retail price variation,” *Rand Journal of Economics*, Vol. 35 No. 1, Spring 2004, 128-146.

Iyer, Ganesh, and Amit Pazgal, “Internet Shopping Agents: Virtual Colocation and Competition,” *Marketing Science*, Vol. 22, No. 1, Winter 2003, 85-97.

Klemperer, Paul, “Markets with Consumer Switching Costs,” *Quarterly Journal of Economics*, Vol. 102, No. 2, May 1987, 375-394.

Lal, Rajiv, and J. Miguel Villas-Boas, “Price Promotions and Trade Deals with Multiproduct Retailers,” *Management Science*, Vol. 44, No. 7, July 1998, 935-949.

Manduchi, Agostino, “Price Discrimination of Buyers with Identical Preferences and Collusion in a Model of Advertising,” *Journal of Economic Theory*, Vol. 116, No. 2, June 2004, 347-356.

McGahan, A. M., and Pankaj Ghemawat, “Competition to Retain Customers,” *Marketing Science*, Vol. 13, No. 2, Spring 1994, 165-176.

Meurer, Michael, and Dale O. Stahl II, “Informative advertising and product match,” *International Journal of Industrial Organization*, Vol. 12, No. 1, March 1994, 1-19.

Moorthy, Sridhar, and Ralph A. Winter, “Price-Matching Guarantees,” *Review of Marketing Science Working Papers*, Vol. 2, No. 1, 2002, Article 2.

Morgan, John, and Martin Sefton, “Information Externalities in a Model of Sales,” *Economics Bulletin*, Vol. 4, No. 7, 2001, 1−5.

Moscarini, Giuseppe, and Marco Ottaviani, “Price Competition for an Informed Buyer,” *Journal of Economic Theory*, Vol. 101, No. 2, December 2001, 457-493.

Narasimhan, Chakravarthi, “Competitive Promotional Strategies,” *Journal of Business*, Vol. 61, No. 4, October 1988, 427-449.

Png, I.P.L., and David Hirshleifer, “Price Discrimination Through Offers to Match Price,” *Journal of Business*, Vol. 60, No. 3, July 1987, 365-384.

Raju, Jagmohan S., V. Srinivasan and R. Lal, “The Effects of Brand Loyalty on Competitive Price Promotional Strategies,” *Management Science*, Vol. 36, March 1990, 276-304.

Robert, Jacques, and Dale O. Stahl II, “Informative Price Advertising in a Sequential Search Model,” *Econometrica*, Vol. 61, No. 3, May 1993, 657-686.

Rosenthal, Robert W., “A Model in which an Increase in the Number of Sellers Leads to a Higher Price,” *Econometrica*, Vol. 48, No. 6, September 1980, 1575-1579.

Roy, Santanu, “Strategic segmentation of a market,” *International Journal of Industrial Organization*, Vol. 18, No. 8, December 2000, 1279-1290.

Salop, Steven, and Joseph Stiglitz, “Bargains and Ripoffs: A Model of Monopolistically Competitive Price Dispersion,” *Review of Economic Studies*, Vol. 44, No. 3, October 1977, 493-510.

Soberman, David A., “Research Note: Additional Learning and Implications on the Role of Informative Advertising,” *Management Science*, Vol. 50, No. 12, December 2004, 1744-1750.

Stahl II, Dale O., “Oligopolistic Pricing with Sequential Consumer Search,” *American Economic Review*, Vol. 79, No. 4, September 1989, 700-712.

Stegeman, Mark, “Advertising in Competitive Markets”, *American Economic Review*, Vol. 81 No. 2, May 1991, 210-223.

Stigler, George J., “The Economics of Information,” *Journal of Political Economy*, Vol. 69 No. 3, June 1961, 213-225.

Van Huyck, John B., Raymond C. Battalio, and Richard O. Beil, “Tacit Coordination Games, Strategic Uncertainty, and Coordination Failure,” *American Economic Review*, Vol. 80, No. 1, March 1990, 234-248.

Varian, Hal R., “A Model of Sales,” *American Economic Review*, Vol. 70, No. 4, September 1980, 651-659.

Villas-Boas, J. Miguel, “Models of Competitive Price Promotions: Some Empirical Evidence from the Coffee and Saltine Crackers Markets”, *Journal of Economics and Management Strategy*, Vol. 4 No. 1, Spring 1995, 85-107.